ELEMENTS OF APPLIED HYDROLOGY

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PREFACE

It is small wonder that hydrology has been looked upon variously as "a science of coefficients" and as a specialty of little general interest. The relationships with which it is concerned are complex and, in many respects, still obscure. The measurement of its variables is difficult and calls for curious and involved techniques; and it is tied up with rather elusive concepts of probability, many of the explanations of which have been characterized more by ingenuity than by careful analysis.

But the authors are convinced that hydrology is much more than a science of coefficients and that its scope is sufficiently broad to warrant its inclusion in the basic training of the civil engineer. Moreover, we believe that, with the proper approach, the study of hydrology offers values entirely apart from its specific practical applications. In what other branch of engineering science, for example, can one learn more of the difficult art of using Nature herself as a laboratory? Or where can one sense more readily the limitations of engineering analysis and develop more firmly the habit of keeping a mental eye out continuously for departures of *assumption* from *fact?* The very precision of our materials-testing machines and our surveying equipment, the mathematical exactness of our approach to structural design, must occasionally lead the unimaginative student to an oversimplified concept of engineering. In hydrology we return again to that stimulating borderland where a recalcitrant Nature persists in ignoring the laws established by man for her behavior.

This concept of the possibilities of hydrology in the undergraduate curriculum has shaped the planning and development of the entire book. Our twin objectives have been (1) to set forth some of the fundamentals of the subject and (2) to encourage an analytical approach to the solution of engineering problems in general. The result is that some of the problems are solved in a somewhat roundabout manner and that many of the old familiar formulas either are missing or are mentioned mainly with a view to analyzing their shortcomings and limitations. The student who goes on to actual practice in hydrology can pick up these special tools as he needs them and, it is hoped, may have cultivated the habit of evaluating them intelligently. On the other hand, the student who takes hydrology merely to round out his curriculum should be encouraged through this approach to apply a little original thinking in whatever line of work he may select.

One difficulty in preparing this text has been to adjust the presentations to a suitable undergraduate level of facility in hydraulics and mathe-

matics. As for hydraulics, we have assumed that the student is not acquainted with the concepts of nonuniform flow and nonsteady flow but that his foundation does include the Bernoulli theorem, the general concepts of uniform flow in open channels, and the concepts of critical depth and energy gradient. For mathematical background, we have assumed nothing beyond elementary calculus and a general acquaintance with the concept of least-squares adjustments. In presenting mathematical analyses we have attempted to emphasize logic rather than legerdemain, with a view to stimulating the kind of thinking required in original analysis. This explains the occasional use of functional notation without any attempt at developing an analytical expression. It also explains the cautious use of empirical formulas, only one of which—the Manning—is employed without some attempt at defining its limitations.

Another difficulty has been that of developing a set of worth-while exercises and problems that do not entail too much routine computation. So many hydrologic problems involve either arithmetic integration or statistical analysis that one can scarcely avoid a considerable amount of tedious computation, and there is relatively little profit from time invested in such routines. Accordingly, in some cases we call for preliminary assembly of the data by the instructor, so that the student's time can be devoted mainly to the less routine parts of the problem. Also we have avoided the inclusion of specific data for problems, in the belief that the beginner in this subject should work with streams in his own vicinity, with which he is already familiar or which he can visit on field trips.

It is hoped that the present text may achieve its objectives at least in part; but the authors are well aware of its shortcomings and will welcome any suggestions for correction of errors, extension of subject matter, or revision of method of presentation.

It is impossible in a work of this type on a young and rapidly growing science to give satisfactory credit for all the ideas and methods presented. The authors have attempted to name the hydrologist responsible for each of several major developments of the science, but there are undoubtedly unintentional major omissions. Contributions to the development of the science of hydrology have been among the major achievements of the Water Resources Branch of the U.S. Geological Survey; and one of the authors must acknowledge his debt not only for the helpful advice and encouragement of associates in this organization but also for the hydrology absorbed during work with it over many years.

It would be difficult, indeed, for the authors to give full and impartial credit to all the many friends who have advised them on one or another section of this book. However, they wish particularly to acknowledge the detailed reviews of the entire manuscript by Arthur W. Harrington, District Engineer, U.S. Geological Survey, Albany, New York; John C.

Prior, Professor of Civil Engineering, Ohio State University; Robert K. Dodson, Assistant Engineer in Charge of Hydrology, Scioto-Sandusky Conservancy District; Lloyd Harrold, Project Supervisor, Soil Conservation Service, Coshocton, Ohio; and the Washington office staff of the Division of Water Utilization, Water Resources Branch, U.S. Geological Survey, including R. W. Davenport, Chief, Walter B. Langbein, and Hollister Johnson. Also, for special reviews of Chapter 7, thanks are due to Ernest B. Lipscomb and R. G. Cox, Engineers with the U.S. Waterways Experiment Station, Vicksburg, Mississippi, and to E. J. Williams, Jr., Chief of the Planning Branch of the Mississippi River Commission; and for a final revision of this chapter, sincere thanks are due Lt. Col. Shapland and Mr. Guillou. For a review of Chapter 8, the authors are indebted to Leonard Schiff, Hydraulic Engineer, Soil Conservation Service, Coshocton, Ohio.

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CHAPTER 1

INTRODUCTION

Hydrology and Related Sciences

Hydrology, in a broad and literal sense, is "the science treating of water, its properties, phenomena, and distribution," but the word as used by scientists and engineers usually has a considerably narrower connotation. For example, we do not think of the mechanical engineer as making use of "hydrologic" principles when he designs a steam engine or of the botanist as doing a piece of "hydrologic" research when he studies the movement of water through the stems of a plant. Even such phenomena as the hydraulic jump are assigned to the province of fluid mechanics rather than to that of hydrology.

A definition broad enough to include most of the ordinary engineering connotations of the word may be stated as follows: "Hydrology is an earth science dealing with the occurrence and movement of water upon and beneath the land areas of the globe." "Land areas" includes, of course, *inland* water surfaces, such as lakes and rivers. With this definition, we may think of hydrology as being bounded above by meteorology, below by geology, and at land's end by oceanography, but with lines of demarcation that are by no means distinct. The several sciences are not blocks to be fitted into individual compartments; and perhaps none of them represents a distinct body of subject matter so much as it represents a different point of view. The entire cycle of water movement, from cloud to earth to sea to cloud is of interest to meteorologist, hydrologist, geologist,

and oceanographer alike; but each treats it from a different aspect. The meteorologist concerns himself primarily with the precipitation phase of the cycle; he analyzes the movements of air masses and makes short-term predictions of temperature and wind and of the occurrence of precipitation. The hydrologist measures the flow of streams and of ground water, analyzes the regimen of rivers, and then, on the basis of his own measurements and the predictions of the meteorologist, makes both short- and long-term quantitative predictions of flood and drought and normal flow; he measures the water intake into the soil and into ground-water reservoirs and is concerned with evaporation and transpiration losses; he also studies the methods of developing water resources and predicts the effects of proposed improvements on the regimen of streams and of the ground-water reservoir. The geologist's interest is the structure of the earth, and water to him is a mechanical and chemical agent which produces changes in physiography and internal structure by glaciation, erosion, transportation and deposition of sediment, freezing and thawing, and chemical action; on him the hydrologist depends for information as to the probable location, extent, and source of recharge of ground-water reservoirs. The oceanographer is concerned largely with tidal movements, wave action, ocean currents, and similar phenomena.

Clearly, all these sciences have a large body of subject matter in common; clearly, also, the list of related sciences may be extended still further if we choose. Thus meteorologic, hydrologic, and geologic data provide the basis for climatology, which, in turn, with the introduction of biological aspects, blends into plant ecology; and from ecology it is but a step to agronomy. And, to complete the cycle, we find the hydrologist making use of his knowledge of ecology to deduce depth to ground water, seasonal fluctuations in the ground-water table, and probable rates of evapotranspiration and working with the agronomist to develop adequate conservation practices that will protect the soil from erosion and the streams from silting and will provide water where it is needed and remove it where it is in excess.

Practical Applications of Hydrology

A brief review of some of the practical applications of hydrology may provide a helpful background for more detailed study of the subject. We shall consider here a few of its uses in connection with structural design, water supply, irrigation, power, flood control, navigation, erosion control, and pollution abatement.

1–1. Structural Design

If flowing water reaches the floor system of a bridge, disaster may result. Except for relatively short and heavy girder spans, it is seldom

possible to design the structure to withstand the lateral pressure resulting from such an occurrence. Preliminary to any bridge design, then, the maximum area required by the stream to be spanned must be determined with reasonable accuracy. This is a problem in hydrology. It is not sufficient to determine the height to which water has risen in the past; rather, an estimate must be made of the greater heights to which it may rise and of the relative probability of such events, so that the additional cost of protection against the occurrence may be balanced against the possible cost of replacing the structure. Moreover, the prediction must often be made in the absence of stream-flow records of any kind and, further, must often take into account the effect on stage of changes in the channel—as when the bridge piers and abutments may obstruct the flow or when the channel is to be shifted or straightened to pass under the bridge.

In highway and railroad construction the cost of culverts adds up to an appreciable part of the total. Sometimes culverts are so located that one need not expect them to be destroyed by a flood that exceeds, even by a considerable amount, their design capacity; in such cases we have simply the problem of inconvenience caused by temporary interruption of service due to flooding of the road or rails. Here again an economic balance must be struck between the losses of occasional interruption of service, on the one hand, and the cost of larger drainage structures, on the other. Hydrology provides the statistical data for such studies, while the principles of hydraulics provide the basis for analyzing the effect of the structures on the flow.

Closely akin to the problem of highway and railroad culvert design are those of storm sewer design, though the hydrology of urban areas is a study in itself, much more complex than the apparent similarity of such areas would suggest.

In any type of reservoir, provision must be made for passing flood flows over or around the dam. The spillway structure is often the most expensive portion of the dam; hence economy demands that it be as small as possible. On the other hand, overtopping of the nonoverflow section is a serious matter and, in the case of an earth dam, is almost certain to cause failure, with destruction and frequently death in the areas downstream. Clearly, the best in hydrologic design is a prerequisite to safe and economical dam design. The hydrologist must evaluate not only the probability of floods of various magnitudes but also the effect of the reservoir upon the distribution of the flood volume. This latter point is worthy of emphasis, for it is sometimes overlooked, with the result that unnecessary expenditures may be made on "overdesigned" structures. For example, the stream might be capable of producing floods on the order of 20,000 cfs, and yet a spillway of 4,000 cfs capacity might be ample to

protect a dam on it from being overtopped, because of the ponding action of the reservoir behind the dam.

The reader should note the distinction between hydrologic design and hydraulic design. The former, as pointed out in the preceding paragraphs, is concerned with determining the quantities of water that must be handled; the latter proceeds from there to determine the form of structure best suited for the job. The engineer is no more warranted in undertaking the hydraulic design of a structure without attention to hydrologic design than he would be in undertaking to determine the size of structural members without first ascertaining the load that may come upon them.

1–2. Municipal and Industrial Water Supply

Even in those regions of the United States where water is in most abundant supply, the location and development of sources adequate to the needs of urban areas and industries is a matter of increasing concern. Most laymen do not realize, and some engineers fail to appreciate, that availability of water is often the determining factor in the location of important industries and the limiting factor in the growth of municipalities. Few cities can go as far afield for their water as does New York, with reservoirs more than 100 mi beyond the city limits. To the small town, development of a near-by source to provide an extra 100 gpm for a new canning factory is just as important as a far-flung system of reservoirs is to the metropolis.

The great drought of the early 1930's brought a realization of the narrowness of the margin of adequacy of water supplies throughout the country and stimulated a commendable development of auxiliary sources. Unfortunately, it may also have brought a false sense of security, for a generation has now grown up in the belief that the worst that can happen has already happened. The hydrologist knows better. He recognizes, first, that the drought was less severe in some areas than in others that are "meteorologically similar"—in other words, that a drought of the same over-all intensity and the same probability of occurrence could, "next time," hit many communities harder than they were hit before. Next, he has at his disposal plenty of geological and botanical evidence that Nature came far from doing her worst in those dry years. And, finally, he knows that in many areas the increased pumping rates in recent years may already have lowered ground-water levels to the point that a drought even less severe than that of the 1930's might now set new minima in stream flow and underground water reserves.* To apply this knowledge in a specific

*As almost all streams draw their low-water flow from the underground reservoir, the study even of surface supplies may often be essentially a problem in ground-water hydrology.

case and produce a reliable quantitative answer constitute one of the most complex and important problems of hydrology.

1-3. Irrigation

The hydrologic problems in irrigation are similar to those in water supply but on a grander scale, for it takes as much water to irrigate fifty medium-sized farms as to supply a city of 100,000 people. Not many years ago the hydrology of irrigation was relatively simple; in many cases the analysis could properly be limited to some such elementary procedure as this: "Here is a reservoir site, capable of holding 100,000 acre-feet. The river, in its year of minimum flood, yields twice that quantity in the week after first snow-melt; evaporation will claim 10,000 acre-feet during the summer, and the 90,000 remaining will water 45,000 acres." Today an equally summary analysis may still be adequate in some regions. But increasingly we find ourselves confronted with limiting conditions, and the complexities of the problem correspondingly increase. On some rivers, appropriations of water far exceed the total discharge, and downstream projects depend on "return flow" from upstream projects. Elsewhere, irrigation projects depend on underground reservoirs, artificially recharged by "water-spreading" on permeable areas; and an elaborate system of hydrologic bookkeeping is required to keep inventory of the supply and determine safe pumping rates. More and more the hydrologist is called upon to evaluate new projects in areas where the margin of safety is already low or to discover new sources of water for projects in difficulty or to develop more economical methods of water use.

West of the 105th meridian, our economy is founded largely upon irrigation. The continued security and development of this entire region will, in years to come, be increasingly dependent on the vision and skill of the hydrologist. In the humid region an increasing number of farmers are finding it profitable to install irrigation equipment for specialty crops.

1-4. Power

Hydrologic studies are essential to the planning of any water-power development, and for many existing plants the operating schedule is dependent upon a perpetual hydrologic inventory and prediction system.

To determine the feasibility of a "run-of-river" plant, operating with pondage just sufficient to tide it over the peak-demand hours of each day, a reliable prediction is needed of the absolute minimum daily flow that may be expected of the stream and of the percentages of time that various other low rates of flow may be expected to exist. The absolute minimum sets the prime rate that can be expected of the plant as an isolated unit; the additional low-flow data permit an estimate of the amount of power

that will have to be obtained from other sources to maintain a higher dependable rate of power output.

For the "storage" plant, low seasonal flows rather than low daily flows are the important item, and reservoir drawdown studies must be made to determine the prime power possibilities of the site and the relative economy of various heights of dam and capabilities of turbine-generator units. Once constructed, the storage plant poses a continuing problem in the economics of operation. Must the reservoir be drawn on sparingly, so that there will be ample reserve to tide over a long dry period, or can it be tapped boldly and thus permit the shutdown of fuel-burning plants in the system? Clearly, the answer depends on reliable hydrologic predictions of stream performance—predictions covering periods perhaps months in the future. Quite a number of large power producers do work along this line, operating extensive snow surveys and keeping continuous records of fluctuations in ground-water level and soil moisture content to provide the basic data.

1-5. Flood Control

Though levees to shut out the flood are almost as old as civilization, it was not until early in the present century that much thought was given to other methods of reducing the likelihood of flood damage. Such methods include reservoirs to hold back flood waters, channel improvements to speed them on their way, and diversions to transfer them to channels not available to them in nature. Flood control projects range from small improvements, such as localized dredging or channel-straightening undertaken by municipalities, to gigantic, basin-wide developments, involving tens or hundreds of millions of dollars, like the reservoir systems of the Miami and Muskingum rivers and the combination levee-cutoff-diversion program on the lower Mississippi.

Design of any flood control project must be based on reliable hydrologic studies. It is necessary, first, to analyze statistically the probable frequency of floods of various magnitudes, so that potential future flood losses may be reasonably predicted. Next, a "design flood" must be synthesized and a variety of preliminary plans prepared for works that might protect against it. After this, a number of the more promising alternatives must be studied in detail, either analytically or by means of hydraulic models, or by a combination of the two methods. Flood control studies are complicated by the fact that any type of flood control project modifies the natural regimen of the stream and thus, in the process of protecting one area, may increase flood damage in another. As a final step, the best one or two of the alternatives must be investigated even more intensively, with a view to seeing how they may be expected to behave when subjected to floods other than the "design flood." Some

types of works, for example, will give complete protection against floods smaller than the design flood, while others may not go into effective action on any flood except a "major" one. Again, some types of works are capable of withstanding floods greater than the design flood and even of giving some protection against them, whereas with other types the occurrence of a "super" design flood can be expected to result in major damage to the works, complete loss of benefits, and possible additional losses.

Continued development of both the basic theory and the technique of flood-routing studies is essential to intelligent, economical planning of flood control projects. Methods of study that were adequate in the day of the "local" project do not suffice for the integrated, basin-wide improvements toward which engineering and political thought are increasingly turning.

Closely akin to flood control is flood prediction. Millions who live in the river valleys scan the daily flood forecasts of the Weather Bureau as religiously as all of us do the weather forecast. In times of high water these forecasts are relied upon in planning the evacuation of threatened areas, in organizing stand-by crews for emergency work on railroads and highways, and in putting emergency controls into effect at municipal water plants and other public utilities. These forecasts, which achieve a remarkably high degree of accuracy, are the joint work of hydrologists and meteorologists.

1-6. Navigation

On the Ohio and the upper Mississippi and on many smaller streams, navigation is made possible by canalization—that is, the provision of a series of dams creating slack-water pools. Hydrologic problems in projects of this type center on such questions as how the structures will affect flood stages, how much water will be required for lockages, and where (on the smaller streams) this water can be obtained and conserved.

On streams with flatter slopes dredging, realignment, and stabilization of the channel may be the basis of navigation improvement. If the stream is nonalluvial, the hydrologic studies are relatively simple and consist mainly of hydraulic computations to determine the effect of the proposed improvements on water surface profiles for various rates of flow. Streams carrying silt pose much more complex problems. For example, in one location a dredge cut may result in permanent channel improvement, while in another the river may fill in the cut so rapidly as to make the dredging an almost useless expenditure. Again, bank protection of a type adequate for one location may be torn out repeatedly by the river in another of almost identical appearance. It has been said that alluvial streams have strong "personalities" and strenuously resent a clumsy effort

to restrain them. Modern engineering practice recognizes this and has been rather successful in taming rivers, now that their "psychology" is better understood. As far as possible, the treatment consists of finding out what the river wants to do and helping it to do it. Where it cannot be given free rein, the river is usually more amenable to suggestion than to command. The use of the expression "training-works" in reference to certain types of dikes and abattis is a well-chosen figure of speech.

Bank-cutting, silt transportation, and silt deposition have been the subject of intensive study for the past several decades, and much still remains to be learned. Models with movable beds, theoretical analyses based on fluid mechanics and the dynamics of suspended particles, and statistical studies of the bed load and suspended sediment of natural streams all have their place in the analyses of these phenomena.

1–7. Erosion Control

Widespread dust storms in the early 1930's focused attention dramatically on the fact that the soil is not an inexhaustible resource and provided the impetus for intensive study and development of soil conservation practices in arid and semiarid regions. Simultaneously, the creation of the Tennessee Valley Authority stimulated scientific investigation of the extent of land deterioration and the means of preventing it in well-watered areas. Despite—or perhaps because of—the violence of the political and economic arguments that accompanied this development, a sound foundation for truly scientific conservation practices was rapidly evolved. In this development the hydrologist, the ecologist, and the agronomist worked hand in hand.

From the standpoint of hydrology, the problems of erosion control center about the phenomena of overland flow and infiltration. How effective is a given cover of vegetation in protecting a soil from erosion? With given soil and given cover on a given slope, what is the critical distance from the crest beyond which erosion may be expected to begin? At what rate of rainfall will surface runoff begin, under given initial conditions of soil and cover? How will the infiltration rate vary as rain continues? How many tons of soil will be lost per acre per year with various crops and cropping procedures? What effects will land conservation practices have on flood flows and low-water flows of streams? What types of structure are best suited for preventing erosion in ditches and arresting the developing of gullies?

Loss of topsoil to the streams means deposition of that soil in other areas. In a state of nature a rough long-time balance of values possibly exists between loss and gain; one need only consider the building-up of alluvial plains and delta lands to see that this may be so. But where there

are existing farms or reservoirs or canalized reaches, deposition of sediment is as much an economic loss as is erosion; and loss from the topsoil of a headwater farm is not offset economically by deposition in river bottoms or in the Mississippi River delta. Every dam creates a settling basin, in which the river, momentarily brought to a halt, drops its suspended load. Thus the life of a reservoir is fixed, to a great extent, by the rate at which erosion is taking place in the tributary drainage area. Estimation of this "useful life" has become an important phase of the hydrologic investigation of proposed reservoir projects; much work remains to be done both in establishing a sound analytical basis for such estimates and in developing procedures for the desilting of existing reservoirs whose life is threatened.

A fascinating corollary of the reservoir-silting problem is the effect of clear water on the channel downstream. For every velocity and depth of flow, a river has a certain capacity for carrying suspended matter, and, when robbed of its load by a reservoir, it tends to pick up a new charge of sediment from its bed and banks after leaving the pool. This may result in a lowering of the bed, radical changes in channel alignment, and steepening of the slopes of tributaries—in short, a complete upset of the physiographic balance that must either work itself out or be arrested by additional control works. Analysis of such problems is still in its infancy and is a stimulating challenge to the combined efforts of hydrologists and geologists.

1-8. Pollution Abatement

The growth and industrialization of American cities has brought about many public health problems, not the least important of which is that of the pollution of streams. Many rivers downstream from cities have become open sewers, dangerous to public health and destructive of fish, other wild life, and natural beauty. In less serious instances stream pollution creates public nuisances. Pollution control is largely a sanitary engineering problem, to be solved by strict laws and enforcement and involving vast expenditures of public funds for sewage and industrial-waste treatment. However, the disposal of a certain amount of sewage by dilution is usually considered permissible, particularly in periods of high water, as a river is a natural water-purification system, functioning through bacterial action and aeration. Complete prevention of stream pollution, although possible on some streams, is not economically feasible. It is here that the hydrologist comes to the assistance of the sanitary engineer. A complete stream pollution control study must include an investigation of stream flow, particularly of the magnitude and duration of low flows. In some instances the augmentation of low flows, by means of reservoirs, has proved to be at least as important to the

control of stream pollution as have investments in additional sewage-treatment plants.

The Hydrologic Cycle

The field of hydrologic science, both theoretical and applied, can perhaps be best conceived in terms of the "hydrologic cycle." Among the descriptions of this cycle, one of the most helpful is that of Horton, which is quoted below:*

The hydrologic cycle is often considered as consisting of three phases, namely, rainfall, runoff, evaporation. It may more properly be considered as consisting of two main divisions, each in turn involving two principal phases: (1) Atmospheric division—vapor phase and rainfall phase; (2) surface division—runoff phase and evaporation phase. It is interesting to note that each division comprises: (1) Transportation of water; (2) temporary storage; and (3) change of state. Thus the atmospheric division comprises (*a*) vapor flow, (*b*) vapor storage in the atmosphere, and (*c*) condensation or change from vapor to liquid or solid state. The surface division comprises (*a*) runoff, infiltration, or groundwater flow, (*b*) surface, groundwater, and soil moisture storage, and (*c*) evaporation, or change from solid or liquid to the vapor state. (Evaporation occurs slightly in the atmospheric division and condensation occurs to some extent in the surface division; freezing and melting occur in both divisions.)

Any natural exposed surface may be considered as a unit area on which the hydrologic cycle operates. This includes, for example, an isolated tree, even a single leaf or twig of a growing plant, the roof of a building, the drainage basin of a river system or any of its tributaries, an undrained glacial depression, a swamp, a glacier, a polar ice-cap, a group of sand dunes, a desert playa, a lake, an ocean, or the Earth as a whole.

Figure 1† is intended to show the principal steps involved in the hydrologic cycle in the most typical case, that of a drainage basin tributary to the ocean.

Beginning at the top and reading counter-clockwise, the path of the water in the course through the hydrologic cycle may readily be traced. Figure 1 is, however, little more than a suggestive picture from which a wealth of detail is necessarily omitted. For example, on the diagram but little space is devoted to evaporation from the soil. This, however, is a very complex process. It involves the flow of vapor from the air into the soil, from the soil into the air, temperature-differences between the soil and air, the diurnal and annual heat waves flowing into the soil, evaporation from the water table, and also evaporation from capillary films in the non-saturated zone. Vapor evaporated in one part of the soil may flow to and be condensed in another. These phenomena involve the effect of

*Robert E. Horton, "The Field, Scope, and Status of the Science of Hydrology," *AGU Transactions*, 1931, pp. 189–202. The quotation is from pp. 192–93.

†Reproduced with slight changes as Fig. 1-1 of this text. The changes made by the present authors do not affect the application of the figure to any of the quoted remarks.

both salts in solution and of the radii of capillary films on the vapor tension of equilibrium. As a further illustration, the little segment labeled "temporarily stored on the surface" includes water some of which goes into the soil as infiltration. In that case the detention or storage is often but a surface film or thin layers

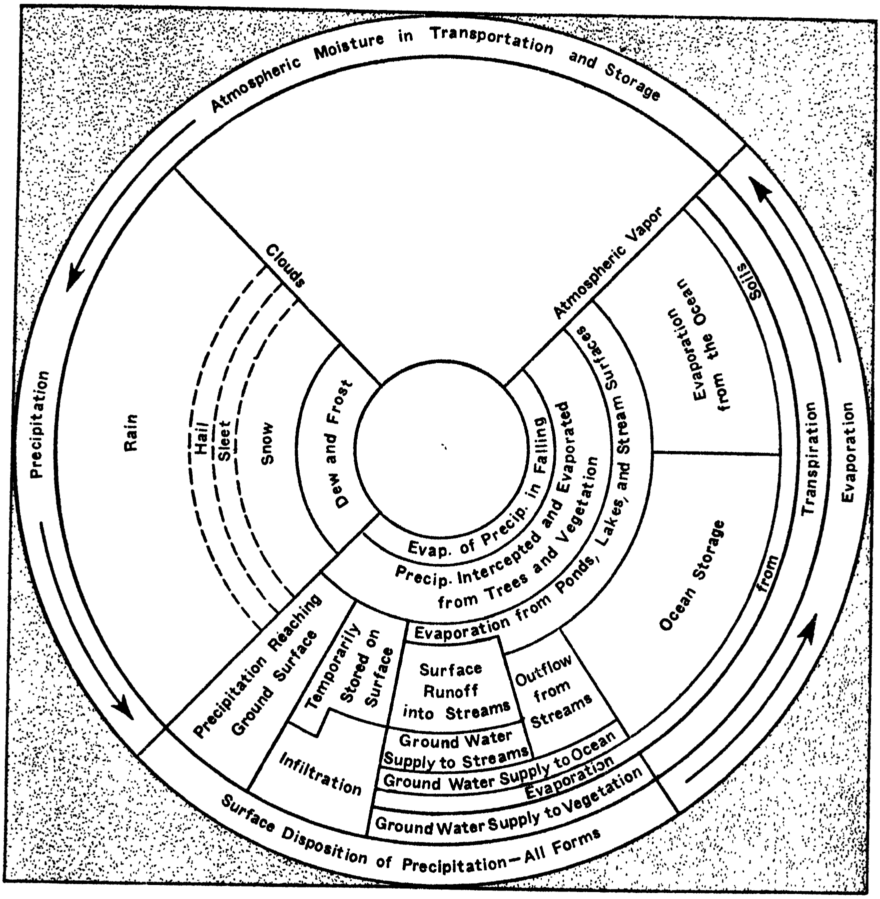

FIG. 1–1. The hydrologic cycle (schematic). (Reproduced with very minor changes, from R. E. Horton, "The Field, Scope, and Status of the Science of Hydrology," *Trans. A.G.U.*, 1931, pp. 189–202.)

in tiny pools. Temporary surface-storage ranges from such transitional phenomena to the extended and long-continued storage of water in lakes, swamps, marshes, as accumulated snow-layers in winter, as glaciers and polar ice-caps. Other details might be similarly amplified.

It is profitable to spend some time with Fig. 1–1, tracing out a few of the myriad paths which a particle of water may follow to complete the

cycle. Any counterclockwise path that does not intersect a solid arc is permissible. (Movement along a radial line, past the end of an arc, does not constitute intersection.) There is no significance in the relative size of the various segments of the cycle. The student should note that Fig. 1–1 is for the case of a "drainage basin tributary to the ocean." When Horton suggests that a "single leaf" may be considered as a unit on which the hydrologic cycle operates, he does not mean that this particular sketch would be applicable to the leaf.

Pictorial representations of the hydrologic cycle like that shown in Fig. 1–2 have become popular in recent years. Unfortunately, such drawings can scarcely avoid giving the cycle a definite spatial connotation ("cloud-land-river-sea-cloud") and suggesting that other alternative paths ("cloud-river-cloud") are short cuts. That such is not the case is clear from Fig. 1–1; any path from cloud back to cloud completes the cycle.

Notes on the Present Text

In an elementary text it would be impossible to present a "balanced" treatment of the field of hydrology and still progress beyond the descriptive (or "survey") level. In the following chapters we have chosen rather to concentrate on a *few* of the principles and on their applications, having in mind primarily the three most common types of problem associated with surface streams: What is the mean annual yield? How will this stream behave in time of drought? How will it behave in flood? It will be noted that one or another of these three problems is involved in every item of the section on "Practical Applications of Hydrology" (p. 2 ff.), and it is for that reason that they have been included here. Omitted, either because of their relatively specialized nature or because of their extreme complexity, are problems of ground-water movement, yield of wells, sediment transportation, consumptive use for irrigation projects, etc.

The student is presumed to be sufficiently advanced to interpret units of rate and volume without special definition, even though he may not have had previous occasion to use them. We do wish to suggest, however, that he form the habit of picturing new units physically in terms of familiar quantities; this practice leads to a clearer physical concept of any problem and also helps prevent the misplacing of decimal points and similar mistakes in computation. For example, a rainfall rate of 1 in./hr can be pictured as almost exactly equal to 1 cfs for every acre on which the rain is falling. Again, the volume unit "second-foot-day" can be pictured as equivalent to the volume of a 1-acre pond, 2 ft deep. To develop a "feeling" for the physical magnitude of various rates of flow, it is helpful to memorize the low-water and bank-full discharge rates of three or four streams with which one is familiar.

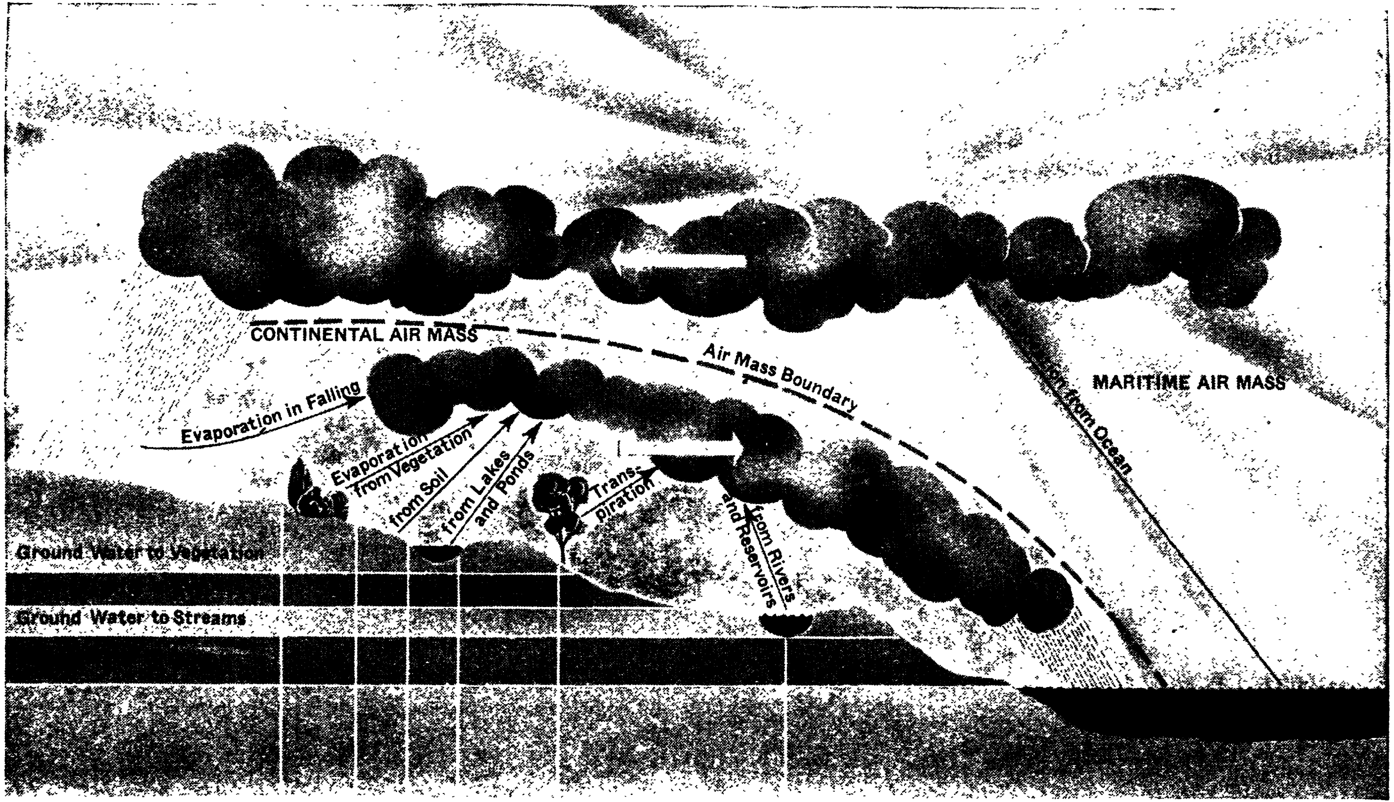

FIG. 1–2. Pictorial representation of the hydrologic cycle. (From Benjamin Holzman, "Sources of Moisture for Precipitation in the United States," *Tech. Bull. No. 589*, U.S Department of Agriculture, Government Printing Office, 1937.)

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CHAPTER 2

COLLECTING AND PRESENTING PRECIPITATION DATA

PRECIPITATION

The word "precipitation" as used in meteorology includes all moisture that reaches the earth, whatever its form—rain, snow, sleet, hail, dew, or frost.

2-1. Causes

Free air is seldom, if ever, perfectly dry, but the amount of water which it contains (in the form of water vapor) bears little relation to the amount of precipitation that may be expected at any point. For example, the average moisture concentration in the atmosphere over desert regions may be greater than that in areas classified as "rain forest." Precipitation can result only when the air containing the moisture is cooled sufficiently for a part of the moisture to condense. If factors capable of producing this cooling are not present, precipitation cannot occur.

Cooling of air masses may be brought about either by radiation or by lifting. Cooling by radiation is the cause of dew and frost—forms of precipitation that are quantitatively so unimportant that they are seldom even measured. Radiation need not be considered further here. Cooling by lifting is the cause of rain, snow, sleet, and hail. The lifting may result (1) from topographic conditions, (2) from frontal action or convergence, or (3) from thermal convection. Along the West Coast, topographic influences predominate; air masses from the Pacific lose their moisture quickly as they are forced up the coastal ranges, with the result that in parts of Washington and Oregon the average annual precipitation exceeds 100 in. From the Rockies east to the Atlantic, frontal action and convergence are the major causes of precipitation. Relatively warm air masses moving northward from the Gulf of Mexico encounter colder masses moving southward from Canada or eastward from the northern Pacific and, being lighter, are forced upward and over the colder masses, giving up their moisture in the process. Over the entire country thermal convection contributes a varying percentage to the total annual precipitation. Its most characteristic manifestation is the convection thunderstorm. Though productive of intense rainfall rates and of heavy local concentrations of rainfall, such storms are spotty and contribute a smaller proportion of the total annual precipitation than might be expected, except along the Gulf Coast and possibly in some desert areas.

2-2. Distribution

The variation in mean annual precipitation over the continental United States is shown by the isohyetal map (Fig. 2-1). It is suggested that the student spend a half-hour or so studying this map in conjunction with a simple relief map, bearing in mind the general statements of the preceding paragraph. If, in addition to this study, he makes a point of memorizing the values to the nearest inch at a few key points—say, Portland (Ore.), San Francisco, Salt Lake City, Denver, El Paso, Kansas City, New Orleans, Columbus (Ohio), Miami (Fla.), Washington (D.C.), and Portland (Me.)—he will find himself able to reproduce the continen-

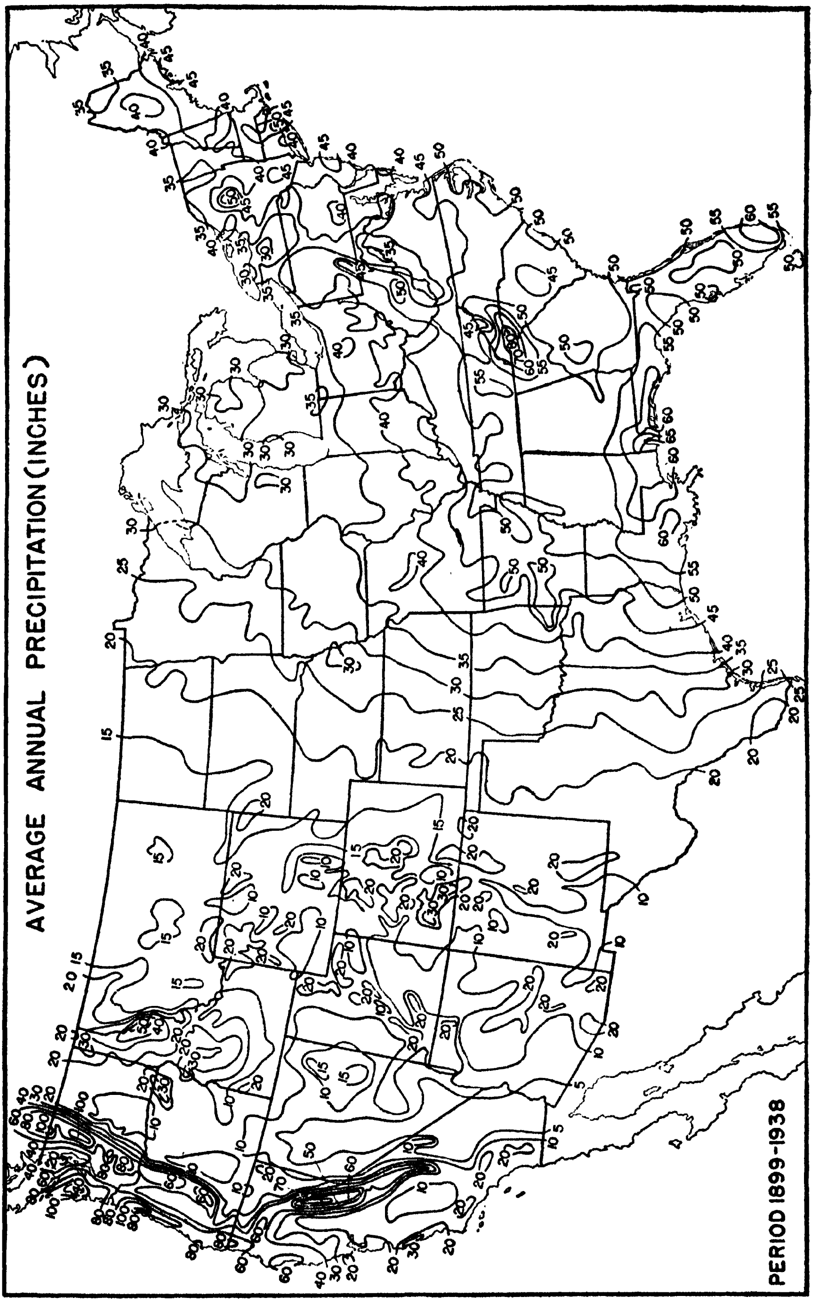

FIG. 2-1. An isohyetal map showing mean annual precipitation for United States. (From *Climate and Man: Yearbook of U.S. Department of Agriculture*, 1941.)

tal pattern of mean annual precipitation *quantitatively* with a surprising degree of accuracy.

Two points with equal mean annual precipitation may be characterized by widely different distributions of precipitation throughout the year. Thus at North Platte, Nebraska, more than 40 per cent of the total annual precipitation of 18.4 in. occurs during June, July, and August, and less than 10 per cent of it during December, January, and February; while at Santa Barbara, California, where the mean annual precipitation is about the same, practically none occurs in the three summer months and more than 60 per cent in the three winter months. Elsewhere, as in New England and part of the Middle West, for example, the quantities falling in the four seasons are very nearly equal. Such differences in the precipitation regimen are highly important to the hydrologist; in fact, knowledge of annual rainfall alone gives only the smallest beginning of a clue to stream behavior.

2–3. Deviations from Mean Values

Also of great significance are the deviations from mean annual and seasonal precipitation at any given point or over any given area. Though the hydrologist may learn considerable about stream behavior from "average" conditions, most of his problems have to do with extremes—droughts and floods that trace back to periods of deficient or excessive precipitation. Studies of long rainfall records (60 to 100 or more yrs in length) indicate that for many stations east of the Mississippi River the minimum year of record is on the order of 60 per cent of the mean and the maximum is on the order of 150 per cent. A number of stations in this region show deviations considerably greater than these, and in some parts of the West deviations of from less than 40 per cent to more than 200 per cent are not uncommon. In general, the percentage deviations from the annual mean increase as the annual precipitations decrease.

Deviations from seasonal means, as might be expected, are greater than deviations from annual means, and monthly deviations are still more pronounced. For periods shorter than a month, "mean" or "normal" precipitation has little, if any, significance.

Absolute extremes of drought or of heavy precipitation are not likely to occur simultaneously at all points in a drainage basin of more than a few square miles in area. Thus deviations from mean annual precipitation *over an area* will, in general, be smaller than deviations at a point; and the larger the area, the smaller the deviations, other things being equal. Over New York State as a whole, for example, the extremes of record are 82 per cent and 126 per cent, as compared with extremes of 71 per cent and 152 per cent at Albany and similar values at a number of other stations. To the hydrologist the concept of mean annual precipitation over a drainage basin is of considerable significance, even if the basin

contains areas of widely diverse precipitation characteristics; but mean annual precipitation over an artificially bounded area, such as a state, means nothing unless the area in question happens to be more or less homogeneous from a meteorological standpoint. Thus, to say that the mean annual precipitation of California is 23.99 in. conveys nothing of significance, for individual subareas within the state have means varying from less than 2 to more than 100 in., and there is no one drainage area of which the state mean is representative (unless by chance).

2-4. A Meteorological Problem

The practicing hydrologist must have at his disposal a great amount of detailed meteorological data on the area with which he is working and far more knowledge of weather phenomena in general than is even suggested in the preceding paragraphs. However, texts on meteorology and climatology are numerous, and it is not proposed to duplicate their contents here. A single question will illustrate the nature of the meteorological problems with which the hydrologist may be concerned and should suffice to put the student on guard against developing an oversimplified concept of the subject. The question we propose is one that has been the subject of much discussion over a considerable number of years: What is the principal source of moisture for precipitation in the United States?

Many hydrologists have answered "Evaporation from continental areas," and have justified their position thus: There is quite general agreement that the rivers and underground discharge areas return to the oceans roughly one-fourth of the total precipitation that falls on continental areas. Throughout recorded history the oceans have remained at approximately a constant level. Thus they must be returning water to the continents at approximately the same rate, which leaves three-fourths of the continental precipitation to be supplied by moisture evaporated from land surfaces.

This argument, of course, depends on the assumption that the rivers and underground discharge areas are the *only* media by which water can move from land to sea; in other words, it assumes that air masses leaving the continent carry no moisture that will be precipitated at sea. In recent years air-mass analysis, made possible by radiosonde observations, has supplied sufficient evidence to cause a number of meteorologists to take the diametrically opposite view—namely, that the principal source of moisture for precipitation in the United States is evaporation from ocean areas, "only a very small part being attributable to continental evaporation."

The student will recognize that the first of these *extreme* positions is essentially indefensible, for its logical basis is destroyed by the presentation of a single case in which an air mass developing over a continental area actually precipitates its moisture over an ocean. But he should also

recognize that the opposite *extreme* (e.g., "only a *very small part* attributable to continental evaporation") can be defended only by the presentation of an overwhelming array of quantitative data. Clearly, it is the meteorologist rather than the hydrologist who has at his disposal the observational equipment and the analytical techniques that will ultimately give the best answer. As for the practical importance of finding the answer, one need only cite the oft repeated claim that irrigation projects, artificial lakes, and soil conservation measures in general, by increasing the opportunity for evaporation, will increase the rainfall in the surrounding area. If such a claim is true, it is of great significance; if untrue, its repetition may well lead to the development of ill-conceived conservation projects. At present it is wise for the hydrologist to maintain an open mind but to recognize that meteorologists as a group tend now to credit ocean evaporation somewhat more than land evaporation as the source of continental precipitation; that no convincing evidence of increased precipitation as a result of conservation measures has yet been presented; that bodies of water even as large as the Great Lakes are not considered by meteorologists as having any great effect on precipitation; and that general rains do not occur in the middle Mississippi or Ohio River valleys except when there have been winds from the Gulf of Mexico for a sufficient time to transport moist air from the Gulf.

Collection of Data

2-5. Agencies

In 1945 the U. S. Weather Bureau was publishing each month in *Climatological Data* records of precipitation from approximately sixty-five hundred stations. Nearly fifty-eight hundred of these stations were maintained by the Weather Bureau, of which approximately three hundred were first-order stations. The remainder were operated by other federal, state, and municipal agencies and private interests. In addition, the records of the hydroclimatic network, available in a monthly *Hydrologic Bulletin*, comprised about twenty-eight hundred recording gages and approximately sixteen hundred nonrecording gages, making a total of about ten thousand four hundred records available in the two publications. Many other records are being collected, but not published, both by the Weather Bureau and by other agencies. The regular stations of the Bureau operate recording rainfall gages and make hourly observations of barometric pressure, wind direction and velocity, temperature, relative humidity, sky conditions, and many other weather phenomena. Co-operative observers, serving without pay or for nominal compensation, make daily observations of precipitation and temperature. Other agencies which regularly collect data on precipitation include the Army and the Navy, the Bureau of Reclamation, the Soil Conservation Service, and a number of the larger flood control districts. Unfortunately, the records

of many of these latter organizations are not easily available to practicing hydrologists, though in each of several states there is a conservation department, water resources board, or other agency that acts as a depository for them.

2-6. Equipment and Methods

Measurement of precipitation is far from as simple as it may at first appear. Among the difficulties in the way of accurate measurement may be noted the following:

(*a*) Almost any object suitable for use as a rain gage extends above the surface of the earth and thus creates eddy currents which may affect the amount of the catch.

(*b*) Relatively few gage sites are sufficiently sheltered from wind to reduce these effects to a minimum and, at the same time, sufficiently clear of obstructions to make them "typical" of the surrounding area for storms coming from all directions.

(*c*) A measurement of precipitation is never subject to check by repetition and seldom (and only approximately) by duplication.

(*d*) The sample constituting the measurement is almost unbelievably small; standard gages located 20 mi apart provide a sample of one part in thirty billion of the precipitation for the total area.

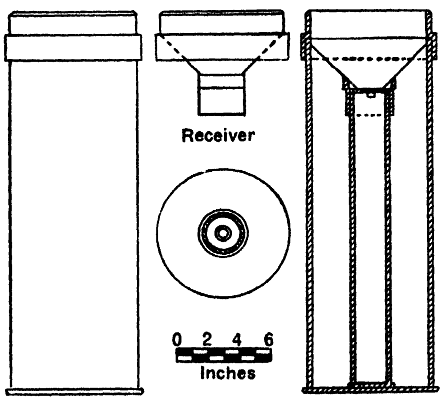

FIG. 2-2*A*. Standard 8-inch nonrecording gage. (Adapted from "Instructions for Cooperative Observers," *Weather Bureau, Circulars B* and *C*, Government Printing Office, 1941.)

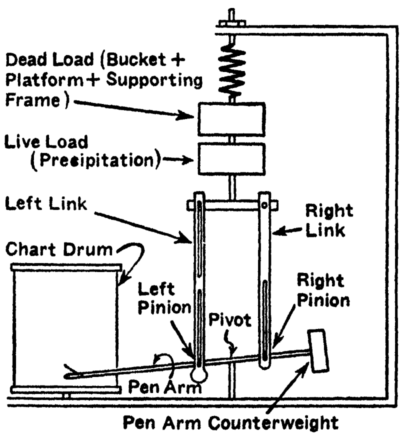

FIG. 2-2*B*. Fergusson type recording gage (schematic). (Adapted from "Soil and Water Conservation Instruments—No. 1," mimeographed bulletin of the Hydrologic Division, Soil Conservation Service.)

The standard 8-in. nonrecording rain gage of the U.S. Weather Bureau is shown in Fig. 2-2*A*. The receiver has a sharp edge exactly 8 in. in inside

diameter and is provided with a funnel-shaped bottom which conducts rain to a measuring tube 2.53 in. in diameter, so that the depth of rainfall is magnified ten times. This tube, being 20 in. high, holds exactly 2 in. of rain. The large outer jacket, known as the "overflow attachment," catches the overflow from the measuring tube. Depth of rainfall is gaged with a thin stick, graduated in inches and tenths (tenths and hundredths of an inch of precipitation), inserted into the measuring tube. If overflow has occurred, the measuring tube is removed and emptied and then refilled with the water from the overflow jacket; this process is repeated as many times as necessary, and the final partial tubeful is measured with the stick.

The standard recording gage of the Weather Bureau is 12 in. in diameter and is equipped with a tipping bucket that meters the flow from the receiver, closing an electrical circuit with each 0.01 in. of rainfall. The recording mechanism consists of a clock-driven drum carrying the record sheet, on which a pen traces a zigzag line, each step of which corresponds to one closing of the circuit and 0.01 in. of rainfall. Corrosion, dirt, and faulty leveling all affect the accuracy of the recording gage, and, even when in perfect operating condition, it tends to underregister during periods of very intense rainfall. Because of these sources of error, the total catch for the day is always measured in the same manner as in the nonrecording gage, and this measurement provides a standard to which the tape record can be adjusted if the discrepancy warrants.

The Fergusson type weighing and recording gage has in recent years come into popular use with the Soil Conservation Service and other agencies. Styles and sizes vary, but the principle of action is that water entering the collecting ring is caught in a bucket resting on a weighing platform, the movement of which is transmitted through a system of links and levers to a pen which makes a trace on a suitably graduated revolving chart. The mechanism is arranged to reverse the travel of the pen after a certain amount of precipitation (say, 3 in.) has accumulated, and reverse again after another equal amount, so that the gage may operate unattended for a week at a time except in regions of very intense precipitation, where totals may exceed the capacity of the gage (usually 12 in.). The mechanics of its operation, particularly of the reversal mechanism, are indicated in Fig. 2–2*B*. Among the advantages of the Fergusson-type gage over the recording gage described in the preceding paragraph are that (1) its action is entirely mechanical, and no source of electric power is needed; (2) it is not quite so subject to error due to corrosion, dirt, and faulty leveling; and (3) such snow as it catches is weighed and recorded automatically. Disadvantages include: (1) difficulties with the reversing mechanism, (2) effects of temperature on the spring balance, and (3) shrinkage and expansion of the chart paper caused by changes in humidity. Another type of weighing gage uses only a single traverse of

the pen, thereby avoiding the difficulties of the reversing mechanism but reducing the scale of the record graph.

Exposure. The literature of rainfall measurements abounds with experiments which prove that the most disturbing agency to a proper collection of rainfall is the wind . . . . It is important to select for the exposure of the gage a position in some open lot, unobstructed by large trees or buildings. Low bushes, shrubbery, fences, or walls that break the force of the wind are beneficial; but in order that these protecting objects may not themselves intercept rain that would otherwise fall into the gage, they should be no nearer to the gage than their own height. Unfortunately the necessity of locating Weather Bureau offices in large cities renders it in some cases impracticable to find a ground exposure, and then the gage must be exposed on a roof. In such cases the middle of a large flat, or nearly flat, roof should be selected, and all possible advantage should be taken of parapets, penthouses, etc., that will break the force of the wind. In some instances, fences 6 or 8 feet square have been built upon the roof to protect the gage. There is no satisfactory exposure possible on a pyramidal roof exposed to the wind, or near the edge of any roof exposed to the wind. A description of the site and surroundings should be made a matter of record.*

Snow Measurements. Regardless of location, most of the standard Weather Bureau gages are not satisfactory for measuring precipitation that falls as snow. For such purposes a number of ingenious devices have been developed—shields for use with the nonrecording and the weighing and recording gage, sampling tubes, snow mats, and the like. Whatever the method, snow depth must always be converted to equivalent water depth. For an extremely rough average value, some hydrologists have taken 10 in. of snow as the equivalent of 1 in. of water; but the departures from this average are so great that, for anything approaching a true record, the actual equivalent must always be determined either by weighing or by melting and measuring. It has been said that the real problem in snow-gaging is to catch the snow; if a gage is used, eddy currents are far more troublesome with snow than with rain; and, if the measurement is made by sampling, the difficulty of finding spots that are truly representative of the average depth—that is, spots that are unaffected by either drifting or wind sweeping—is great indeed. In mountainous areas, particularly in the West, an inventory of snow resources is of great importance in planning the operation of irrigation and power reservoirs for the following summer, and a great network of "snow courses" is maintained to provide the necessary data, as discussed in the section on "Snow and Snow Surveys" (p. 51 ff.).

*"Measurement of Precipitation," *Weather Bureau No. 771* (U.S. Department of Agriculture, 1936), p. 1. It may be noted that in recent years quite a number of weather stations in cities and towns have either been moved to near-by airports or supplemented by the airport stations; thus the necessity for roof exposures is perhaps not so common as formerly.

Presenting Point Rainfall Data

The student should at this point familiarize himself with a few of the more common methods of presenting precipitation data graphically. Fig. 2–3*A*, *B*, *C* shows three different plottings of a simple chronological chart of annual precipitation for the 67-yr period of record at Columbus,

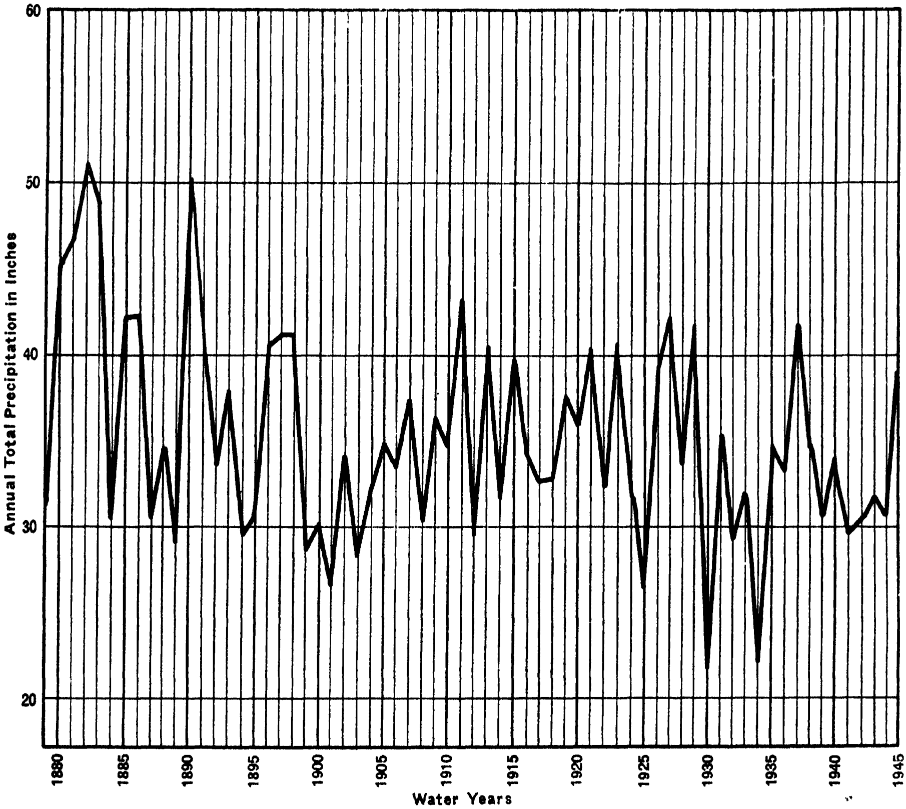

Fig. 2–3*A*. Annual total precipitation, Columbus, Ohio, illustrating a common form of graph for presenting precipitation data.

Ohio, with a graph of the 5-yr moving mean superimposed in Fig. 2–3*B*. Fig. 2–4 is a mass curve of rainfall at three stations near Wooster, Ohio, during the storm of June 6–17, 1946. The record chart for the weighing-type gage is such a mass curve or graph of accumulated precipitation with respect to time. Rainfall intensities (expressed in in./hr) are calculated for any time interval by the following formula:

$$\text{Intensity} = \frac{\text{Rainfall amount (in.) for the time interval}}{\text{Duration of time interval (hr)}}.$$

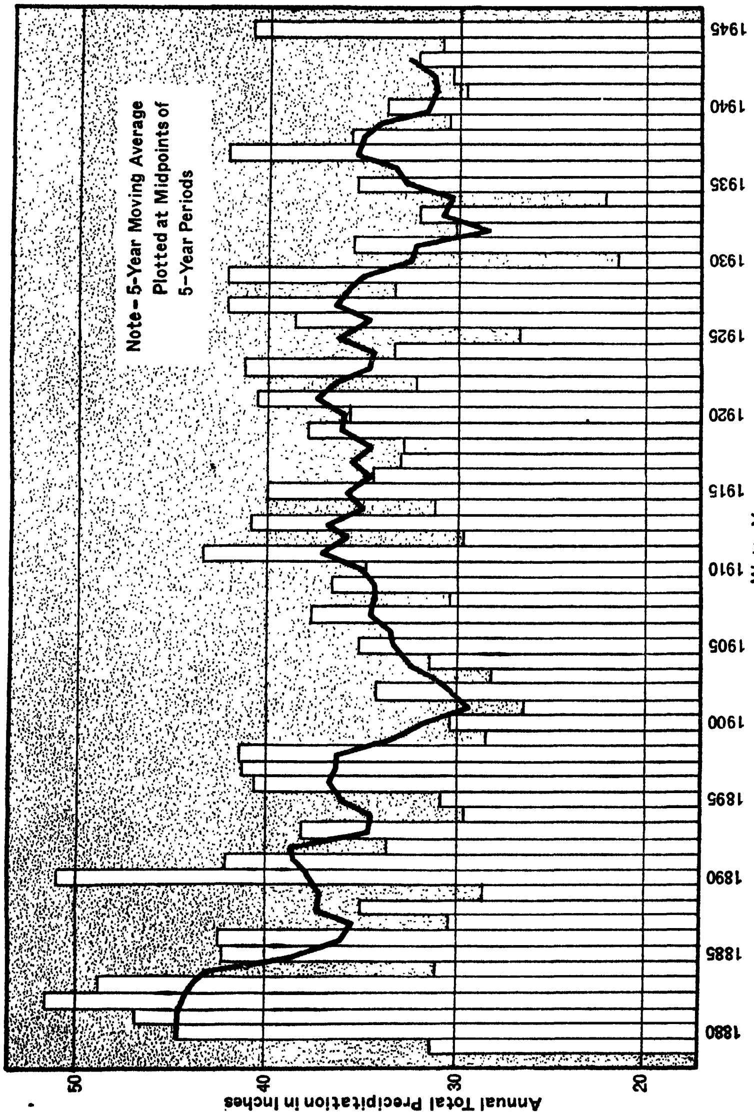

FIG. 2-3*B*. Annual total precipitation, Columbus, Ohio, plotted in bar diagram form, with "moving average" superposed.

Fig. 2–5 is a so-called "frequency curve" of annual rainfall, featuring a rearrangement in order of magnitude of the annual data of Fig. 2–3*C*.

2–7. Chronological Charts (Including Mass Curves)

It should be recognized that the chronological chart of annual precipitation (Fig. 2–3*A*) is, properly speaking, a succession of dots. The lines joining the dots have no mathematical significance and are introduced

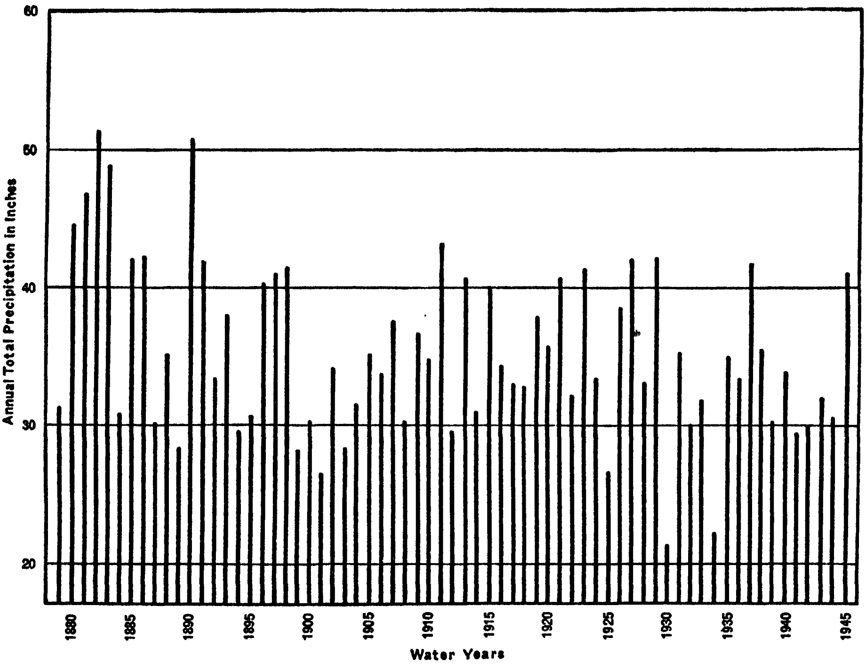

FIG. 2–3*C*. Annual total precipitation, Columbus, Ohio, illustrating another form of bar diagram.

merely to guide the eye. For example, one dot represents the total precipition between October 1, 1937, and September 30, 1938.* and the next dot to the right represents the total precipitation in the following 12-month period; however, the midpoint on a line joining the two dots does *not* represent the rainfall between April 1, 1938, and March 30, 1939. The same applies to chronological charts of other intervals, such as a month or a day. For this reason the writers prefer either the form of Fig. 2–3*B* or that of Fig. 2–3*C* to the form of Fig. 2–3*A*, for either 2–3*B* or 2–3*C*

*For the reason for ending the year on September 30, see Art. 4–4.

puts the reader on notice that he is not dealing with a continuous curve. Nevertheless, joining the dots with straight lines is easier and is by no means to be condemned, for the chance of misunderstanding is probably small. When using the form of Fig. 2-3*A*, the important thing to watch

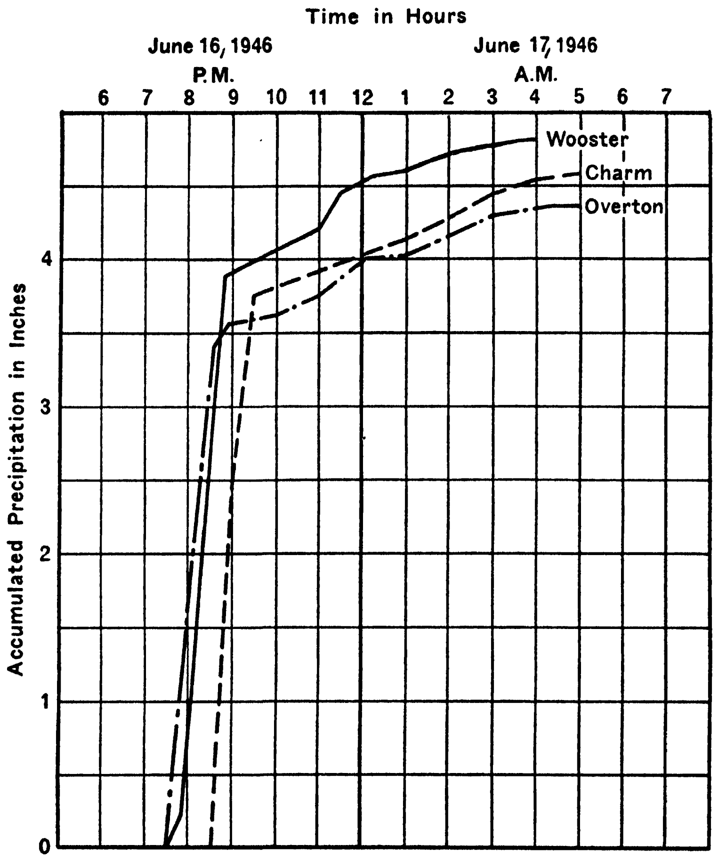

FIG. 2-4. Recording rain gage graphs, storm of June 16–17, 1946, near Wooster, Ohio. (From "The Flood of June 1946 in Wayne County," *Bull. 9, Ohio Water Resources Board.*)

is that the year (or month or day) labels are put *on the lines* and *not* on the spaces—or else that the dots are plotted in the middle of the spaces. If the spaces are labeled and the dots are plotted on the lines, it is often difficult to be sure whether the label refers to the dot on the left or the dot on the right.

The 5-yr moving mean is of value in smoothing out extreme annual variations and indicating trends more clearly; 3-yr and 10-yr means are

also commonly used. Statisticians prefer to plot the moving mean at the midpoint of the period to which it refers, while some meteorologists and hydrologists customarily plot it at the end of the period; it makes little difference which is done, but, because of the two conventions, it is always desirable to state on the graph "mean plotted at midpoint" or "mean plotted at end of period," as the case may be.

Mass curves (Fig. 2-4) are truly continuous curves; the ordinate at any plotted point is the summation of precipitation from time zero. However, it is the usual practice to draw straight lines between plotted points, rather than a smooth curve, and to plot sufficient points to represent the record with reasonable accuracy.

2-8. Frequency Curves

The "frequency curve" of Fig. 2-5 is an elementary statistical device that may be found useful for a number of purposes. It is based on a simple array of numerical data in order of magnitude. The example given is derived from the rain gage record of Fig. 2-3*A*, *B*, *C*. The 67 years of record are considered as a 67-item sample of the whole "universe" of annual precipitation values that may occur at Columbus. Accordingly, each year is considered as representing $^1/_{67}$ of "all years," or 1.49 per cent of "all years," at the station. As no sample can be expected to represent the "universe" perfectly, a reasonable amount of smoothing is justifiable in drawing a curve through the plotted data. From this curve we may make such observations as the following:

(*a*) Nine years out of ten, on the average, the annual precipitation will be equal to or greater than 29 in.

(*b*) One year out of ten, on the average, the annual precipitation will be greater than 45 in.

(*c*) Eight years out of ten, on the average, the annual precipitation will be something between 29 and 45 in.

A similar curve, with the data arranged in reverse order, might also have been drawn, from which should be read values "equal to or less than . . . ," or the percentages might be reversed on Fig. 2-5 with the caption changed to "percentages—equal to or less than—." In the latter case the curve is unchanged, for the sum of the "percentages equal to or greater than" plus the "percentages equal to or less than" must in every case be 100 per cent.

There is a paradox in the construction of these two types of frequency curves that has caused a great deal of discussion, and the student should be familiar with it. To take a very simple case, let us assume that we have a sample consisting of only ten items, so that each item represents "10 per cent of all years." The magnitudes of the items are from 32 to 7,

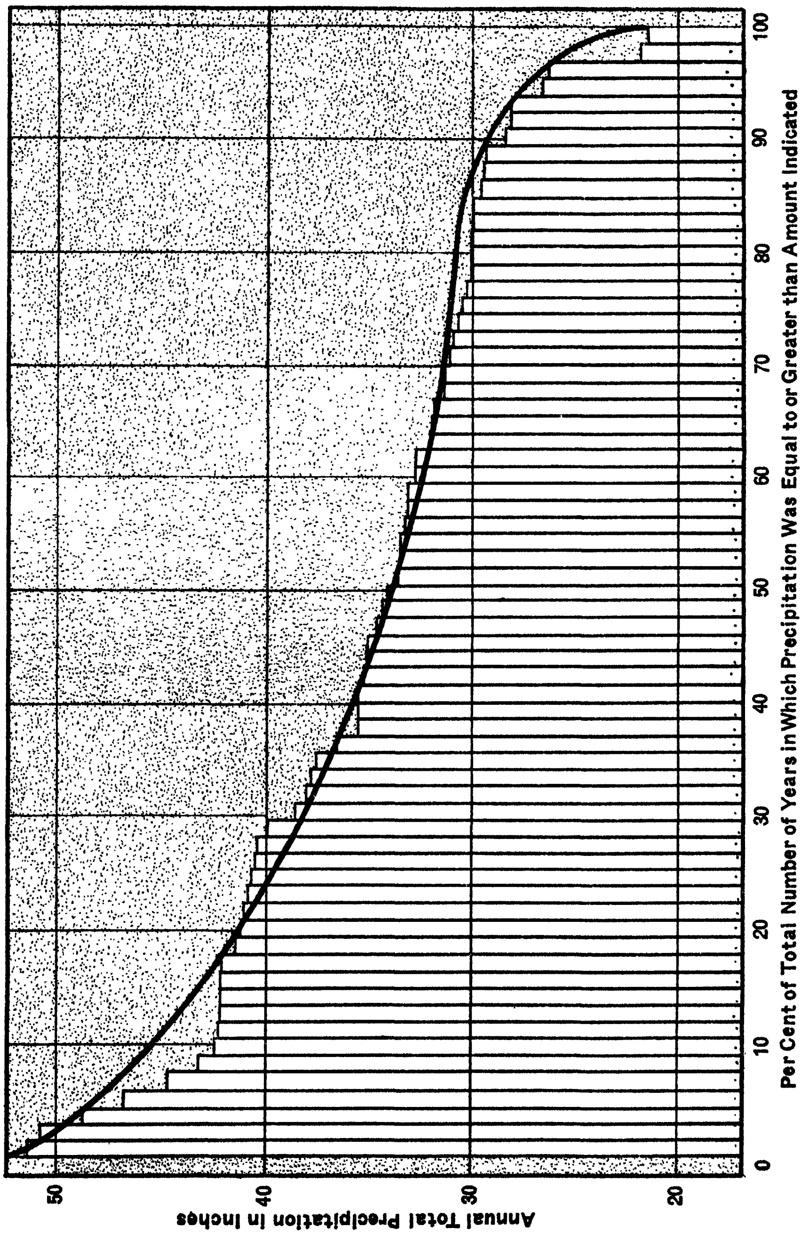

FIG. 2-5. Annual total precipitation, Columbus, Ohio, with years arranged in decreasing order of precipitation amount.

as plotted on Fig. 2-6. The question is where to draw the frequency line or curve. Obviously, there can be only one curve, regardless of whether

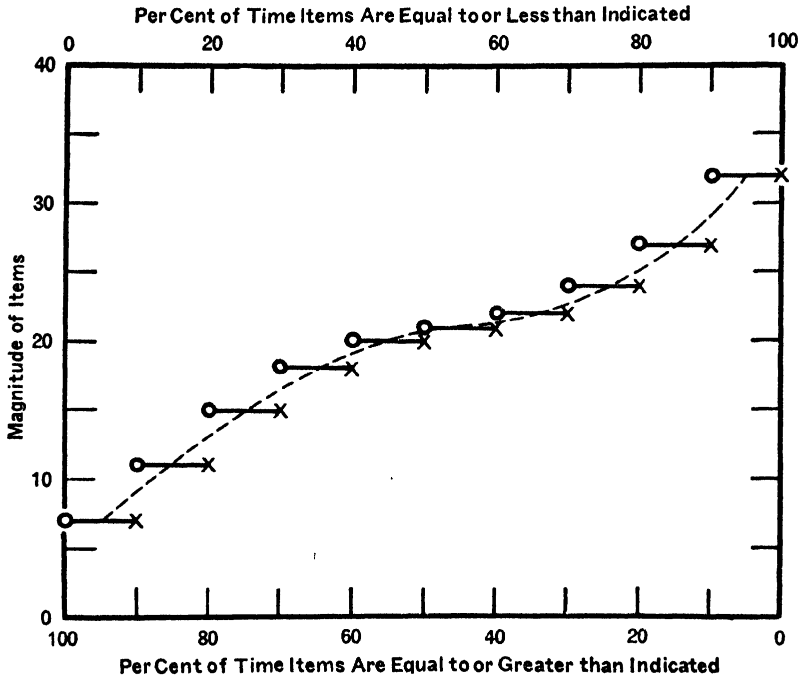

FIG. 2-6. The plotting position paradox.

we use the "equal to or greater than" percentages at the bottom of the diagram or the "equal to or less than" percentages at the top. But if we list the items as in Table 2-1, assigning 100 per cent of the "equal to or less than" to the first item, and reduce the percentage by 10 per cent as we go down the list of items, we get the arrangement of the third column

TABLE 2-1

Items in Order of Magnitude	Order of Magnitude	Per Cent of Time Equal to or Less than—	Per Cent of Time Equal to or Greater than—
32........	1	100	10
27........	2	90	20
24........	3	80	30
22........	4	70	40
21........	5	60	50
20........	6	50	60
18........	7	40	70
15........	8	30	80
11........	9	20	90
7........	10	10	100

below. If we start at the bottom with 100 per cent of items "equal to or greater than," we get the arrangement of the fourth column.

The third column would give a line through the right ends of the bars for each item (X's) and the fourth column a line through the left ends of the bars (0's). A suggested compromise is to draw the line at the midpoint of each bar, as shown by the dotted line of Fig. 2-6; and this is satisfactory for most purposes. This treatment may be justified by considering each of the ten items in the sample as representing a 10 per cent portion of the "universe," and each 10 per cent portion as consisting of a large number of items, some larger and some smaller than the sample value but all lying somewhere between the sample value and the sample values next larger or smaller.

Another compromise which has more statistical justification than the above is as follows: If the ten sample items are distributed uniformly from 0 to 100 per cent, then there must be eleven intervals or spaces between 0 and 100. This plotting system avoids the paradox, is simple to apply, and approaches the correct statistical solution. Plotting positions may be computed from the formula $P = m/(n + 1)$, in which P is the portion of total time expressed as a fraction, m is order of magnitude of a given item, and n is the total number of items.

The greater the number of items, the less important the paradox becomes, and in the sixty-seven items of Fig. 2-5 it is almost entirely obscured. However, when a frequency curve expressed in percentage of time is converted to a scale of return periods, the paradox may become important. For example, if the ten items of Fig. 2-6 are considered to be maximum annual occurrences and the first or "midpoint" compromise is used, then the maximum item is plotted at 5 per cent of time. Converting to return period, 5 per cent of time equals once in 20 yr, or once in double the length of record. The actual occurrence was once in 10 yr, and the second suggested plotting method would place the maximum at once in 9.1 yr, which is fairly close to the actual occurrence. It should be noted, however, that the resulting curve will not be affected by the plotting method except near the ends, where its precise location is of little practical significance because of the large sampling errors involved.*

Frequency of Intense Rainfall at a Point

2-9. Frequency Data as a Basis for Sewer and Culvert Design

It is seldom, if ever, economical to design sewers, drains, and certain types of culverts to handle the maximum runoffs that may be expected

* There is no precipitation record anywhere in the world long enough to warrant any significant distinction between a "100-yr occurrence" and a "200-yr occurrence"; for all we know, either of them might be actually a "75-yr occurrence" or a "300-yr occurrence."

to occur. Rather, they are designed with the expectation that they will be overcharged once in 10, 15, or 20 yr, on the average, an economic balance being struck between the average annual damages resulting from these occasional floods, on the one hand, and the cost of providing larger capacity, on the other. Knowledge of the frequency with which various rates of runoff may be expected to occur is thus of primary importance to economical sewer design. For small areas it is often possible to compute a time interval known as "concentration time," after which all parts of the area are contributing to runoff and after which runoff will remain constant as long as the rainfall rate remains constant. Also the portion of rainfall that appears as surface runoff can be estimated by means to be discussed later (Chaps. 6 and 8). When the concentration time and the runoff coefficient are known, it becomes a simple matter to compute the runoff rate corresponding to any given rainfall rate, provided only that the rainfall continues for a period at least equal to the concentration time. If the runoff coefficient of a given area were constant, the frequency curve for rainfall intensities for a period equal to its concentration time could then be converted directly into a frequency curve of runoff from the area. Actually, the runoff coefficient of any area varies from day to day, and hence the problem is not solved so simply; nevertheless, the rainfall intensity–frequency curve still provides the basis for solution.*

The relationships between frequency, intensity, and storm duration vary sufficiently from place to place that local studies should be made wherever important work is contemplated. Moreover, periodic revisions are desirable in each locality—say, every 3 or 4 yr—to take advantage of the constantly accumulating data. The hydrologist should therefore be familiar with the principles and methods involved in such studies.

2–10. A Typical Analysis

Current practice follows largely the method which was so well described by Charles W. Sherman in his analysis of a 50-yr record at Boston, Massachusetts.† (Similar methods had been used previously, notably by engineers of the Miami Conservancy District.) Results of the Boston study are presented graphically in Fig. 2–7. Sherman extended his curves to 72 hr, with the thought that they might be useful in estimating flood flows of streams with concentration periods as long as 3 days. However,

* A complete solution of the problem of frequency of runoff from small areas, taking into account variations in runoff coefficient, is beyond the scope of this text. The interested student is referred to the comprehensive study by W. W. Horner and F. L. Flynt for a detailed treatment ("Relation between Rainfall and Runoff from Small Urban Areas," *ASCE Transactions*, 1936, pp. 140–20).

†"Frequency and Intensity of Excessive Rainfalls at Boston, Mass.," *ASCE Transactions*, Vol. 95 (1931), pp. 951–968.

point rainfall intensities are properly applicable only to areas for which the concentration periods are but a few hours in length; hence the labor of working out the data for the longer periods may usually be dispensed with.

Sherman tabulated the quantities of precipitation and the corresponding intensities for periods of 5, 10, 15, 20, 25, 30, 45, 60, 80, 100, 120, 150,

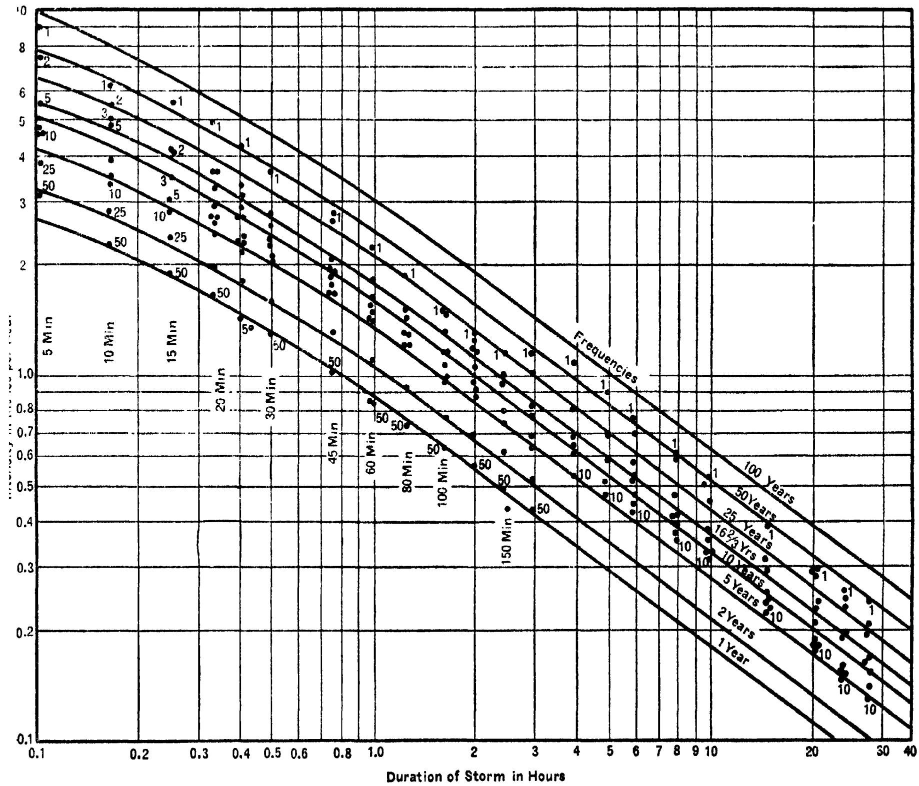

FIG. 2-7. Intensity of precipitation, at Boston, Massachusetts, for various durations and frequencies. (From Charles W. Sherman, *Trans. A.S.C.E.*, 1931.)

and 180 min for all the significant storms that had occurred during the 50 yr of record. Then the fifty greatest intensities for each period were selected and arranged in order. Quoting Sherman:

If the 50-year record were a fair representation of the average 50-year period at Boston, then the greatest intensity for each of the periods should be that which is likely to recur once in 50 years, or that which is likely to have a frequency of 50 years. The points of second magnitude represent intensities which might be expected to be equalled or exceeded twice in 50 years, or to have a frequency of

25 years; and the points of the fifth magnitude would have a frequency of 10 years. It is reasonable to suppose, however, that some of the maximum points may be high enough so that similar intensities will not recur for 100, or even 500 years; the 500-year maximum must occur within some 50-year period, and it may have been in that studied. It is probable also that some of the maximum points may be lower than they would be for an average frequency of 50 years, if the record had covered a longer term, such as 200 years. That is, the curve showing the relation of intensity to duration for a frequency of 50 years can not be very satisfactorily determined from the maximum points of a 50-year record. The 25-year curve is much more definitely fixed, because the record covers two full periods of 25 years; and curves of greater frequency are still more definitely determined.*

Following this line of reasoning, Sherman temporarily ignored the first- and second-magnitude points and plotted the third-magnitude points on log-log paper, using intensity in inches per hour as ordinates against duration as abscissae. A smooth curve through these points was taken as representing, with a fair degree of accuracy, the intensity-duration relationship for a 16⅔-yr frequency. A similar plotting of the fifth-magnitude points yielded the 10-yr frequency curve; the tenth-magnitude points the 5-yr curve; the twenty-fifth magnitude points the 2-yr curve; and the fiftieth magnitude points the 1-yr curve—that is, the curve of intensities that could be expected to be equaled or exceeded once a year, on the average.

These curves were not straight lines but, when they were assembled on one sheet (Fig. 2–7), it became apparent that all were of the same general form. The student should recognize that the study could have stopped at this point, for the graphical presentation is clear, convenient, and complete. It seemed desirable, however, to develop an empirical equation representing this family of curves. One should look on such an equation as a mathematical description of the original chart, by means of which one can reproduce the original faithfully and to any scale desired or can interpolate curves for other frequencies. No greater significance should be attached to it than that; it is purely empirical, and, though it may be used as a "pattern" for similar studies elsewhere, it is, of itself, no more suitable for *general* use than are the curves on which it is based.

Had the lines been straight, each of them would have corresponded to an equation of the form

$$\log i = \log K + d \log t,$$

in which i represents intensity in in./hr, t the duration in min, d the slope of the curve, and K the ordinate corresponding to a duration of 1 min. The curvature prevented use of quite so simple an equation, but *by trial* it was found that, if a constant value 7 were added to all the t's,

* *Ibid.*

the replotted points for each frequency would fall approximately on a straight line. The corresponding equation, of course, is

$$\log i = \log K_1 + d_1 \log (t + 7),$$

and d_1, the slope, proved to be approximately -0.7 for all lines. The equivalent algebraic form is

$$i = \frac{K_1}{(t + 7)^{0.7}}. \tag{2-1}$$

Next, the values of K_1 for the several curves were plotted on log-log paper against the corresponding frequencies F, expressed in years. This plotting resulted directly in a straight line,

$$K_1 = 16F^{0.27}. \tag{2-2}$$

Combining Eqs. (2-1) and (2-2), we derive

$$i = \frac{16F^{0.27}}{(t + 7)^{0.7}}. \tag{2-3}$$

It was believed that curves for the 25- and the 50-yr frequencies could be constructed more properly from this equation than from the actual first- and second-magnitude points, the latter being in question for the reason already stated. This was accordingly done, and a curve for 100-yr frequencies was also constructed in the same manner. Sherman points out that there is doubt as to the propriety of this last step but that the curve is "probably of significance as giving some indication of what may occur sometime in the way of phenomenal precipitation."

There remains to be mentioned one point designated by Sherman as the "extended duration" principle. By this principle, all storms in which the total precipitation was sufficient to show significant average rates for periods longer than the actual duration of the rainfall were considered as though they had continued for the longer times. Thus a storm which lasted for 50 min and yielded a total precipitation of 1.50 in. was counted not only as a 50-min storm with an intensity of 1.80 in./hr but also as a 60-min storm with an intensity of 1.50 in./hr. Of the 50 highest intensities for each duration from 5 to 180 min, 10 of the 650 figures were supplied in this way. For the longer storms (up to 72 hr), a much larger proportion of the entries was affected by this principle. The logic of the concept of extended duration is clear, and the student will observe that neglect of it would result in curves lower than those actually developed.

2-11. Special Techniques

Seldom does one have exactly 50 yr of record to work with in a study of this kind; hence seldom is it possible to plot the 2-, 5-, and 10-yr fre-

quency curves directly from points of twenty-fifth, tenth, and fifth magnitudes, respectively. It is almost always desirable to have curves for these frequencies; and, if they are to be derived without mathematical analysis, it is necessary to begin the study somewhat differently. After the greatest intensities for each period are arranged in descending order of magnitude, the frequency of each intensity is computed by dividing the number of years of record by the magnitude (ignoring the paradox, as is the usual practice); intensity is then plotted against frequency (preferably on log-log paper); curves are fitted to these points—one curve for the 5-min data, one for the 10-min, and so on—and, finally, the intensities corresponding to the desired frequencies are read from these curves and plotted against duration to give work sheets similar to those which which the Sherman study began.

Some investigators in other regions have found that their data did not yield a family of parallel curves and hence have derived an individual equation for each of several different frequencies. For example, S. D. Bleich, studying rainfall in New York City, developed a set of six equations applicable to 1-, 2-, 5-, 10-, 25-, and 50-yr frequencies, respectively.* In many cases it may be a matter of personal judgment whether the improvement in the fit of the curves achieved by the use of individual equations warrants the additional complication. Certainly, there is much to be said for the simplicity of a single equation embodying all factors; moreover, from a statistical standpoint, it may sometimes be questioned whether the better fit of the multiple equations is of any significance.

Some investigators consider it worth while to develop the frequency equations by the method of least squares, thus insuring that the resulting curves are actually the "best fit" for the data. Others consider this refinement a waste of time because the departures of individual points even from lines of best fit are so great. The student should, in any case, be familiar with the method; the specific example that follows is from the previously referenced studies by S. D. Bleich.†

For the 10-yr frequency, Bleich found that a nearly straight-line plot resulted from adding 12 to t (cf. the value of 7 used by Sherman on the Boston data). The logarithmic equation, then, is

$$\text{Log } i = \log K - d \log (t + 12)$$

or

$$\text{Log } i - \log K + d \log (t + 12) = 0. \tag{2-4}$$

Instead of determining K and d graphically from a "best-fit-by-eye" curve, it is proposed to derive them by least squares.

*S. D. Bleich, "Rainfall Studies for New York, N.Y.," *ASCE Transactions*, Vol 100 (1935), pp. 609–644.

†*Ibid.*, pp. 625–627.

If any actually observed pair of values of i and t be inserted in Eq. (2-4), the left-hand member will not, in general, equal 0 but will equal some quantity x, whose value depends on K and d and measures the departure of that particular observation from the line defined by K and d. By the theory of least squares, the line of best fit is the one for which the sum of the squares of the departures of all the observations is a minimum. That is,

$$\Sigma\,[\log i - \log K + d\log(t + 12)]^2 = \Sigma x^2 \tag{2-5}$$

must be a minimum. This will occur when the first partial derivatives of Eq. (2-5) with respect to log K and d are equal to zero. Now

$$\frac{\partial(\Sigma x^2)}{\partial \log K} = -2\Sigma\,[\log i - \log K + d\log(t + 12)], \tag{2-6}$$

and, equating to zero, transposing, and introducing n, the number of observations, Eq. (2-6) becomes

$$\Sigma \log i = n\log K - d\Sigma\log(t + 12), \tag{2-7}$$

which is known as the "first normal equation." Similarly,

$$\frac{\partial[\Sigma x^2]}{\partial d} = 2\Sigma\{[\log i - \log K + d\log(t + 12)]\log(t + 12)\},$$

whence, equating to zero and transposing, we obtain

$$\Sigma\,[\log i \cdot \log(t+12)] = \log K \cdot \Sigma\log(t+12) - d\Sigma\,[\log(t+12)]^2, \tag{2-8}$$

which is the second normal equation.

The data and computations from the observed values are shown in columns (1)–(7) of Table 2-2. Substituting in the normal equations, we obtain

$$3.1907 = 6\log K - 9.6053d$$

and

$$4.6457 = 9.6053\log K - 15.9460d;$$

TABLE 2-2

Calculation of Factors in Normal Equations

(1)	(2) $t+12$	(3) Log $(t+12)$	(4) $[\text{Log}\,(t+12)]^2$	(5) Observed i	(6) Log i	(7) Log i . Log $(t+12)$	(8) Computed i	(9) x	(10) x^2
5.....	17	1.2304	1.5139	6.98	0.8439	1.0383	6.69	−0.29	0.0841
10.....	22	1.3424	1.8020	5.28	0.7226	0.9700	5.46	+0.18	0.3240
15.....	27	1.4313	2.0489	4.78	0.6794	0.9725	4.62	−0.16	0.0256
30.....	42	1.6232	2.6348	3.22	0.5079	0.8244	3.26	+0.04	0.0016
60.....	72	1.8573	3.4495	2.12	0.3263	0.6060	2.13	+0.01	0.0001
120.....	132	2.1206	4.4969	1.29	0.1106	0.2345	1.32	+0.03	0.0009
Sums.	...	9.6053	15.9460	...	3.1907	4.6457	...	...	0.1447

and, solving them simultaneously for K and d, we find $K = 63.75$ and $d = 0.795$, whence

$$i = \frac{63.75}{(t + 12)^{0.795}}. \tag{2-9}$$

To complete the least-squares study, columns (8), (9), and (10) have been added to Table 2–2. Column (8) lists values of i computed from Eq. (2–9); column (9) gives the difference between the computed values and the observed values listed in column (5), and column (10) gives the squares of these differences. The average departure of the observed values from the computed values is

$$\sqrt{\frac{\Sigma(x^2)}{n}} = \sqrt{\frac{0.1447}{6}} = 0.155 \text{ in./hr.}$$

It should be noted that, with the aid of a computing machine, all operations necessary in the least-squares adjustment just described can be performed in well under an hour.

2–12. Inapplicability to Large Areas

In the point rainfall studies just discussed, it has been assumed that the frequency with which a particular intensity of rainfall occurs over an area is equal to the frequency with which the same intensity occurs at a point in that area. This is not precisely correct; actually, the probability of a given intensity over a homogeneous area is always somewhat less than the probability of the same intensity at a point in the area. The larger the area, the greater the error in the assumption of equivalence; clearly, it is not even approximately true for areas larger than those covered by the storms producing the significant intensities. Moreover, for any given area, the greater the intensity of rainfall, the greater is the error in the assumption; meteorological conditions may occasionally produce intensities *at a point* greater than any possible combination of circumstances could produce over an area of more than a few acres. Frequency studies taking area into account have only recently been undertaken; the statistical problems involved are beyond the scope of this text, and few quantitative results are available at present. It seems probable, however, that the difference between point intensity and areal intensity can be ignored in many problems; tentatively it is suggested that point intensities may be applied to areas with concentration times of less than 1 or 2 hr, for frequencies of up to once in 20 or 25 yr. These limits are arbitrary and may need revision when more area-intensity-frequency data are available.

2-13. Available Studies

Attention should be called here to the valuable collection of rainfall intensity-frequency data in *Miscellaneous Publication No. 204* of the United States Department of Agriculture, prepared by David L. Yarnell in 1935. The principal feature of this publication is a set of fifty-six isohyetal maps of the continental United States, covering the range of durations and frequencies shown in Table 2-3. One of these maps is

TABLE 2-3

Frequency, once in—	Duration in Minutes				Duration in Hours					
	5	10	15	30	1	2	4	8	16	24
2 yr.....	X	X	X	X	X	X	...	...	...	...
5........	X	X	X	X	X	X	X	X	X	X
10........	X	X	X	X	X	X	X	X	X	X
25........	X	X	X	X	X	X	X	X	X	X
50........	X	X	X	X	X	X	X	X	X	X
100........	X	X	X	X	X	X	X	X	X	X

reproduced here as Fig. 2-8. All available Weather Bureau records through 1933 were taken into account in this study. However, a great

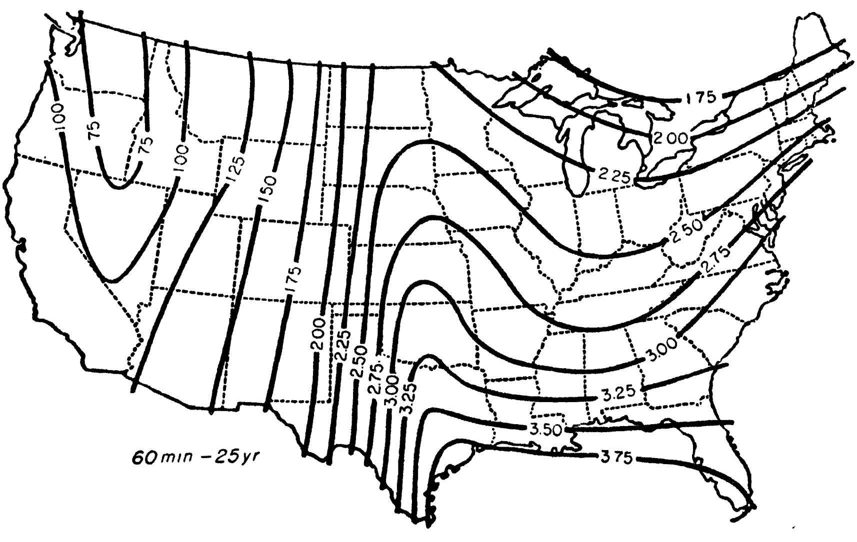

Fig. 2-8. One-hour rainfall, in inches, with 25-yr frequency. (From David L. Yarnell, "Rainfall Intensity-Frequency Data," *Misc. Pub. No. 204*, U.S. Department of Agriculture, Government Printing Office, 1935.)

amount of "smoothing" was resorted to in drawing the isohyetals, with the result that local peculiarities in rainfall may be obscured, particularly in mountainous regions. Although of inestimable value for preliminary work, these charts should not be used for final design if adequate data are available for making a frequency study of the specific area under consideration.

Another extensive study of point rainfall intensity is contained in "Storm Rainfall of the United States," published by the Miami Conservancy District as Part 5 of the District's *Technical Reports*. The latest edition (1936) is based on all available data through 1934. The area covered is limited to that part of the continental United States east of the 103d meridian. A set of twenty-four isohyetal maps covers the range of durations and frequencies shown in Table 2-4.

TABLE 2-4

Frequency Once in—	Duration in Days					
	1	2	3	4	5	6
15 yr.	×	×	×	×	×	×
25.	×	×	×	×	×	×
50.	×	×	×	×	×	×
100.	×	×	×	×	×	×

It will be noted that there is little overlap between the Yarnell and the Miami studies and that durations covered by the latter are mostly beyond the range for which point intensities are primarily useful.

Isohyetal Maps

Isohyetal maps are convenient for depicting the distribution of precipitation over an area, and they also provide an accurate means for computing the equivalent uniform depth of precipitation over the area. Fig. 2-1 is an isohyetal map of *average annual precipitation;* the hydrologist frequently has occasion to use such maps showing the precipitation for a given year or month or even for such short periods as a day or less. In detailed studies of storms, isohyetal maps may even be constructed to show the distribution of precipitation by successive 15-min intervals.

2-14. Description

An isohyetal map resembles a topographic map in form and in method of construction. However, there is one important difference in the basic data. The "shots" for a topographic map are taken on peaks and ridge lines, at breaks in grade, and at the low points of depressions; hence the map is a recognizable reproduction of the topography, and, if the field work has been done thoroughly, no major topographic feature is omitted.

On the other hand, the data for an isohyetal map are the records of precipitation from gages located essentially at random with respect to critical points in the "precipitation topography." The result is that there is about the same relation between map and actuality as there would be on a topographic map for which the shots had been taken, say, at regular distances from the instrument instead of at breaks in grade. In other words, the "topography" of an isohyetal map over any small area does not necessarily bear any recognizable relationship to the actual precipitation pattern of that area. Further, there is no guarantee that the extreme peaks or depressions are plotted; for the probability that any gage catches either the maximum or the minimum precipitation that falls anywhere in the area during the period depicted is extremely remote. On the other hand, if the gages are reasonably close together and fairly uniformly distributed, there is every reason to believe that (1) the *general* pattern for the period is adequately reproduced except for relatively small subareas and (2) the *amount* of precipitation over all but relatively small areas is accurate within limits that can be fairly well defined.

2-15. Rules for Construction

To construct an isohyetal map, a number of more or less arbitrary rules of procedure are needed. The rules prepared for the Ohio Water Resources Board in 1946, for a set of annual maps covering the state of Ohio, are listed and discussed below. These maps were to be drawn to a scale of 1:500,000, with an isohyetal interval of 1 in., and were to be in sufficient detail that they could be used for computing the equivalent uniform precipitation for the year over any desired subarea of 100 sq. mi or more.

(*a*) For each annual map, make use of all gages that have a complete record for that year, even though this results in a highly irregular spacing of gages.

As a consequence of this rule, some of the maps gave the impression that the "precipitation topography" was considerably rougher in one area than in another, when actually the additional roughness was only the result of the delineator's having more data to work with. However, for the purpose, neither appearance nor over-all uniformity of accuracy was as important as was the attainment of the highest possible accuracy in each part.

(*b*) Do not use any gage with an incomplete record for the year, unless the estimating necessary to complete the record involves only a few days of relatively light, winter-type precipitation that is known to have been reasonably uniform in areal distribution.

This rule was based on statistical studies that indicated that points on annual isohyetals could be located by straight-line interpolation between

gages as much as 20 mi apart, with a standard error of 2.8 in., and that with the gage spacing reduced to 10 mi, the standard error of the isohyetals was still on the order of 2.1 in. Since the estimation of missing records for summer months could easily introduce errors larger than 3 in. in the total annual precipitation for a gage, it appeared that the maps would be at least as accurate without the use of estimated records as they would be with them.

(*c*) The record of each gage is to be taken as correct for the location of the gage.

This rule had its application chiefly where two gages close together showed widely different amounts of precipitation. It required that all isohyetals needed to show this spread in values be passed through the "neck" between the gages (see Fig. 2–9). As an alternative, the records of the two gages could have been averaged, the resulting value plotted midway between the gages, and the gages ignored in drawing the isohyetals. This would have given the isohyetals a smoother appearance, and probably without much sacrifice of accuracy. However, the final maps would have been somewhat confusing, for the isohyetals would then have appeared to contradict, in places, the evidence of the actual gage records. Moreover, it seemed desirable to limit as much as possible the opportunities for personal interpretation on the part of the draftsman.

(*d*) Between adjacent gages, isohyetal lines are to be located by straight-line interpolation, *by eye*.

No other procedure than straight-line interpolation seemed justifiable in the absence of sharp topographic features, such as mountain peaks, that might be accepted as producing definite maxima or rain shadows at points intermediate between gages. However, statistical studies indicated that deviations from the straight-line assumption were so great as to make a *scaled* interpolation unwarranted.

(*e*) Between points located by straight-line interpolation, the isohyetals are to be faired in by eye to produce smooth curves, giving reasonable consideration to the evidence of near-by gages not involved in the interpolation.

For example, where four gages lay approximately at the corners of a square, the primary interpolations were made along the edges of the square. However, interpolation along the two diagonals might (and usually did) give conflicting values at the midpoint. In such cases the average of the two midpoint values was taken as an intermediate control point, or hypothetical gage, and a sort of secondary straight-line interpolation was applied along the diagonals in all directions from that point to help define the isohyetals.

(*f*) Where the evidence of the gages is such as to permit of more than one solution, choose the one that gives the smoother or better-rounded contour.

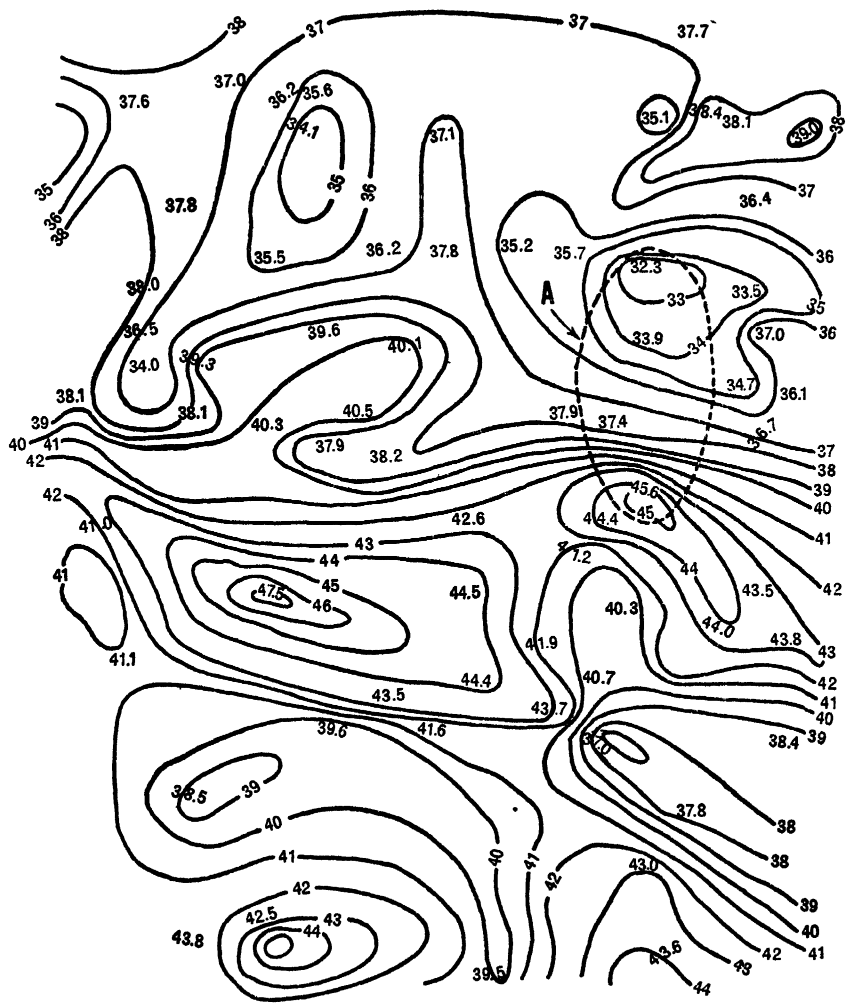

FIG. 2-9. Portion of an isohyetal map of Muskingum drainage basin, Ohio, showing total precipitation during water year 1939–40. Scale 1 in. = approx. 12 mi.

The student should recognize that such a set of rules is not to be applied indiscriminately. For any given problem, suitable variations would have to be made, taking into account (1) the scale and purpose of the map, (2) the spacing of the isohyetals, (3) the number of gages avail-

able and the adequacy of their records, (4) the topography, and (5) the time interval.

2–16. Evaluation of the Personal Factor

Despite the rules, maps constructed by two draftsmen from the same data will not look alike in all subareas. To evaluate this personal factor, an experiment was conducted with two draftsmen, one fairly experienced and the other a beginner. Each of them prepared, independently, an isohyetal map from data consisting of 230 gage records distributed over an area of about 8,000 sq mi. The "precipitation topography" was fairly rough—minimum gage reading was 29.6 in., maximum was 47.7 in., and the standard deviation from the equivalent uniform precipitation was 3.42 in. Plotting was done to a scale of 1:500,000. Both draftsmen used the rules of construction given above. The comparison of the maps is as follows:

(*a*) For the total area of 8038 sq mi, the equivalent uniform depths of precipitation as planimetered from the two maps differed by 0.04 in.

(*b*) For seven subareas of 933 to 1408 sq mi, the equivalent uniform depths differed by from 0.01 to 0.23 in. and averaged a difference of 0.08 in.

(*c*) For twelve subareas of 85 to 140 sq mi, the equivalent uniform depths differed by from 0.02 to 0.63 in. and averaged a difference of 0.22 in.

For areas of all sizes, the average difference was on the order of one-fourth of the estimated standard error inherent in the map. Thus it appears that the difference between maps drawn by two different draftsmen is of little, if any, significance, provided that they follow the same rules of construction.

Computing Equivalent Uniform Depth of Precipitation over an Area

It is never possible to determine exactly the equivalent uniform depth of precipitation over a given area.* Rain gages give, at best, a very small sample. There are three methods of treating their records to arrive at an approximate answer, and, in general, the three methods give three different approximations.

2–17. The Unweighted Mean

The simplest approximation to the equivalent uniform depth of precipitation over an area is given by the unweighted mean of the records of all gages within the area. If the gages are not too irregularly spaced, this method probably gives results that are not enough worse than the

*The authors have chosen the expression "equivalent uniform depth" in preference to "mean precipitation" to avoid confusion. The term "mean" is reserved for use in connection with time series, except where the significance is unmistakable, as when referring to the "mean value" of a set of readings.

results of the other methods to make much difference for many practical purposes. A little additional refinement can often be obtained, particularly for small areas, by including the records of a ring of gages surrounding the area but outside it. The ring should, of course, be complete, with the several sectors more or less equally represented.

2-18. The Thiessen Method

The "Thiessen method" takes its name from A. H. Thiessen, who first suggested it in the *Monthly Weather Review* for July 1911. It attempts

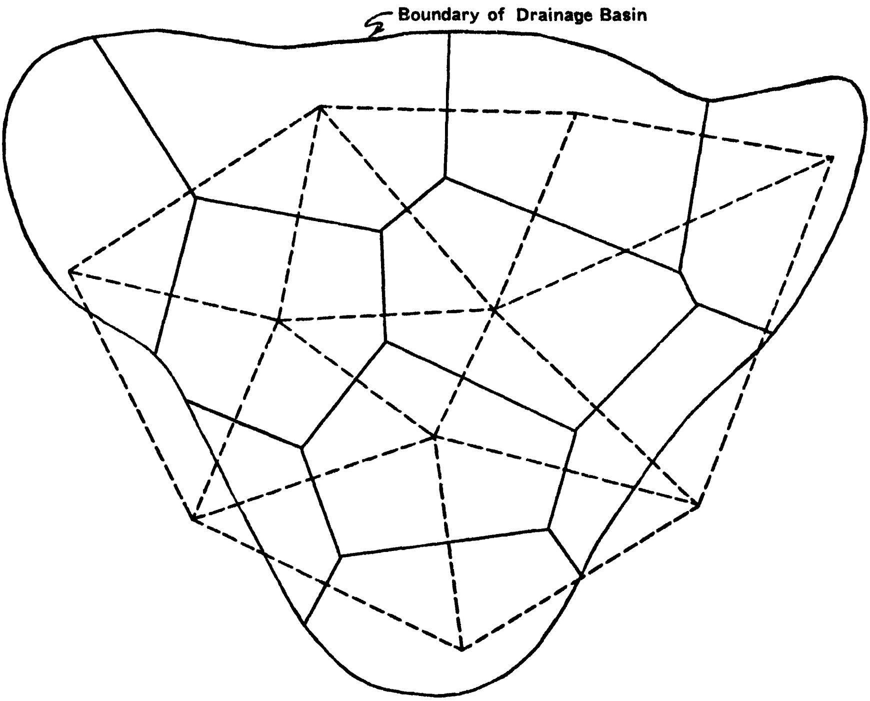

FIG. 2-10. Thiessen polygons.

to make allowance for irregularities in gage spacing by weighting the report of each gage in proportion to the area which that gage is assumed to represent. Thus better use is made of the data, and, in general, the results should more nearly approximate the true value than do those computed without weighting. Of course, if a gage representing a relatively large subarea happens to give a reading considerably different from the true equivalent uniform depth over that subarea, the Thiessen method may give a result *worse* than an unweighted mean. The assumption is that in a system of gages such discrepancies tend to balance out.

The area that any specific gage is assumed to represent is the area comprising all points that are closer to that gage than to any other. These areas are determined graphically as follows (see also Fig. 2–10):

(*a*) Draw a set of triangles connecting adjacent gages, including in the system a ring of gages outside the area at interest.

(*b*) Construct the perpendicular bisectors for all sides of all triangles. These bisectors define a set of polygons, one for each gage, and each polygon contains only points that are closer to the gage at its center than to any other.

Although there is a multiplicity of choice in drawing the triangles initially, there is no ambiguity in the finally resulting polygons and no option as to which gages are included and which are excluded. The work will be easier if flat triangles are avoided, and this is usually possible (see Fig. 2–10). If the polygons are drawn correctly, there will be no re-entrant angles.

The area of each of the polygons is determined by planimetering or otherwise. These areas are the coefficients by which the respective gage readings are to be multiplied. There is no need to convert them to percentages of total area or to square miles; the planimeter readings can be used direct. The answer sought is the sum of the products (coefficient *times* gage reading) divided by the sum of the coefficients.

2–19. Use of Isohyetal Maps

The isohyetal map makes still better use of the gage data than does the Thiessen method, because it takes into account the actual spatial relationship of the gages. In other words, it does not accept blindly the Thiessen assumption that each gage reports the equivalent uniform depth of precipitation over the surrounding area. Rather, it takes into account the evidence of other near-by gages and makes corrections accordingly. In general, it should give a closer approximation to the true value than would either of the other methods. The computation procedure is simple.

(*a*) Planimeter the total area over which the equivalent uniform depth of precipitation is desired and the area within each isohyetal. (It is not necessary to convert these readings to any other units.)

(*b*) Multiply the total area by the minimum depth of precipitation occurring anywhere within it.

(*c*) Add the total area to the area within the lowest isohyetal, divide by 2, and multiply by the difference between the low point and the lowest isohyetal. To this product add half the area within the lowest isohyetal.

(*d*) To the sum of steps (*b*) and (*c*), add the area within each of the remaining isohyetals, and divide the sum by the total area.

The work is best arranged in tabular form like that of Table 2–5, which shows the computations for area *A* of Fig. 2–9.

TABLE 2-5

CALCULATION OF EQUIVALENT UNIFORM DEPTH ON AREA A OF FIG. 2-9

Item	Precipitation (In.)	Area in Arbitrary Units	Extension	Equivalent Uniform Depth (In.)
Lowest point in area........	32.3	123	3973	...
Lowest isohyetal...........	33	111	137*	...
Succeeding isohyetals.......	34	87	87	...
	35	71	71	...
	36	61	61	...
	37	46	46	...
	38	32	32	...
	39	26	26	...
	40	21	21	...
	41	19	19	...
	42	14	14	...
	43	11	11	...
	44	8	8	...
Highest isohyetal..........	45	4	4	...
Total................	....	123	4510	36.67

*(33.0 − 32.3) (123 + 111)/2 = 82
111/2 = 55
137

2-20. Comparison of the Three Methods

The standard errors of the various methods can be computed statistically, under certain assumptions, but the procedures are somewhat beyond our present scope. It may be of interest here, however, to see quantitatively how much the results of the three methods actually varied when applied to specific areas. Table 2-6 shows typical results selected from the study already referred to.

Although the preceding discussion has indicated the general superiority of the isohyetal map for determining the equivalent mean depth of precipitation over a given area, it should be recognized that in any one specific case it cannot be proved to be the best.* It is easy to set up hypothetical conditions under which the Thiessen method will give better results, and others under which even the use of unweighted gage records will give a closer approximation to the true value.

*In Table 2-6 the three values for each area are arranged in numerical order, with prefixed T, I, and U for identification. Since the true value of the equivalent uniform depth is not (and cannot be) known, we cannot tell from the numerical order which of the three approximations is most nearly correct. All that can be said in any given case is that the middle value must be closer to the true mean than at least one of the other two values. Thus if the numerical order is TIU, I must either be better than T or better than U, but it is not necessarily better than both of them. Depending on the actual value of the true mean, any of the following four orders of accuracy for the three methods is possible when they appear in the above numerical order: ITU, IUT, UIT, TIU. In twenty areas studied it was found possible to state definitely in only ten that the isohyetal method was not the worst; in any of the other cases it *might* have been the worst, and in *all* cases it might have been no better than second best.

To illustrate this point, let us take an exaggerated and stylized example. Assume an area containing seven gages, with the central gage having a Thiessen coefficient of 100 and each of the peripheral gages having a Thiessen coefficient of 10. Precipitation reported by the central gage is 36 in.; all other gages report 30 in. From these data, the precipitation

TABLE 2-6

COMPARISON OF TYPICAL RESULTS OF THREE METHODS FOR COMPUTING EQUIVALENT UNIFORM DEPTH

Designation	Area (Sq Mi)	Number of Gages*	Equivalent Uniform Depth (In.)†	Difference between Consecutive Values	Difference between Extreme Values
A......	144	9–11	T—41.36 U—40.48 I—39.87	0.88 0.61	1.49
B......	128	7–10	U—37.18 I—36.61 T—36.60	0.47 0.01	0.48
C......	106	10–11	I—36.67 T—36.47 U—36.47	0.20 0.00	0.20
D......	982	31	T—37.80 I—37.64 U—37.03	0.16 0.61	0.77
E......	834	32	U—38.14 I—38.11 T—38.00	0.03 0.11	0.14
F......	1345	32	U—39.27 T—39.07 I—39.06	0.19 0.02	0.21
G......	8038	230	T—38.85 I—38.76 U—38.62	0.09 0.14	0.23

*For areas A, B, and C the first number is the number of gages included in the Thiessen computation. The second is an estimate of the number of gages that probably influenced the shape of the isohyetals.

†Arranged in numerical order for each area. This is not necessarily or even probably the order of accuracy in any given case. Prefixes denote method of computation: "U" = Unweighted mean; "T" = Thiessen method; "I" = Isohyetal.

amounts by the three methods are: T, 33.75; I*, 32.00; U, 30.9. Now, if the precipitation reported by each of the seven gages actually happened to be the *true* equivalent uniform depth for its respective subarea, the Thiessen value is exactly correct, and the order of accuracy of the three methods is TIU. If, however, the precipitation reported by the central

*Assuming a conical set of isohyetals: Altitude of cone is 36 − 30 = 6; volume is one-third base *times* altitude; average depth of precipitation is one-third altitude *plus* 30 in.

gage is 2 in. higher than the true equivalent uniform depth for the area it represents,* the true mean for the whole area becomes 32.50 in., and the order of accuracy becomes ITU. If the central gage is 3 in. "off," the true mean is 31.9 and the order of accuracy is IUT; if it is 4 in. off, the true mean is 31.3 and the order of accuracy is UIT.

These comments are not intended to condemn the use of isohyetal maps, for, as previously stated, the assumptions on which they are based make the best possible use of the data. All that is desired here is to point out that for any *specific* case, the best that can be said is that the isohyetal method is *most probably* the nearest correct of the three.

Studies of Individual Storms

In predicting the magnitude of possible floods on drainage basins too large to be treated by point rainfall methods, a common procedure is

TABLE 2-7

Excerpt from Miami Conservancy District Data Sheet, Tabulating Precipitation in Storm of March 23-27, 1913

(1) Station	Daily Precipitation (2) 23	(3) 24	(4) 25	(5) 26	(6) 27	Maximum Precipitation (7) 1	(8) 2	(9) 3	(10) 4	(11) 5
Camp Dennison	0.02	1.92	2.25	2.85	0.40	2.25	5.10	7.02	7.04	7.44
Canal Dover	0.30	2.70	1.35	0.75	...	1.35	2.10	5.80	5.10	5.10
Canton	1.03	2.20	3.00	1.62	0.60	3.00	4.62	6.82	7.85	8.45

to "transpose" to that drainage basin a number of the major storms that have occurred in that general section of the country. The justification and limitations of such procedure will be touched on briefly in Chapter 6. It is desired here, however, to describe the construction of the time-area-depth curves that are basic to the method. The discussion is based on the Miami Conservancy District's studies as reported in "Storm Rainfall of the United States."†

For each storm the basic data consisted of the daily rainfall records at all stations touched by the storm. The stations were listed in alphabetical order, and opposite each station were copied the daily rainfall records during the period considered, as indicated in columns (2) through (6) of Table 2-7. Next, each of these daily columns was totaled, and the day with the greatest total was considered the date of maximum 1-day precipitation. The period of maximum 2-day precipitation was taken as

*Such a departure is thoroughly reasonable. If a major storm had centered over this gage, the record might very well show the maximum precipitation that occurred anywhere in the subarea rather than the equivalent uniform depth over the area.

†*Miami Conservancy District Technical Reports*, Part 5 (1917; 2d ed., 1936).

consisting of the day of maximum precipitation and the day immediately preceding or following, depending on which had the greater total sum of precipitation at all stations. The 3-, 4-, and 5-day maximum periods were similarly determined. After these periods were fixed, the total amounts of precipitation at each station for each period were entered in columns (7) through (11). In the example shown, March 25 was the day of maximum total precipitation. Thus it is the March 25 precipitation that is entered in column (7) for each station, regardless of whether or not that was the day of maximum precipitation at that particular station. Similarly, the sum of March 25 and 26 is entered in column (8), and so on.

Five isohyetal maps were next drawn, one each for the maximum day, the maximum 2 days, and so on. From each of these maps, data were obtained by planimeter for constructing an "area-depth" curve for the corresponding period of the storm. Note that the ordinate of any point on such a curve shows the *average* depth over the area indicated by the abscissa of the point. The computations involved may be indicated by the following equation:

$$\bar{d}_m = \frac{I\left(\sum_{i=m+1}^{n-1} A_i\right) + \frac{IA_n}{2}}{A_m} + m + \frac{I}{2},$$

in which

$\bar{d}_m$ is the average depth over the area A_m;

I is the isohyetal interval;

A_i is the total area within any isohyetal i;

$\sum_{i=m+1}^{n-1} A_i$ indicates the summation of the areas within all isohyetals lying inside A_m (except the highest);

m is the isohyetal bounding A_m;

n is the highest isohyetal within A_m.

Area-depth curves for areas up to 10,000 sq mi for the maximum 3-day periods of twelve major storms are shown in Fig. 2-11. In the Miami report these curves are carried out to 110,000 sq mi, the horizontal scale being one-tenth as great for areas from 10,000 sq mi up.

The student should note that the concept of frequency has not been introduced in the foregoing discussion. Statisticians are only now developing an adequate treatment of this problem. A rough (and perhaps oversimplified) idea of the approach may be gained, however, from the following: Suppose that a specific drainage area of 2000 sq mi is situated within the boundaries of a "meteorologically homogeneous" area of

100,000 sq mi. Suppose also that in a period of 30 yr there has been only one storm anywhere in this larger area that has produced an average depth of rainfall as great as 9 in. over a subarea as large as 2000 sq mi. Then, if this 30-yr period is a "typical" one, a storm of similar size would center over a *specific* 2000 sq mi only once in fifty such periods, on the

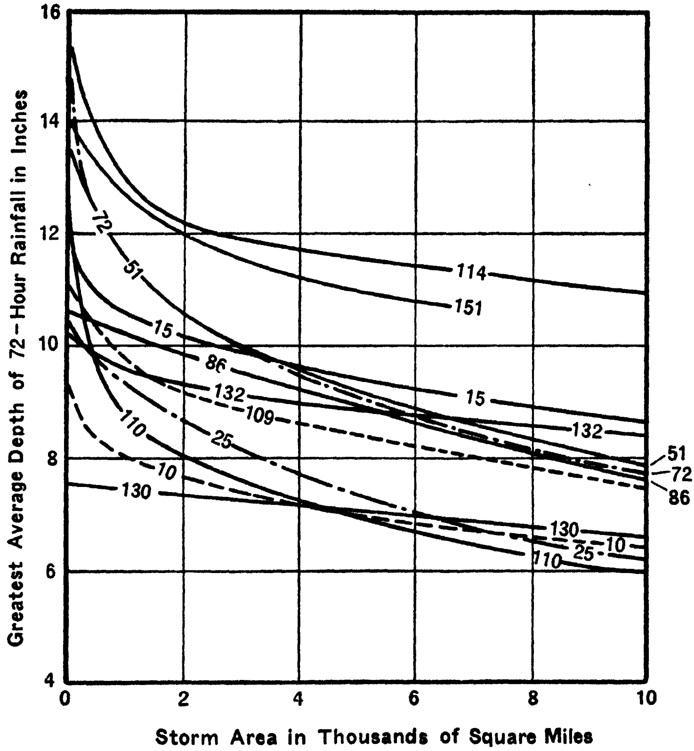

FIG. 2-11. Time-area-depth curves for storms over northern states, showing greatest average depth of rainfall during 3 days. (From Miami Conservancy District, *Technical Reports*, Part 5.)

average; hence the frequency of such a storm with respect to a specific 2000-sq mi drainage area might be said to be roughly on the order of once in 1500 years.

Snow and Snow Surveys

The storage of appreciable quantities of water on the ground in the form of snow complicates hydrologic studies. Generally, in areas where more than 10 per cent of the annual precipitation falls as snow, special methods

of measuring snow precipitation and storage are desirable. In the United States this would include New England, New York, Pennsylvania, the higher parts of the Appalachians, Michigan, Wisconsin, Minnesota, and all the states north and west of Kansas. In other areas snow may be a significant factor at times, as in studies of winter floods.

The weather records obtained by the Weather Bureau do not contain all the necessary information on snow, largely because the snow density may change rapidly and the snow may evaporate or melt rapidly, so that a record of the accumulated precipitation in the form of snow, plus a record of existing snow depths, does not give an indication of the water content of the snow remaining on the ground. The density of freshly fallen snow varies greatly, from less than 1 per cent water content to more than 50 per cent. Windblown snow rapidly increases in density. The density is further increased as the surface absorbs heat. Usually snow melts only from above, and no percolation takes place until the snow has reached its maximum power of water suspension, when it is said to be "ripe." The density of ripening snow from top to bottom is remarkably homogeneous. The density of ripeness depends on the initial density of the snow and therefore on the character of the snow crystal; thus some snows are ripe at lower densities than other snows. The rate of melting after the snow is ripe depends on the heat absorbed, or, in other words, the runoff from melting snow is roughly proportional to degree days above 32F. A warm period or a warm rain may melt ripened snow with dramatic suddenness. Rains on unripened snow are generally absorbed, increasing the density and hastening the ripening.

Snow surveys consist essentially of measurements of the depth and density of the snow. A long metal tube, with a cutting edge and a shoulder inside the cutting edge is driven vertically into the snow to the ground and is lifted with the core of snow inside the tube (see Figs. 2–12*A*, *B*, *C*). The depth of snow is measured, and the tube is weighed with the core and without, thereby determining the weight and the density of the snow. A series of measurements are made at each site, usually at equal intervals along a course. Snow survey sites must be carefully selected, so that the results are representative of the average conditions over the surrounding area. This may be quite difficult, as drifted areas may give erratic results, measurements under trees may give too low determinations because of interception losses, and open areas may have more evaporation and more melting than is typical of general conditions. Snow surveys in the West are often made at the same carefully selected sites and on the same dates each year.

Snow surveys are, of course, useful to any hydrologic study in which the amount of snow storage is a factor, but the method was developed

FIG. 2–12A. Snow survey equipment in use, determining water content of snow core. (Photo by courtesy of G. C. Clyde.)

and used in the mountainous West to enable predictions to be made of the amount of spring runoff from a basin, so that the water could be allocated judiciously. In parts of the West a large portion of the available water

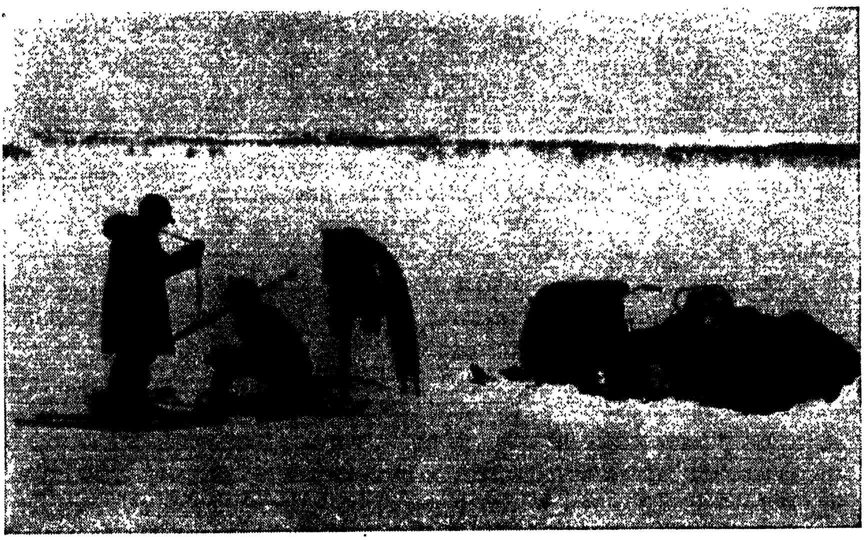

FIG. 2–12B. Snow survey equipment, including snowmobile. (Photo by courtesy of G. C. Clyde.)

comes as runoff from melting snow. In New York, New England, and Pennsylvania similar surveys have proved well worth while. Here the actual quantity of water in the form of snow may be estimated, but in the rough mountains of the West it has proved more practicable to use a percentage method of forecasting. The snow survey determinations are compared with the normal or average for previous years. The percentage figure obtained is applied to the normal spring runoff to give the expected or forecasted runoff. Certain refinements, such as weighting the snow survey determinations according to the altitude of the measurement site,

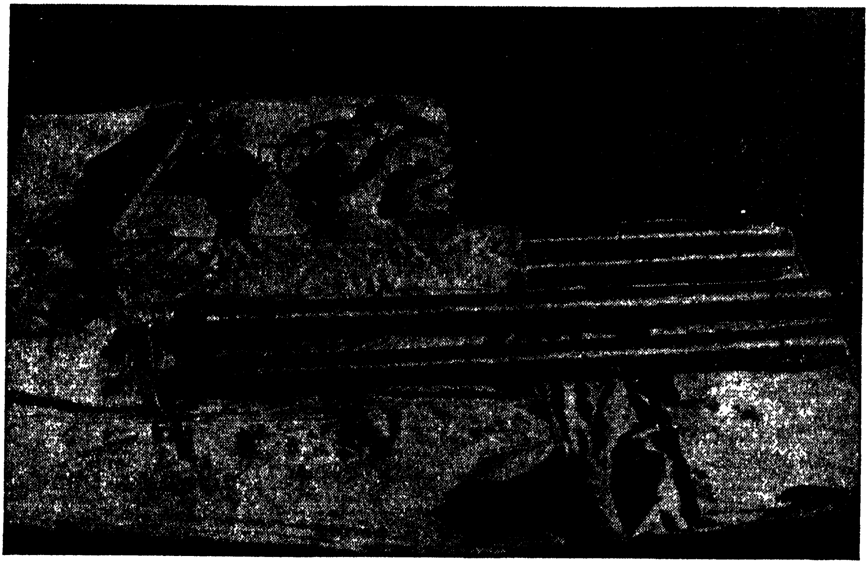

Fig. 2–12*C*. Snow survey equipment, showing cutting edge, tubes, scales, and wrench for assembling tube sections. (Photo by courtesy of G. C. Clyde.)

have been used to increase the accuracy of the above method. The accuracy of the forecasts is surprising; for some areas the forecasts are within 10 per cent. Other areas give more erratic results, particularly where spring rains complicate the forecast.

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CHAPTER 3

COLLECTING AND PRESENTING RUNOFF DATA

Elementary Concepts of Stream-Gaging

3–1. Current-Meter Measurements

The discharge of a stream is the volume of water flowing past a cross section of the stream in a unit of time and is equal to the cross-sectional area multiplied by the average velocity perpendicular to the cross section. Discharge is usually expressed in cubic feet per second (abbreviated "cfs" or "second-feet"). Though many methods are available for determining discharges in open channels, the velocity-area method, using a current meter to measure velocity, is of widest applicability and is generally used in stream-gaging.

A current meter contains as one of its essential parts a cupped or vaned bucket wheel which revolves at a rate proportional to the velocity

of the water impinging upon it. By placing a current meter at a point in moving water and counting the number of revolutions of the bucket wheel in a given interval of time, the speed of the water at that point can be determined from the calibration of the meter. The small Price meter now in general use (Fig. 3-1) is a vertical-axis cup-type current meter, with the bucket wheel attached to a vertical shaft which revolves as

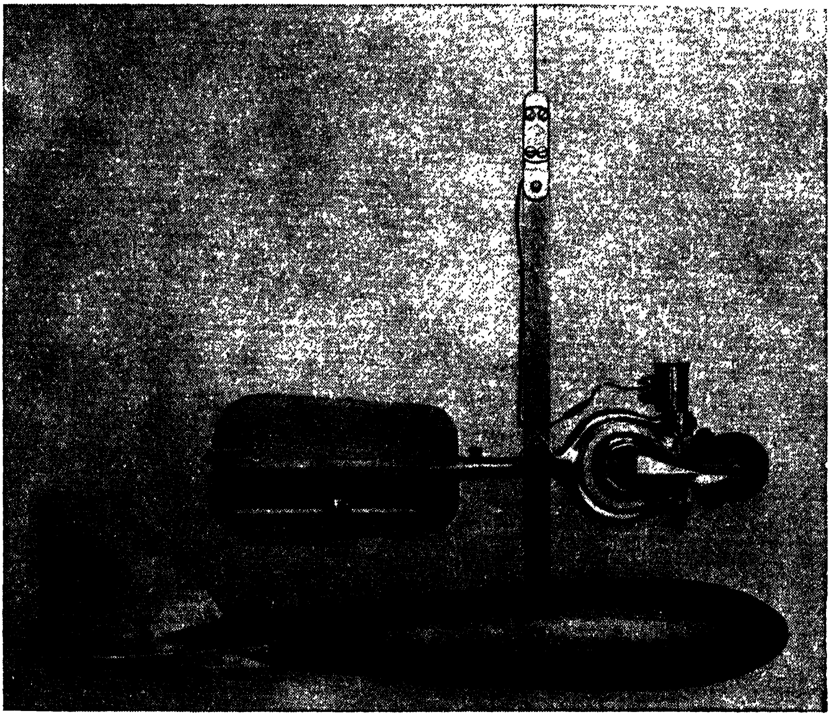

FIG. 3-1. Small Price current meter (cable suspension, Columbus-type weight) (Photo by courtesy of A. W. Harrington.)

the bucket wheel is turned by impinging water, making and breaking an electrical circuit in a commutator head at every revolution (or, with some types of commutator heads, at every fifth revolution). The circuit is usually energized by a flashlight battery and carried by the cable on which the meter is suspended, the revolutions of the bucket wheel being registered as clicks in head phones connected to the circuit.

A current meter is calibrated by determining the number of revolutions per unit of time when the meter is towed at a constant speed through still water. The determination of velocity at a point, then, consists of

counting the number of revolutions of the meter (clicks in the head phones) in a given time and, from the meter calibration table or "rating," determining the corresponding velocity. It should be noted that a current meter actually measures speed rather than velocity and that the horizontal

FIG. 3–2A. Current meter measurement being made from bridge. (Photo by U.S. Geological Survey.)

angle which the flowing water makes with the cross section must be observed in order to compute the downstream velocity component.

The processes of measuring the cross-sectional area and of determining the average velocity in the section are combined in making a current-meter measurement. The meter is usually suspended from a cable which carries the electrical circuit above a streamlined weight heavy enough to permit vertical determinations of depths, and the measurements of depths and velocities are made from a bridge or other structure spanning the

stream (Figs. 3-2A, B). For low flows such that the stream can be waded, the meter is mounted on a rod, and the hydrographer determines the depths with the rod and the widths with a graduated line or tape stretched across the stream. In either case the measuring section is divided into a number of panels or partial sections, the widths of these subsections are measured, and depths are determined at the ends of each subsection. Velocity readings are taken to determine the average velocity in the vertical at each point where depths are measured. A page of discharge-measurement notes, illustrating a method of computation, is reproduced as Fig. 3-3. The discharge of a partial section is the product of the width

FIG. 3-2B. Current meter measurement being made through ice cover. (Photo by courtesy of A. W. Harrington.)

by the average depth by the average velocity. The total flow is the sum of the flows in all the partial sections.

The analysis of a large number of velocity readings shows that, for channels with reasonably smooth bottoms and no disturbing conditions, the shape of the vertical velocity curve is approximately parabolic, with the horizontal axis at the point of maximum velocity a short distance below the surface. Advantage is taken of this characteristic in stream-gaging. From the geometry of the parabola it can be shown that, for a parabolic-shaped vertical velocity curve, the average of the velocities at 0.2 depth and at 0.8 depth is approximately the mean velocity. Similarly, if the maximum velocity is at 0.15 of the depth below the surface, then the mean velocity is at 0.6 depth. Vertical velocity curves in natural

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September 1943

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES BRANCH

.85

DATE JULY, 5 19 46 **DISCHARGE MEASUREMENT NOTES**

AUGLAIZE RIVER AT Near DEFIANCE, OHIO

Angle Coefficient

Dist. from Initial Point	Depth	Observation Depth	Revolutions	Time in Seconds	Velocity At Point	Velocity Mean in Vertical	Velocity Mean in Section	Area	Mean Depth	Width	Discharge
2	0			0							
							0.36	6.40	0.8	8	2
10	1.6	0.6	15	46		0.73					
							1.08	16.50	1.65	10	18
20	1.7	0.2	30	42	1.58	1.43					
		0.8	30	52	1.28		1.66	17.00	1.70	10	28
30	1.7	0.2	40	42	2.10	1.88					
		0.8	30	40	1.66		1.98	17.50	1.75	10	35
40	1.8	0.2	50	49	2.25	2.08					
		0.8	40	46	1.92		2.26	18.00	1.80	10	41
50	1.8	0.2	60	48	2.75	2.45					
		0.8	40	41	2.15		2.48	18.50	1.85	10	46
60	1.9	0.2	50	40	2.75	2.50					
		0.8	50	49	2.25		2.38	16.50	1.65	10	39
70	1.4	0.6	50	49	2.25	2.25					
							2.01	18.00	1.80	10	36
80	2.2	0.2	40	42	2.10	1.77					
		0.8	30	46	1.44		1.68	22.00	2.20	10	37
90	2.2	0.2	40	50	1.77	1.58					
		0.8	30	48	1.38		1.51	22.00	2.20	10	33
100	2.2	0.2	33	44	1.51	1.44					
		0.8	25	40	1.38		1.06	22.00	2.20	10	23
110	2.2	0.2	15	41	0.82	0.67					
		0.8	10	43	0.52		1.42	22.50	2.25	10	32
						1.18					

.90 .92 .94 .96 .97 .98 .99 .99 .98 .97 .96 .94 .92 .90 .85 .80 .75

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FIG. 3-3. An example of current meter measurement notes. (NOTE: The numbers on the right margin, with the center point on the left, are a convenient means of observing horizontal angle "coefficients.")

streams usually approximate this condition. In making a current-meter measurement, therefore, velocity readings are taken at 0.2 and 0.8 of the depth and averaged to determine the mean velocity in the vertical. For depths so shallow that the 0.8 reading cannot be made because of the distance of the meter above the bottom of weight, the 0.6 depth method is used.

There are minor pulsations in velocity in open-channel flow, even though flow is uniform and the banks and bottom smooth. These fluctuations have periods varying from a few seconds to several minutes, and, to reduce inaccuracies from this cause, velocity readings are usually timed for from 40 to 60 sec.

Measuring sections must be carefully selected. The small Price current meter has high accuracy from about 0.5 to 15 fps; velocities outside this range should be avoided. Eddies and cross currents should also be avoided. The spacing of the observations (widths of partial sections) should be such that the essential variations in velocity and in contour of the bottom are measured. Generally, at least twenty observations of depth and velocity should be made, even in a narrow stream. The number of observations should be increased where the bottom is uneven or the velocities are irregular, the necessity for additional observations depending on their effect on the accuracy of the total discharge measured. In some cases irregularities or disturbances near the bottom may affect 0.8 depth velocity readings, in which case additional readings in the vertical will be required to obtain the mean velocity in the vertical. Here again the necessity for additional readings depends on whether or not the inclusion of these readings will affect appreciably the total discharge figure. During rapidly changing stages it may be desirable to reduce the number of observations, as explained on page 70 below.

3-2. Stage-Discharge Relation

For hydrologic work a day-by-day record of stream flow is essential. Generally, current-meter measurements are far too costly and time-consuming to be repeated at anything like daily intervals. River stage, on the other hand, can be measured daily or even continuously at relatively small cost. Thus to obtain a satisfactory record of stream flow it is necessary to have (1) a permanent gage installation, (2) a number of discharge measurements covering a considerable range of stages, and (3) knowledge of the relationship between stage and discharge.

Since the cross-sectional area at any point is a function of gage height alone, the possibility suggests itself of relating discharge to stage alone. However, this will theoretically be possible only at points where velocity also is uniquely determined by stage—and it is not always so determined, for, in a given channel, velocity is a function of slope as well as of stage, and slope and stage bear no fixed relationship *except under certain conditions*.

What are the conditions under which there will be a fixed stage-discharge relation? By the concept of critical depth, any point on the water surface profile in an open channel is on a dropdown curve (supercritical velocity, subcritical depth), on a backwater profile (subcritical velocity, supercritical depth), or at a drop or other local phenomenon (critical velocity, critical depth). The "shooting" velocities of a dropdown curve should be avoided in locating gages. A gage will therefore be on a backwater profile, and often there will be a drop or other feature downstream which will "control" the stage-discharge relation. For example, a gage located on a deep pool above a fixed overflow dam high enough not to be subject to submergence from downstream conditions will have a definite, fixed stage-discharge relation. If the deep pool is replaced by a permanent channel and there is no variable flow entering the channel between the gage and the dam and the flow is steady, then the dam will still "control" the stage-discharge relation. If there is no point on the stream where flow is at critical depth, then, in the strict hydraulic sense, there is no control. Nevertheless, there will often be a stage-discharge relation approaching the definite ideal case, and the physical feature downstream from the gage upon which this stage-discharge relation depends is called by hydrographers the "station control."

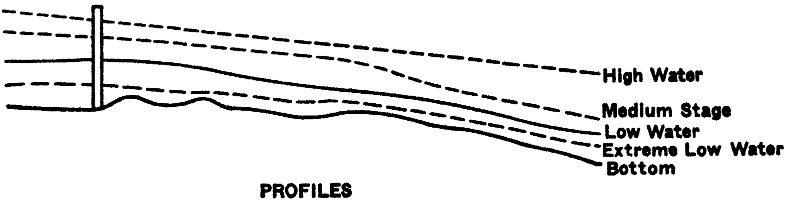
A control may be either natural or constructed and may be a ledge of rock, a gravel riffle, an overflow dam, or some other constriction or obstruction governing the stage-discharge relation. If there is no constriction or obstruction for a considerable distance downstream, the channel features, such as shape, slope, and frictional resistance, are assumed to govern the stage-discharge relation, and the condition is called "channel control." Usually the low-water control is drowned out by some downstream feature at higher stages, and often there are several partial controls, as illustrated in Fig. 3–4.

The control concept is an important one and has many applications other than to stream-gaging. In planning channel improvements, dredging, and other operations in which the objective is to change the stage-discharge relation, careful studies must be made of downstream features to ascertain what the control will be after completion of the project and what the new stage-discharge relation will be.

A control is said to be "permanent" if, with steady flow and no submergence (no backwater from downstream), there is a fixed stage-discharge relation. If the control is subject to scour or fill, it is called a "shifting" control.

The requirements for a fixed stage-discharge relation are (1) absence of "backwater" and (2) steady flow. Since it has previously been stated that any gage is on a "backwater profile," one may well wonder about the first of these requirements. Unfortunately, two connotations of the

word "backwater" have developed—one in hydraulics and one in stream-gaging. Four paragraphs above, the word was used in the hydraulic sense, which is clear from the context. In the present case the stream-gaging connotation applies, namely, the effect of *unusual* and *temporary* conditions downstream, such as ice in the channel or on the control, unusually high stages on downstream tributaries, or other abnormal conditions which render the stage-discharge relation temporarily inapplicable. As for the second requirement for a fixed stage-discharge relation, it is clear that with unsteady flow there will be variable slopes and variable slope-stage relationships. Steady-flow conditions are rare in nature.



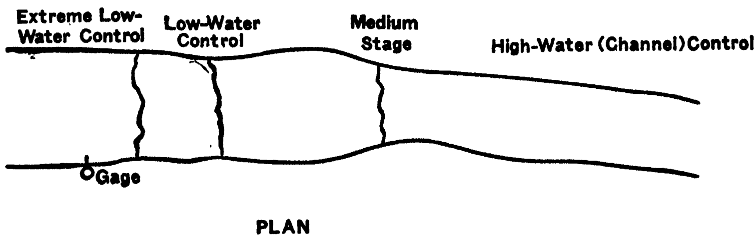


FIG. 3-4. Reach of stream below gage, showing partial controls.

From the above discussion of controls, backwater, and variable flow it might be assumed that stage-discharge relationships would have limited use in stream-gaging, but such is not the case. The sites selected for gaging stations and controls are such that, for most stations operated in the United States, the errors involved in computing discharges from a stage-discharge relation, due to variations in slope, velocity, or channel conditions other than backwater, are so small that they may be neglected. The exceptions, though not numerous, are important ones and are discussed in the section on "Taking Account of Variable Slope" (p. 72).

3-3. Selection of a Gaging Station Site

It is apparent from the foregoing that the careful selection of a gaging station site is important. An ideal location for a gage is one in which

the velocity is uniquely defined by stage; but this would require (1) a permanent control not subject to backwater and (2) a large pool or reservoir upstream from the control, so that unsteady flow would have no appreciable effect on the slope. It is generally impossible to find conditions that approach the ideal, but the best site available should be chosen. Detailed discussion of this subject may be found in references in the bibliography; a summary of the more important considerations is as follows:

1. The control should be as permanent as available.
2. The control should be subject to a minimum of "backwater" from downstream.
3. The gage should be near and preferably in the first low-water pool upstream from the control.
4. The inflow between the gage and the control should be negligible—that is, so small that the stage-discharge relation is not appreciably affected.
5. The measuring section should be near enough to the control that the discharge measured is essentially the same as that flowing past the control, in order to avoid channel storage corrections.
6. The measuring section should be on a straight reach of channel, of regular cross section, so that accurate measurements can be made with normal care and without obstructions near the section which might cause undue turbulence, boils, eddies, and negative flow. Velocities should be within the accurate measuring range of a current meter.

3-4. Gage Installations

At this point it would be well to consider the physical installations at a gaging station. A graduated staff set on the river bank or a weight and chain which can be used to measure the distance to water from a fixed point on a bridge or other structure can be used to obtain records of stage. Many such installations are in use and give satisfactory records when the rate of change of stage is slow enough that one or two gage-height readings per day completely define the stage hydrograph. Generally, rivers rise and fall too rapidly for this to be true—even rivers with drainage areas of 7000 sq mi or more—and recording gages that give a complete and accurate record of stage are much more satisfactory.

A recording gage installation consists essentially of a stilling well, connected to the river by an intake pipe, with an instrument housed in an instrument shelter above the well. The instrument is actuated by a float on the water surface in the well, to which it is connected by a wire cable. In some models the cable passes over a wheel on the end of a drum on which the gage chart is mounted, and a clock mechanism moves the recording pen at a uniform rate along a path parallel to the axis of the drum. In other models the clock moves the drum at a constant speed of

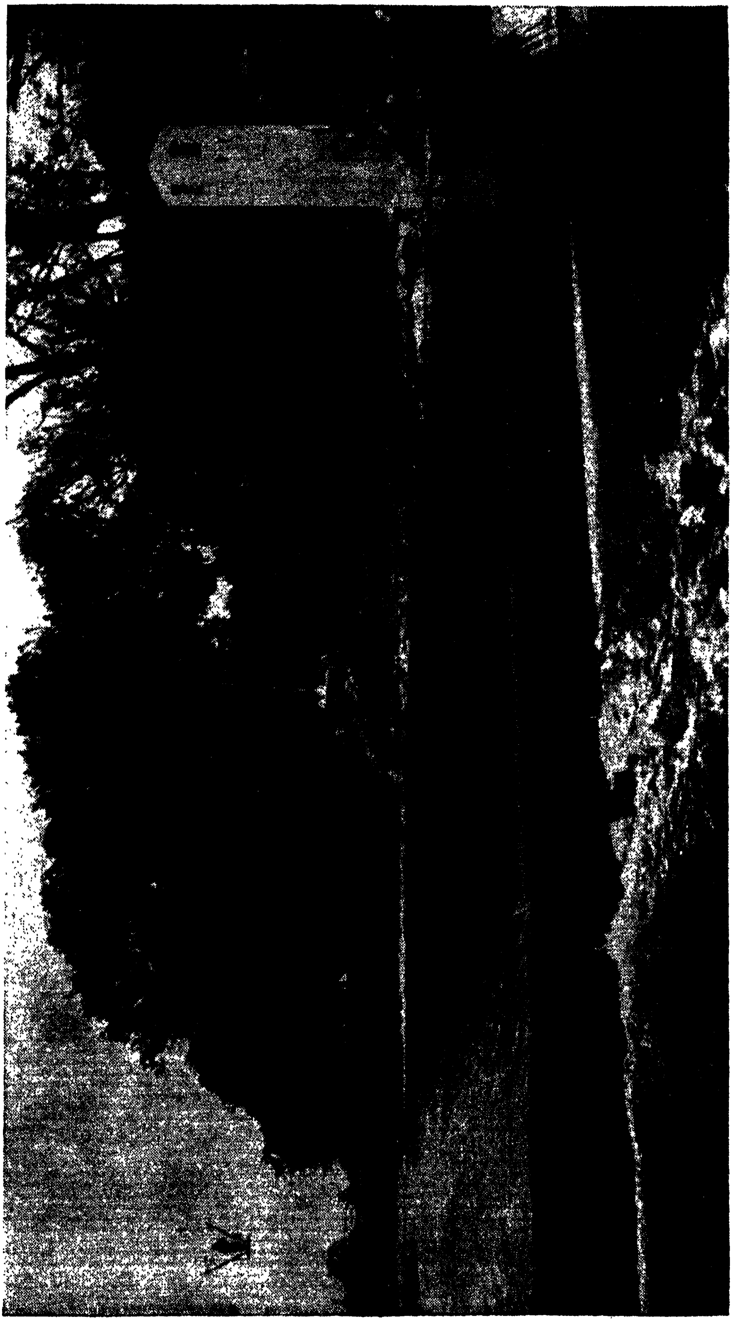


FIG. 3-5A. Gaging station on the Olentangy River, at Stratford, Ohio, showing concrete well and gage shelter, artificial control, and cable installation. (Photo by courtesy of C. V. Youngquist.)

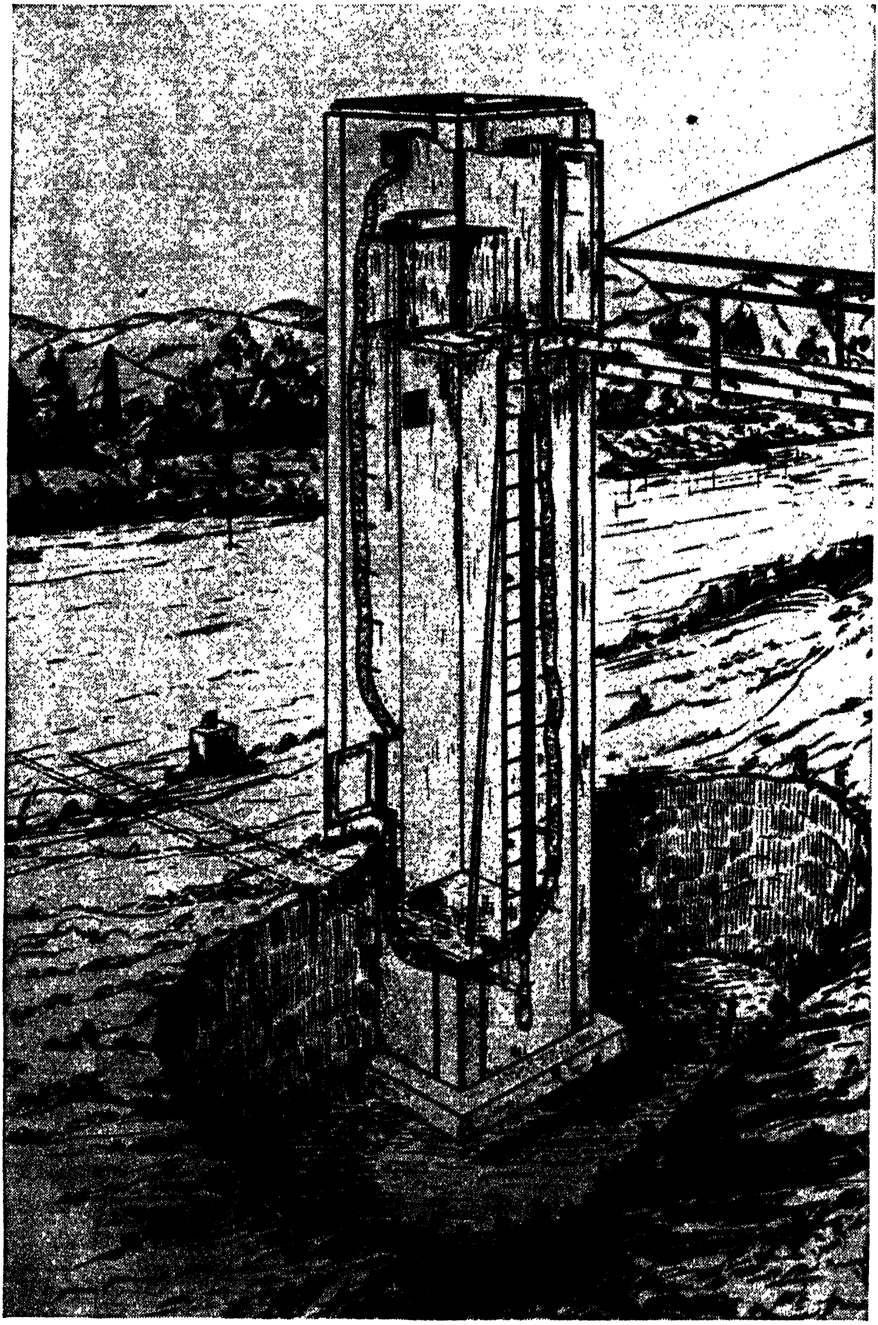


FIG. 3-5B. Sketch of a typical recording gage installation.

rotation, while the float cable, by means of a gear arrangement, moves the pen to right or left as the water surface level rises or lowers. In either type the resulting record on the paper is a graph of stage against time. If the cable actuates the drum, the chart paper must be removed and replaced as soon as the pen has moved the length of the drum, which usually means servicing once a week. If the clock drives the drum, a

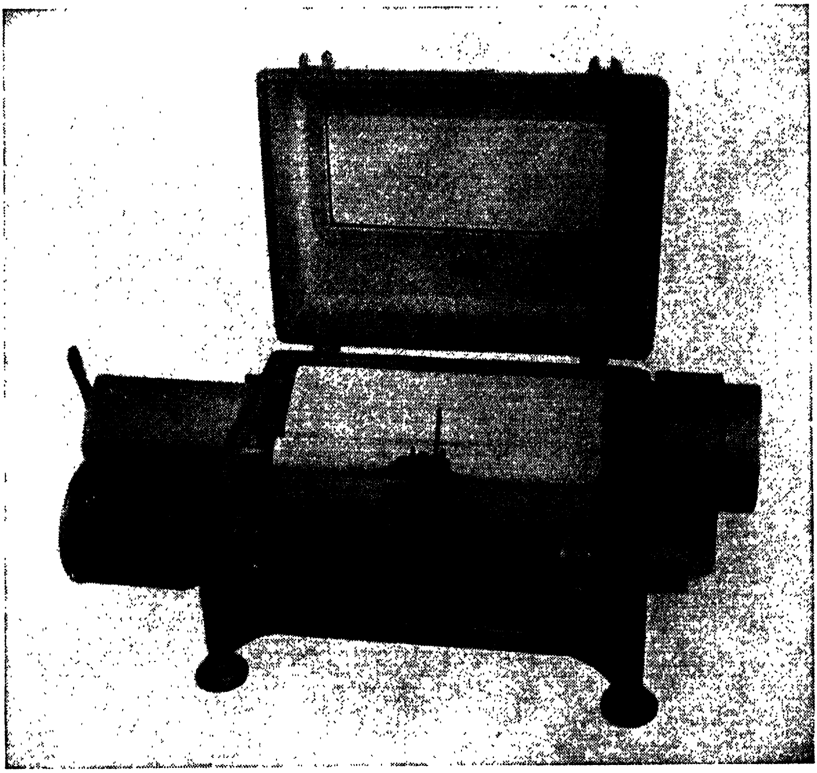


FIG. 3-6A. Friez continuous water-level recorder.

continuous roll of paper can be used, of sufficient length to record a month or more of record. A recorder installation has the disadvantages of a higher first cost and the expense of maintenance of the recorder mechanism and the well and intake, but it gives a complete and accurate record of gage heights. Figs. 3-5*A*, *B* show typical recording gage installations, and Figs. 3-6*A*, *B* show continuous water-stage recorders.

Artificial controls are often constructed to stabilize the stage-discharge relation. Usually they are designed to improve the sensitivity when a small increase in gage height gives a large increase in discharge, that is,

when the rating curve is "flat." A wide horizontal dam is extremely insensitive at low flows. Artificial controls could be designed to give a linear gage-height discharge relationship, but the limitations of cost and of allowable increase in stage or backwater above the control seldom, if ever, permit this. A control weir should approach a V-notch in sensitivity at low stages; at higher stages, where the increase in sensitivity

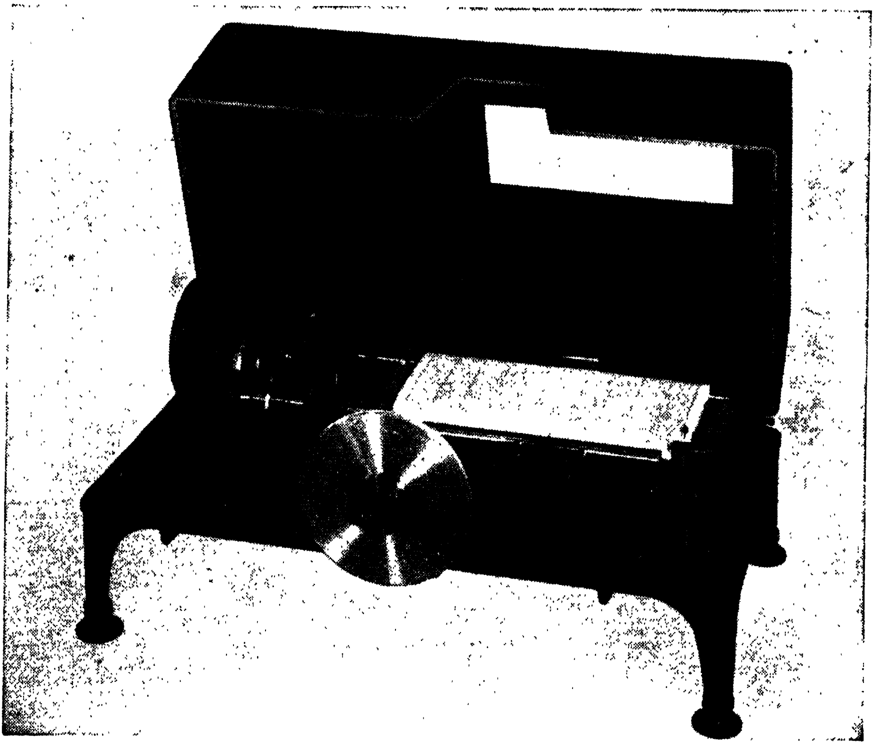


FIG. 3–6B. Stevens continuous water-level recorder.

is less important, the structure can be wider and flatter. In general, an artificial control should (1) eliminate as much as possible the effects of variable downstream conditions; (2) stabilize the low-water rating curve; (3) be sensitive, so that normal errors in the gage-height record will not affect the accuracy of the discharge record beyond allowable limits; (4) result in a rating curve of such slope that it can be extended accurately to peak stages; and (5) be permanent. Each control must be designed to fit conditions at the site and the available funds. Figs. 3–7A, B show artificial controls of types used by the U.S. Geological Survey.

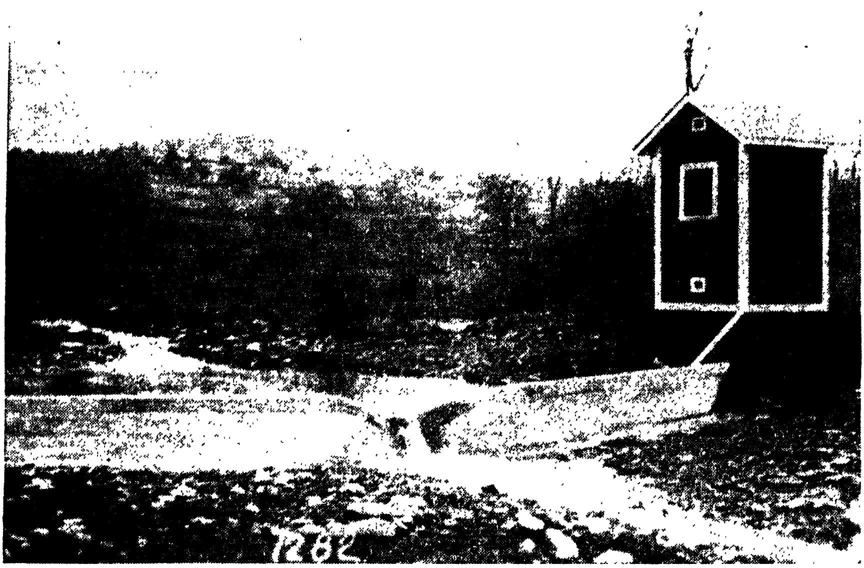


FIG. 3-7A. Artificial control (90° V-notch plate) and gage house, Cold Spring Brook, at China, New York. (Photo courtesy of A. W. Harrington.)

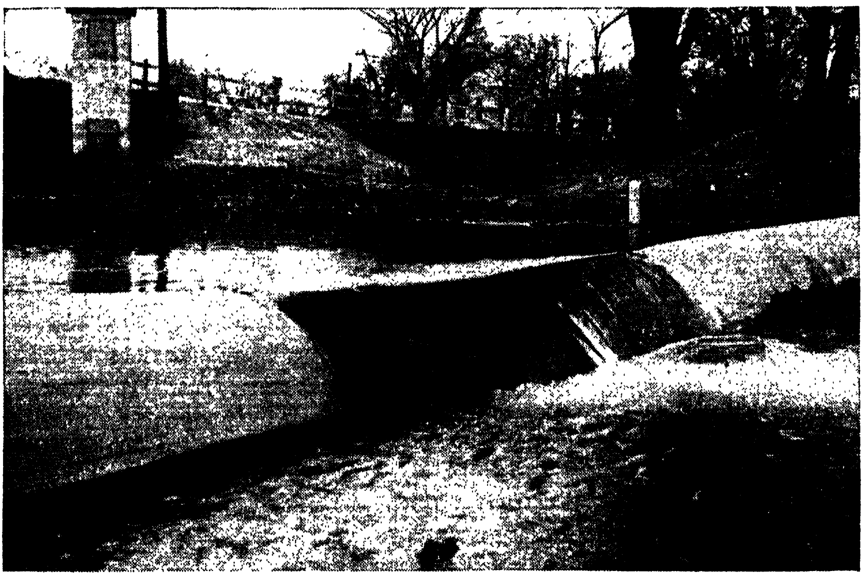


FIG. 3-7B. Artificial control and gage shelter, Ottawa River, at Allentown, Ohio. (Photo courtesy of A. W. Harrington.)

3–5. Simple Rating Curves

The stage-discharge relation at a gaging station is usually determined experimentally by measurements of discharge and observations of stage. Generally, the resulting rating curve is a simple one, parabolic in shape, with slight "breaks" where one partial control is superseded by another. The rating curve shown in Fig. 3–8 is typical of many gaging stations. The result of each measurement is plotted, and a mean curve is drawn through the plotted points. In Fig. 3–8 the areas and mean velocities for each measurement are also plotted, and curves are drawn through them for illustration; but this is not necessary to the development of a rating curve. The measurements were all made at a bridge section about 33 ft wide. Below a stage of about 1.0 ft the channel width is less than 33 ft, and above 5.3 ft there is flow in an overflow channel. A study of the rating curve would indicate that the low-water control was effective from zero flow at 0.6 ft gage height to about 1.5 ft gage height; then there is a gradual change to a medium stage control, effective to about the 4.0-ft stage; the additional curvature suggests that a high-water control is effective above the 4.0-ft stage; overbank flow possibly causes the "break" to the right above the 5.3-ft gage height.

Even with a well-defined rating curve and an apparently permanent control, periodic measurements are desirable, as "shifts" in the rating may occur whenever the channel scours or fills. During winter months there may be backwater from ice, and during the summer there may be weeds or aquatic growth on or above the control. Shifts in ratings may develop from scour or fill in the approach channel or downstream from the control, with no change in the control itself. One partial control may shift without affecting a control effective at other stages. At best, stream-gaging is, therefore, a never ending problem.

In developing a rating curve the mean or average stage during the time of the measurement is plotted against the measured discharge for each measurement. If the rise or fall in gage height is slight, the average of the gage heights at the beginning and end of the measurement may be used; but, if the change in stage is large, the resulting gage height will be erroneous because of the curvature of the rating. In these cases a "weighted" mean gage height is computed by multiplying the discharge in each partial section or group of subsections by the average gage height which was observed during the measurement of the subsection, then the products for all the subsections are added and are divided by the total discharge. To avoid as much as possible this complication and the errors which may be involved in assuming that a weighted mean gage height applies to the discharge as measured, as well as the complications of variable slopes discussed in the next section, it is often desirable to reduce

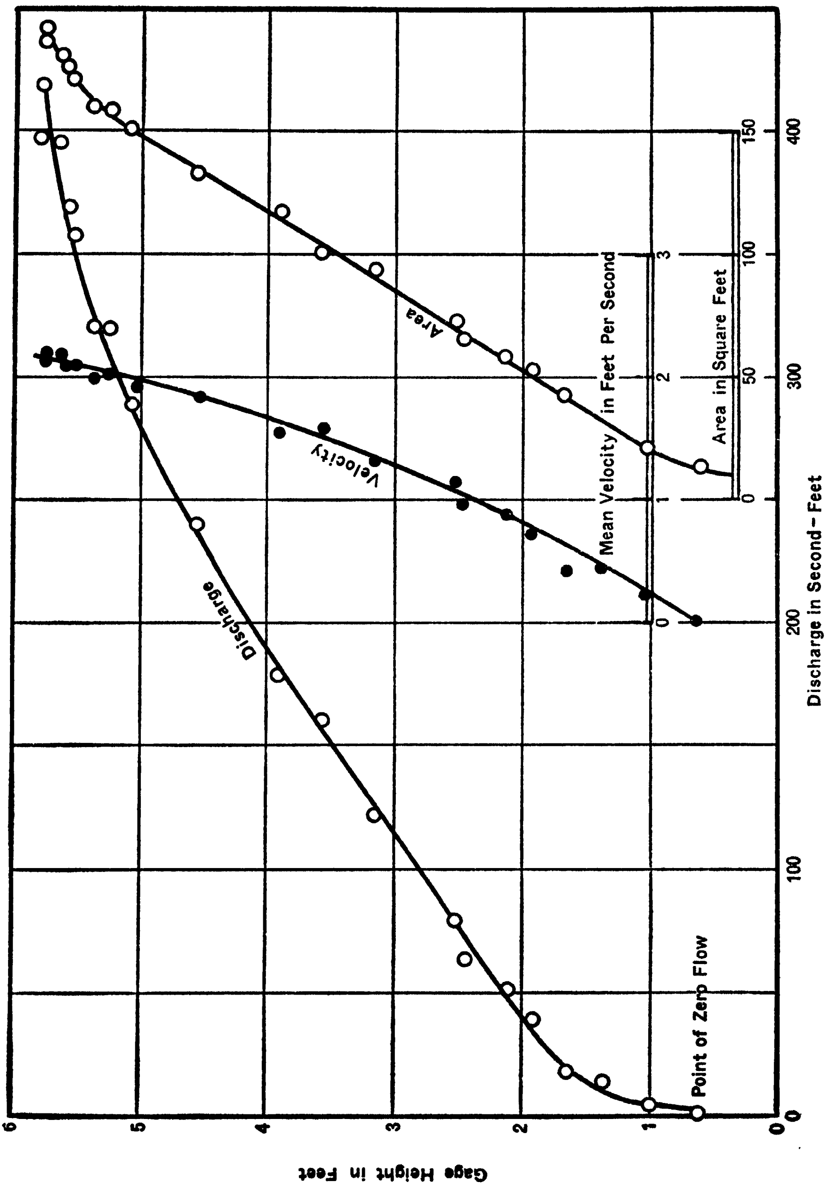


FIG. 3-8. Typical rating curve.

the number of observations and thus hold to a minimum the change in stage during a measurement.

If the plottings of discharge measurements scatter at sites where the channel and controls are permanent, so that the plotting cannot be explained by shifts, then the effect of variable slopes must be investigated.

Taking Account of Variable Slope

3-6. Stage-Slope-Discharge Relations

Variable slope will be a factor in the stage-discharge relation if the slope corresponding to a given stage is not always the same. Variable

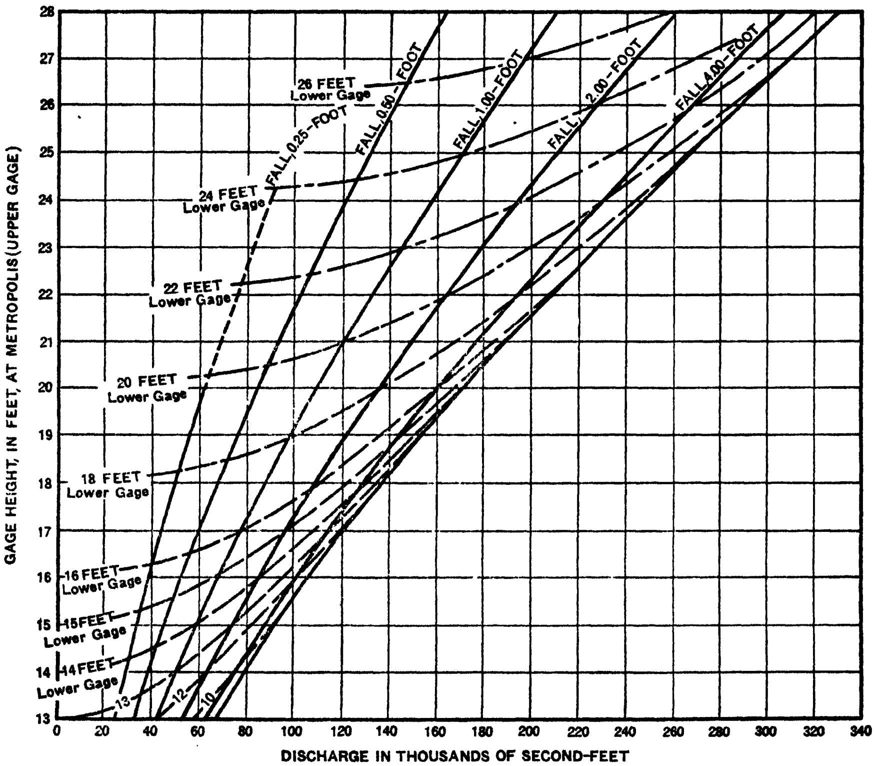


FIG. 3-9. Curves of equal fall, Ohio River, at Metropolis, Illinois. (From *Water Supply Paper 888.*)

slopes may be caused by (1) variable backwater, (2) changing discharge (unsteady flow), or (3) a combination of (1) and (2).

When variable slopes are involved, two gages are necessary, installed far enough apart that the measured fall will not be appreciably affected by normal errors in gage readings, but close enough together that there is

negligible inflow in the "reach" between the gages. The upstream gage, usually called the "primary" gage, should be as close as possible to the control, if there is a control at some stages. The gages preferably are set to the same datum. For all stages at which there is no effective control in the reach, either of two conditions may exist: (1) if the flow is uniform, the difference between simultaneous gage readings divided by the distance between gages is a direct measure of the slope and of the energy gradient; (2) if the flow is nonuniform, the slope measured is that of a chord connecting the water surfaces at both ends of the reach, but the measured fall is a function of the energy gradient at the primary gage.

A three-dimensioned plot of discharge measurements, with gage heights at the primary gage as ordinates and discharges as abscissas and the measured fall noted beside each plotted point, would make possible the sketching of contours of equal fall, resulting in a series or family of curves of equal fall, such as shown that in Fig. 3-9. However, it is generally impossible to obtain measurements under enough varied conditions of slope, stage, and discharge completely to define a family of curves. The possibilities of shifts in the channel reach and inaccuracies in the measurements also present problems, and for these reasons it is usually best to adjust all measurements to a constant fall, such as 1.0 ft, or to a normal fall.*

3–7. Variable Slope Caused by Variable Backwater

We shall consider, first, the case of steady flow and variable backwater. Manning's equation,

$$v = \frac{1.486}{n} r^{2/3} s^{1/2},$$

applies, where v is the velocity, considered as uniform throughout the cross section, n is the roughness coefficient, r the hydraulic radius, and s the energy slope. If, at a given stage, two discharges, Q_1 and Q_2, occur at different times with the corresponding energy slopes, s_1 and s_2, then

$$\frac{Q_1}{Q_2} = \frac{Av_1}{Av_2} = \frac{s_1^{1/2}}{s_2^{1/2}}.$$

If the reach is uniform, then the observed fall is a measure of the energy slope, and measurements can be adjusted to some arbitrary constant fall by the equation

$$Q_c = \frac{\sqrt{F_c}}{\sqrt{F}} Q,$$

*"Normal fall" may be defined as the fall under conditions of steady flow and a minimum of backwater. It is not to be confused with the term "normal slope," which in open-channel hydraulics is the water surface slope parallel to the bottom slope.

where Q_c is the adjusted discharge and F is the existing fall between gages corresponding to the measured discharge Q. Experience has shown that the exponent may not be exactly the theoretical ½, and more precise practical results can be obtained by using the exponent which best fits the data. The equation then becomes

$$Q_c = \frac{F_c^p}{F^p} Q.$$

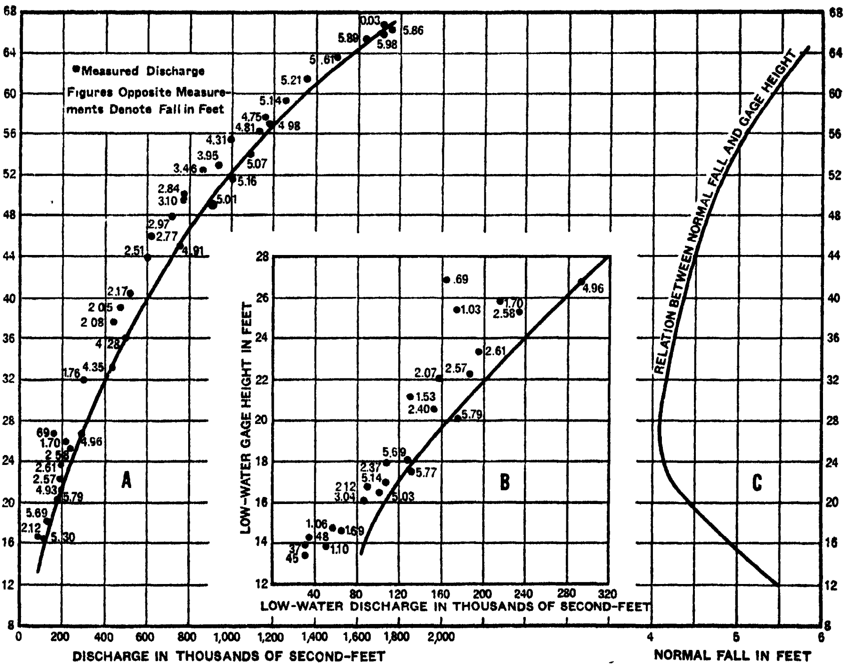


FIG. 3-10. Curves showing relations of stage to discharge and stage to fall, Ohio River, at Metropolis, Illinois. (From *Water Supply Paper 888.*)

In the usual case the reach is not uniform, and there may be an effective control at some conditions of stage and fall. A curve of stage-discharge relation for the condition of least backwater or normal fall can be developed from the plotting of measurements apparently not affected by backwater, and the other measurements may be adjusted to this curve by the relation

$$\frac{Q}{Q_n} = \phi\left(\frac{F}{F_1}\right).$$

Figs. 3-10, 3-11, and 3-9 illustrate how this may be done. First, a trial curve for normal fall is drawn through the measurements that plot

farthest to the right, as shown in Fig. 3–10*A*, *B*. The ratios of measured fall to normal fall and of measured discharge to normal discharge are then computed for each measurement, and the ratios are plotted against each other, as shown in Fig. 3–11*C*. Some readjustment of the normal stage-discharge curve may be necessary to smooth the ratio curve. The discharges for each measurement are adjusted to the normal discharge and are replotted, as shown on Fig. 3–11*A*, *B*. From the final curve for

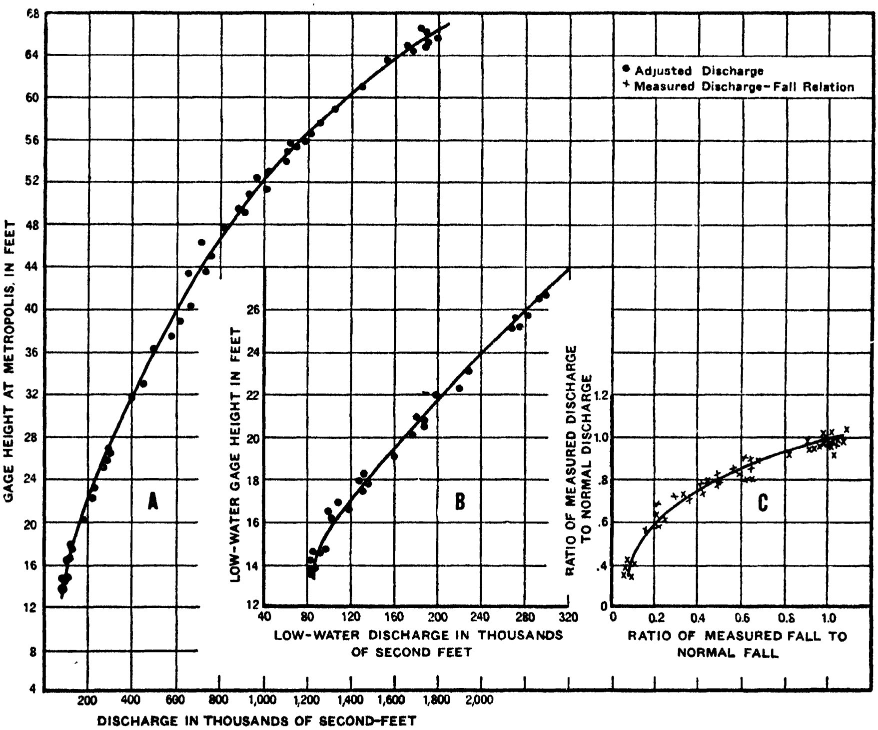


FIG. 3–11. Curves showing relations of stage to normal discharge and discharge ratios to fall ratios, Ohio River, at Metropolis, Illinois. (From *Water Supply Paper 888.*)

normal stage discharge and the final curve for the ratios, the family of curves shown in Fig. 3–9 may be computed. It should be noted that the exponent for the fall differs slightly from ½ for this example.

In case there is no effective control at any condition of stage and fall, the measurements will scatter to the right as much as to the left, and there will be no normal stage-discharge relation. A contour or curve of constant fall can be drawn, however, and the measurements adjusted as in the normal fall case.

It should be noted that the slope term in Manning's equation is the energy gradient. If the cross sections at the two gages are not approxi-

mately equal in area and similar in shape, then adjustments must be made for differences in velocity heads, and corrections for velocity distribution may be required. These adjustments can generally be avoided by careful selection of the slope reach and gage locations.

3–8. Variable Slope Caused by Unsteady Flow

In the second case—that of unsteady flow or changing discharge—the stage-discharge relation may be affected by variable slopes and may also be affected by changing channel storage. If discharge measurements are made at a considerable distance from the control, measurements may appear to be affected by variable slopes when actually they are affected by channel storage alone. For example, a discharge measurement of 100 cfs is made on a stream 1000 ft above the control. During the measurement the gage height at the gage rose at a rate of 0.2 ft/hr, and the channel is such that this rate of rise may be assumed to apply to the entire 1000-ft reach. The average width of channel in the reach is approximately 100 ft. Then the rate of increase of channel storage in the reach is $1000 \times 100 \times 0.2 = 20{,}000$ cu ft/hr, or 5.6 cfs. The discharge measurement should be plotted on the rating curve as 94.4 cfs, as this is the discharge past the control corresponding to the mean gage height for the measurement.

When measurements made during changing stages scatter about the steady-flow stage-discharge curve and the discrepancies cannot be laid to variable backwater or to changing channel storage, then the variable slope of nonsteady flow is the probable cause. The rating curve for the Ohio River at Wheeling, West Virginia, shown in Fig. 3–12, is a typical example of the way that measurements plot during changing stages for stations affected by variable slope. Discharges for rising stages plot to the right, and for falling stages to the left, of the normal steady-flow curve. In some instances the maximum discharge occurs at a lower gage height than does the peak stage. The problem is the same as for variable backwater, except that the variable slopes caused by variable discharge usually change more rapidly than do the variable slopes caused by variable backwater. Because of the rapid variations of the shape of the water surface profile, it is more difficult to make proper use of records of stage at both ends of a slope reach. If it is possible to obtain enough measurements under various rates of change of stage, a family of curves can be drawn, with rate of change of stage as a parameter. However, it usually is impossible to obtain sufficient data; and, as in the case of variable backwater, it is desirable that all measurements be adjusted to the normal curve for steady-flow conditions.

The problem, then, is to find a relationship between the observed data and the effect of changing stage, so that measurements can be cor-

rected to steady-flow conditions. In Fig. 3-13, h represents the initial stage, Δh the increment in stage, V the initial velocity, and Δv the incre-

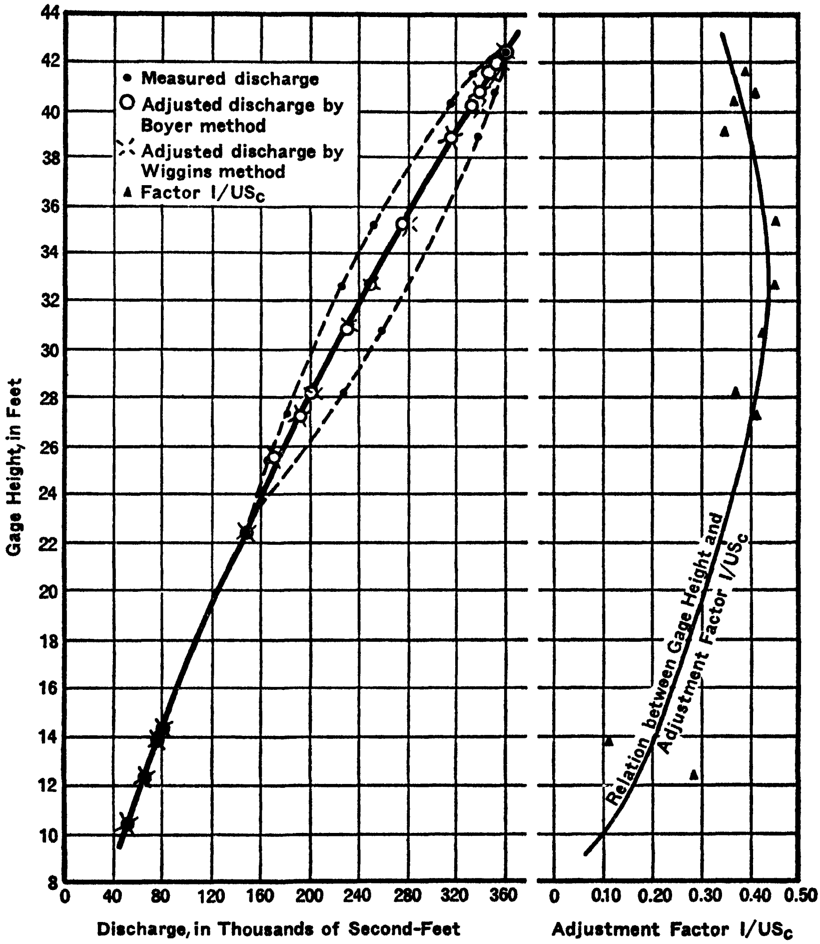


FIG. 3-12. Adjustment of discharge measurements for changing discharge, Ohio River, at Wheeling, West Virginia, during the period of March 14-27, 1905. (From *Water Supply Paper 888.*)

ment in velocity corresponding to the increase in stage Δh in the time Δt. Then the increase in slope Δs is seen to be

$$\Delta s = \frac{\Delta h}{(V + \Delta v)\Delta t} = \frac{\dfrac{\Delta h}{\Delta t}}{V + \Delta_v};$$

that is, the increase in slope equals the rate of change of stage divided by the velocity. From this it follows that

$$\frac{Q_c}{Q_m} = \frac{\sqrt{S_c}}{\sqrt{S_c + \cfrac{\cfrac{dh}{dt}}{V}}},$$

Where Q_c is "normal" steady-flow discharge, Q_m is measured discharge, S_c is the steady-flow slope, dh/dt is the rate of change of stage, and V

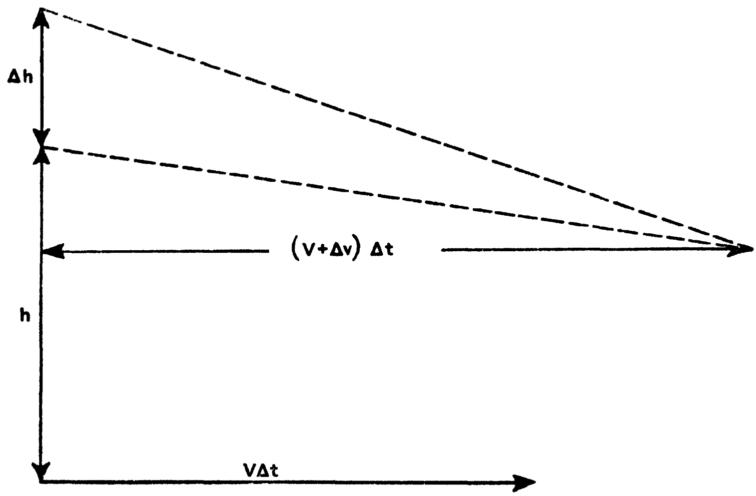


FIG. 3–13. Effect of changing stage.

is the velocity term. This is the Jones formula.* For a slope station with two gages, all the terms are observed data except Q_c (the required answer), and the velocity term V. From theoretical considerations beyond the scope of this course, the velocity term is the velocity of the flood wave rather than the measured velocity of the water. Substituting U, the velocity of the flood wave, for V and rearranging terms, we may reduce the equation to the form

$$\frac{Q_m}{Q_c} = \sqrt{1 + \frac{1}{US_c}\frac{dh}{dt}}.$$

*B. E. Jones, "A Method for Correcting River Discharge for a Changing Stage," *Water Supply Paper 375* (U.S. Geological Survey, 1916), pp. 117–130.

The estimation of the flood-wave velocity U presents a major difficulty, and the slope term also must be estimated if there is but one gage. Boyer* in his method avoids the necessity of computing the velocity of the flood wave and of computing or observing the slope. If a sufficient number of discharge measurements have been made under conditions of both rising, falling, and steady stages at a gaging station, then, from a study of the plotting of the discharge measurements against stage in the usual manner, it is possible to draw a curve of the approximate stage–constant discharge relation. Rising stage measurements will plot to the right, and falling stage measurements to the left, of the curve. The normal discharge, Q_c, for each measurement can be determined from this curve. The measured discharge Q_m and the rate of change in stage dh/dt in ft/hr are observed data, and, by substitution in the equation, the term $1/US_c$ is computed. This term for each measurement is plotted against stage, and a curve is drawn to average the points. From this curve of relation the value of $1/US_c$ is determined for each measurement, and the constant-stage discharge Q_c is computed by substitution in the formula. Application of the Boyer method is shown for the Ohio River at Wheeling, West Virginia, in Fig. 3–12.

Many gaging stations that are affected by the variable slope of changing stages are operated without a second gage, the measurements being adjusted to a normal curve, which is used to determine discharges from the gage-height record. If the loop rating is approximately symmetrical about the normal curve, the discrepancies resulting from disregarding the loop effect should not seriously affect the accuracy of the computed mean discharge for periods comprising a complete cycle of rising and falling stages.

3–9. Variable Slopes from Combined Causes

No satisfactory general solution to the case of variable backwater in conjunction with changing discharge can be stated. Often at gages operated as slope stations (two gages), the effects of changing discharge may be absorbed in the slope or fall as measured by simultaneous readings or gages at the ends of the reach. In other cases, particularly where slopes are extremely flat and backwater changes rapidly with time, no accurate record of discharge can be obtained without an adequate number of discharge measurements being made during every period of backwater. The Ohio River, which under low-water conditions is essentially a series of pools between movable dams, is a typical example. The effects of changing discharges and of variable slopes due to changes at the movable dams

*M. C. Boyer, "Stream-Gaging Procedure," *Water Supply Paper 888,* (U.S. Geological Survey, 1943), pp. 161–163.

present great difficulties to stream-gaging on many of the larger rivers, such as the Ohio, the Tennessee, and the Mississippi. These streams are important because of their size; unfortunately, they are among the few exceptions to the general rule that the effects of variable slopes may be disregarded in stream-gaging. Locations which might require special treatment for variable slopes should be avoided whenever possible in selecting gage sites.

Special Methods of Measuring Discharge

3–10. General

Although of wide applicability and use, the current-meter method of measuring discharges in open channels is but one of many reliable methods. Gravimetric and volumetric methods are of value on streams small enough that the entire flow can be directed for a short interval into a weighing tank or measuring box, and these methods are the most precise. Chemical gaging has greatest application on steep, rocky, swift streams, where a current meter cannot be used. Weirs, orifices, and Parshall flumes may give reliable results; but differences between field and laboratory conditions usually are such that these devices must be rated or checked by other gaging methods. Floats are of doubtful reliability because they fail to measure the average velocity in the vertical, but they may be used for rough determinations when no other method is readily available, as, for example, the rough estimate of flow when no equipment other than a watch is at hand. The student is referred to any standard hydraulics handbook for a discussion of other methods of measuring flows in open channels.

It is frequently desirable to estimate peak flows that have passed without being measured. Rating curves cannot always be extended accurately without reliable estimates of peak flows, and it is difficult to get to gaging stations at the times of peaks, particularly on small, flashy streams. Moreover, estimates of flood discharges are often required at points other than at gaging stations. In general, the methods of determining flood discharges are based on surveys of high-water marks and channel cross sections, estimates of friction losses or roughness coefficients, and the application of a hydraulic formula to the observed data to determine the discharge.

Easily discernible high-water marks, consisting of a line of debris or mud on the banks and on bridge piers and other structures that were reached by a flood, can usually be found immediately following the flood. Sometimes these marks can be identified years afterward, but generally it is necessary to survey or stake out the high-water line within a few days, or the marks are forever lost. Because of turbulence, standing waves, and splashing, high-water marks will not line up in a straight-line

profile; but generally the marks will lie within a narrow belt, so that a reasonably correct profile can be drawn.

3–11. Flow over Dams

Perhaps the most reliable method of estimating flood discharges after the flood has passed is by computing the flow over a dam, because so many data are available upon which to base flow coefficients. The general formula for the flow over dams is

$$Q = CLH^{3/2},$$

in which Q is the discharge in cfs, C the coefficient for the dam, L the length of crest in ft, and H the total energy head in ft. The total effective head is the static head plus the effective velocity head. Hydraulics handbooks contain values of C determined from experiments on various types and shapes of dams.

If the coefficient C is based on actual measurements, the computed discharge should be accurate; but good results can be obtained without a measured coefficient if conditions are favorable. A careful survey of high-water marks above and below the dam and measurements of the dam itself are required. Some of the factors to be considered in computing flows over dams are: angle of channel with dam; alignment of crest of dam; approach and getaway conditions; velocity of approach and extent of submergence of the dam; condition of flashboards at time of crest; uniformity of cross section of crest; flow through gates and around ends of dam; and selection of coefficients for the discharge over the dam and through the gates and wasteways.

3–12. Contracted-Opening Measurements

If a stream passes through a contracted opening, such as a bridge or culvert, having a cross-sectional area smaller than the area upstream and downstream from the opening, there is a sharp drop in the water surface at the entrance to the opening, where a part of the potential energy is converted to velocity head. The discharge can be computed by the application of the Bernoulli theorem, as follows:

Let A = Area of cross section in sq ft,
d = Depth of flow in ft,
z = Elevation of bed, in ft, above an arbitrary datum,
v = Velocity in ft/sec,
H = Drop in water surface in ft,
Q = Discharge in cfs,
h_f = Friction loss in ft in the reach between *1* and *2*,
L = Length of reach, in ft, between *1* and *2*.

Then, referring to Fig. 3–14, and using subscripts *1* and *2* to denote, respectively, the upstream area and the contracted area, we obtain

$$z_1 + d_1 + \frac{v_1^2}{2g} = z_2 + d_2 + \frac{v_2^2}{2g} + h_f. \tag{3-1}$$

Or, transposing, combining, and introducing H, we find

$$z_1 + d_1 - z_2 - d_2 \equiv H = \frac{v_2^2 - v_1^2}{2g} + h_f.$$

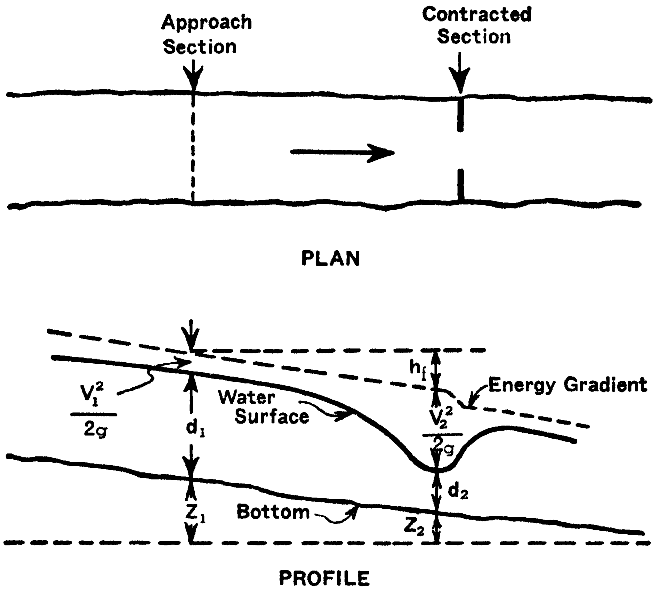


FIG. 3–14. Plan and profile of contracted opening.

But, from $Q = A_1v_1 = A_2v_2$, we have

$$v_1 = \frac{A_2v_2}{A_1},$$

whence

$$H = \frac{v_2^2\left(1 - \frac{A_2^2}{A_1^2}\right)}{2g.} + h_f;$$

and, solving, we obtain

$$v_2 = \sqrt{\frac{2g(H - h_f)}{1 - \frac{A_2^2}{A_1^2}}},$$

or

$$Q = A_2 \sqrt{\frac{2g(H - h_f)}{1 - \frac{A_2^2}{A_1^2}}}.$$

Experience has shown that the discharge is seldom as great as that given by the preceding equation, the difference being due to local losses of energy at the contraction. For any given type of contracted opening, these losses are nearly directly proportional to v_2 over a considerable range of the latter; hence it is customary to introduce an empirical coefficient and write

$$Q = kA_2 \sqrt{\frac{2g(H - h_f)}{1 - \frac{A_2^2}{A_1^2}}}. \tag{3-2}$$

The value of k for any given case may be estimated from the following approximate values:

k = 0.65 for sharp-edged orifices
= 0.80 for sharp square edges of piers and submerged structural members
= 0.95 for round-edged orifices, rounded (small radius), and chamfered edges of piers, and 45° wing walls
= 0.98 for rounded (large-radius) embankments
= 1.00 for continuous bed and banks which cause no disturbance.

If the sides of the contracted opening are of one nature and the bottom is of another, it is proper to combine two values of k from the above list, as illustrated in the example below (p. 85).

Field surveys should determine the profile of high-water marks upstream, through, and downstream from the contracted section; the cross-sectional area of the contraction and of a typical approach section; notes on channel conditions upon which to base an estimated roughness coefficient; notes on sharpness of the corners of the opening upon which to base the selection of k; and such additional information as seems desirable to determine approach and getaway conditions.

From the high-water profile it is a simple matter to determine H; values of A_1 and A_2 are available directly from the field survey; and k can be determined by the method discussed above. This accounts for all quantities in the right-hand member of Eq. (3-2), except h_f; and, for a first-trial solution, h_f may be considered to be zero. The resulting value of Q is somewhat too large but is accurate enough to permit preliminary computation of $v_1(= Q/A_1)$ and $v_2(=Q/A_2)$, which are needed for determining h_f.

For computing h_f, it is convenient to use the Manning formula,

$$s^{1/2} = \frac{nv}{1.486r^{2/3}},$$

and to consider that the average energy slope through the approach reach is the geometric mean of the slopes at its ends, so that

$$h_f = l\sqrt{s_1s_2} = \frac{ln^2}{(1.486)^2} \cdot \frac{v_1}{r_1^{2/3}} \cdot \frac{v_2}{r_2^{2/3}}. \tag{3-3}$$

The value of n, the roughness coefficient, must be based on experience and judgment. In natural channels the values range from about 0.035 to 0.060 but may be higher if there are trees and brush in the channel. Hydraulics handbooks list values of n for various conditions and may be followed if experience is lacking. A relatively large error in estimating friction losses will usually have but slight effect on the accuracy of the result in contracted-opening determinations, as friction losses are but a small part of the total energy head.

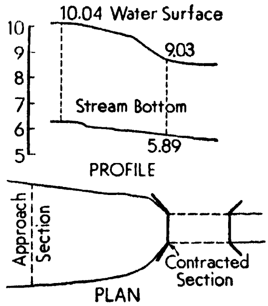
The numerical value of h_f found by solving Eq. (3–3) can now be introduced into Eq. (3–2) and a closer approximation found for Q. It may then be desirable to get new values of v_1 and v_2, recompute h_f, and use this new value for a still closer approximation of Q, this series of steps being repeated until successive approximations cause no appreciable change in the answer. It will be noted that this method of successive approximation gives results alternately too high and too low and converges rapidly, so that more than three solutions should seldom be needed.

If the shape or roughness of either the approach section or the contracted opening is highly irregular, it may be necessary to subdivide the channel, compute the hydraulic radius separately for each subsection, and assume different roughness coefficients for the various subsections. A "correction for velocity distribution" is also required, for in such an irregular section the unit kinetic energy of flow is greater than the velocity head corresponding to the mean velocity over the section. To handle these complications it is convenient to introduce the concept of "conveyance factor," which is defined as

$$K = \frac{1.486}{n} Ar^{2/3}.$$

The student will recognize that the conveyance factor is simply a shorthand way of writing all the factors except the slope factor in the right-hand member of the Manning formula,

$$Q = \frac{1.486}{n} Ar^{2/3}s^{1/2},$$



	Left	Right	Average
Water surface elevation at approach section	10.06	10.02	10.04
Water surface elevation at contracted section	9.05	9.01	9.03
Fall in water surface			*H* = 1.01

Distance between sections $l = 20.0$ ft

APPROACH SECTION

Subsection	n	A	r	$r^{2/3}$	K	K^2/A
Main channel	0.040	80.2	3.50	2.30	6,850	586,000
Right overflow	0.045	21.3	1.45	1.28	900	38,100
Totals		101.5			7,750	624,100

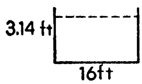
$$Cm_1 = \frac{\Sigma K^2/A}{K^2_{Total}/A_{Total}} = \frac{624{,}100}{(7{,}750)^2/101.5} = 1.055$$

CONTRACTED SECTION

Subsection	n	A	r	$r^{2/3}$	K	
Main Channel	0.025	50.2	2.25	1.72	5,130	$Cm_2 = 1.00$

COMPUTATION OF ENTRANCE LOSS COEFFICIENT

45° Wing Walls No Contraction on Bottom



6.28 × 0.95 =	5.97	
16.00 × 1.00 =	16.00	
22.28	21.97	

$$k = \frac{21.97}{22.28} = 0.99$$

COMPUTATION OF DISCHARGE

$$Q = kA_2\sqrt{\frac{2g(H-h_f)}{Cm_2 - Cm_1\dfrac{A_2^2}{A_1^2}}}$$

First Approximation, assume $h_f = 0$ .... $Q = 0.99 \times 50.2\sqrt{\dfrac{2g \times 1.01}{1.00 - 1.055 \times \dfrac{(50.2)^2}{(101.5)^2}}} = 465$ cfs

$$h_f = \frac{l\,Q^2}{K_1 K_2} = \frac{20 \times (465)^2}{7{,}750 \times 5{,}130} = 0.11\text{ft}$$

Second Approximation .... $Q = 0.99 \times 50.2\sqrt{\dfrac{2g \times (1.01 - 0.11)}{1.00 - 1.055 \times \dfrac{(50.2)^2}{(101.5)^2}}} = 438$ cfs

$$h_f = \frac{20 \times (438)^2}{7{,}750 \times 5{,}130} = 0.10\text{ft}$$

Third Approximation .... $Q = 0.99 \times 50.2\sqrt{\dfrac{2g \times (1.01 - 0.10)}{1.00 - 1.055 \times \dfrac{(50.2)^2}{(101.5)^2}}} = 442$ cfs

$h_f = 0.10$ft, as assumed

Fig. 3-15. Contracted opening measurement data and computations.

and hence is really nothing new or unfamiliar. The advantage of the shorter method of expression will become apparent in what follows.

The proper method of correcting for unequal velocity distribution is a moot question, but the differences in results by the various methods commonly used are generally of minor consequence. One commonly used correction factor, known as the "momentum coefficient" (designated C_m), is computed from the formula

$$C_m = \frac{\Sigma\left(\frac{K_i^2}{a_i}\right)}{\frac{(\Sigma K_i)^2}{\Sigma a_i}},$$

in which K is the conveyance factor, a the area, and the subscript i denotes a quantity pertaining to a specific subsection i, and the summations are to be taken over all subsections. The coefficient is applied to the velocity-head term, so that Eq. (3–1) becomes

$$z_1 + d_1 + C_{m_1}\frac{v_1^2}{2g} = z_2 + d_2 + C_{m_2}\frac{v_2^2}{2g} + h_f,$$

and the ensuing development results in

$$Q = kA_2\sqrt{\frac{2g(H - h_f)}{C_{m_2} - C_{m_1}\frac{A_2^2}{A_1^2}}}. \tag{3–4}$$

The solution is accomplished by successive approximations, as before. The computation of h_f is simplified by the use of K, for, since $Q = Ks^{1/2}$,

$$s^{1/2} = Q/k, \quad \text{and} \quad h_f = l\sqrt{s_1 s_2} = \frac{lQ^2}{K_1K_2}. \tag{3–5}$$

The example in Fig. 3–15 illustrates the above methods. Computations are self-explanatory.

3–13. Slope-Area Measurements

The slope-area method of measurement of flow consists of the determination of the slope of the energy gradient in a reach of channel, the measurement of the average cross-sectional area and length of the reach, and the estimation of the roughness factors applicable to the channel reach, so that friction losses can be computed. When these factors are known, the discharge can be computed by a flow formula such as Manning's. Perhaps the most important application of the slope-area method is in determining flood discharges after the flood has passed. The following are some of the factors to be considered in selecting a site for a slope-area determination and in making field surveys:

(1) The high-water marks should be of good quality, preferably on the ground or at points where velocities of the water were not appreciable (as on the trunks of trees in quiet backwater areas). Profiles of high-water marks on both banks should be surveyed upstream from, through, and downstream from the selected reach.

(2) The channel should be straight, approximately uniform, and preferably contracting—that is, with the downstream cross section smaller than the upstream. Generally, the more nearly a slope-area determination approaches in character a contracted-opening measurement, the more reliable the result. (In a contracting section, errors in estimating roughness coefficients are minimized. Head losses around bends or through an expanding reach are difficult to estimate. Moreover, in an expanding section there is some "loss" of velocity head, and it is always a moot question what percentage of velocity head is "recovered." The possible range is from 0 to 100 per cent, and it is common practice to assume 50 per cent recovery; but this uncertainty is sufficient reason to avoid large expansions.)

(3) The reach should be long enough that slight errors in determining stage will not have too adverse an effect on the results. For example, consider a uniform reach in which the high-water marks are such that the elevation of high water may be determined within ± 0.1 ft, and assume that it is desired to limit to ± 5 per cent the error in computed discharge. Clearly, s_{computed} may lie anywhere between $s_{\text{true}} + (0.2/l)$ and $s_{\text{true}} - (0.2/l)$, where l is the length of the reach. But the permissible limits of Q_{computed} are 1.05 Q_{true} and 0.95 Q_{true}; whence, since $Q \propto s^{1/2}$, the permissible limits of s_{computed} are 1.1025 s_{true} and 0.9025 s_{true}. The minimum permissible value of l, consistent with these limits is found from $0.9025 s_{\text{true}} = s_{\text{true}} - (0.2/l)$ to be $2.05/s_{\text{true}}$. Then, under the given conditions, if the slope is 0.01, the length l should be about 200 ft or longer. If a number of high-water marks are noted and an average high-water profile is drawn through them, the probable errors are reduced, and the length l may be shortened.

(4) The reach should be such that the roughness factors can readily be estimated. Irregularities from section to section in the reach make the selection of a roughness coefficient difficult.

(5) Several cross sections should be taken, so that independent computations can be made for several reaches. Generally, the high-water profile will be a series of approximately straight lines. Sections should be taken at the breaks in the profile.

The computation of a slope-area measurement in a completely uniform reach consists simply of substituting the observed quantities in the Manning formula,

$$Q = \frac{1.486}{n} A r^{2/3} s^{1/2},$$

for in this case the slope of the water surface is the energy gradient. In a nonuniform reach the water surface slope must be corrected for velocity head, and the computation is similar to that for a contracted opening. In general, however, Eqs. (3–2) and (3–4) will not be suitable for use, because A_2/A_1 will be too nearly equal to unity. Instead, the solution of the Manning formula can be made by successive approximations. Corrections for velocity distribution can be handled as in the case of contracted-opening measurements. If, in accordance with suggestion (5), a series of cross sections has been taken, it is customary to make independent calculations for flow in each of the subreaches and then average the results. The range of results from the separate computations should give some indication of the reliability of the final average.

The example in Fig. 3–16 illustrates a method of computing a slope-area observation. The section properties are computed from the observed data as shown. The average high-water surface is determined from a study of the high-water profiles, and the area of each cross section is computed up to its high-water elevation. (For some cases it may be desirable to warp the high-water surface; in this case the surface is assumed level.) The wetted perimeter may be scaled from the plotted cross section or computed by adding the wetted lengths between points on the section, which are equal to the square root of the sum of the square of the horizontal distance plus the square of the difference in bottom elevation. In the example, one roughness coefficient applies throughout each section and throughout the reach, and one hydraulic radius amply defines each section, so that no velocity-distribution correction is required. However, columns are shown for computations of C_m, as a reminder that it may sometimes have to be taken into account. Where needed, it may be computed, as in the example for contracted-opening computations.

Having determined the section properties, the next step is the computation of discharge through each reach. This may be carried out by successive approximations, as in the example. In the first approximation the fall in the reach F is assumed to be a measure of the energy gradient, and the energy slope is computed and substituted in Manning's formula to obtain the first approximation of discharge. This discharge is used in the second approximation to compute the velocity heads at each end of the reach. In this example the reach is contracting, and the difference in velocity heads is subtracted from the fall to obtain the energy head. (In cases of expanding channel it is customary to assume 50 per cent recovery of velocity head. In other words, the energy slope available for overcoming friction in expanding reaches has been taken as the drop between sections *plus* half the difference in velocity heads, all divided by the length of the reach.) The discharge obtained in the second approximation is used in the third, and so on, until the assumed discharge and computed discharge agree. The last step is to average the discharges

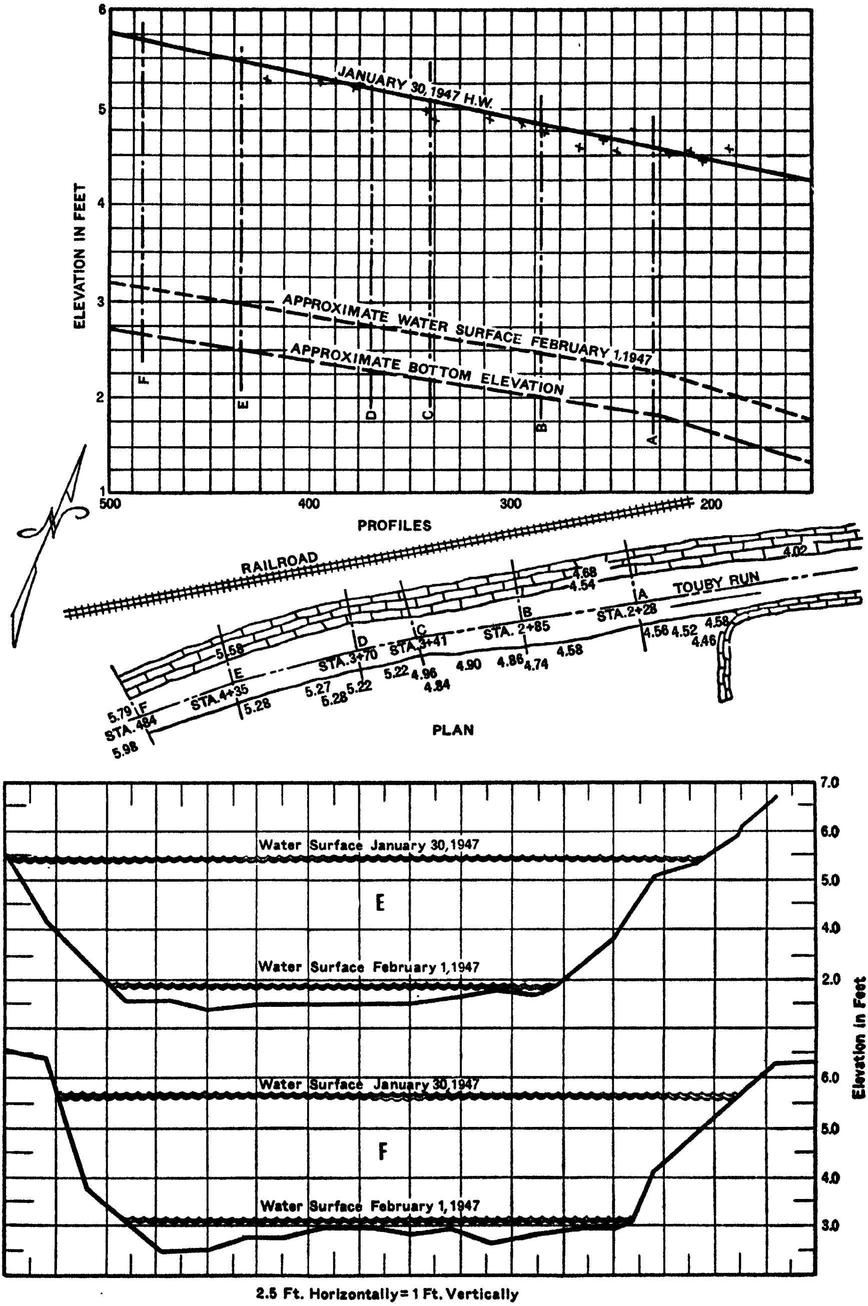


FIG. 3-16. Slope-area determination, flood of January 30, 1947, Touby Run, at Mansfield, Ohio. *(Illustration continued on next page.)*

	SECTION F, STATION 4+84, HIGH WATER ELEVATION 5.69										
	Sta.	Dist	Elev	W.S. Elev	Depth	Sum	Double Area			WP.	
L.B.	0		66	57							
	2		6.4	57							
	3		5.7	57	0						
	4	1	3.8	5.7	19	19	19			21	$R=\frac{A}{WP}=\frac{819}{35.1}$
L.W.E.	6	2	31	5.7	2.6	45	90			21	
	8	2	2.5	5.7	3.2	5.8	11 6			21	=2 33
	10	2	25	57	32	6.4	12 8			20	
	12	2	2.8	57	29	61	12.2			20	
	14	2	28	5.7	29	58	11 6			20	
	16	2	30	5.7	2.7	5.6	11 2			20	
	18	2	30	57	27	54	10 8			20	
	20	2	29	5.7	28	55	11 0			20	
	22	2	30	5.7	2.7	5.5	11.0			20	
	24	2	2.7	5.7	3.0	57	11 4			20	
	26	2	2.9	5.7	28	58	11 6			20	
	28	2	30	57	27	55	11 0			20	
	30	2	30	5.7	2.7	54	10 8			20	
R.W.E.	31	1	31	5.7	26	53	53			10	
	32	1	41	5.7	1.6	42	4.2			14	
	34	2	49	5.7	.8	2.4	48			22	
	36	2	5.7	5.7	0	.8	1.6			22	
	38		64	57							
R.B.	40		6.4	5.7							
	Totals	33					163 8			35 1	
							81 9				

Excerpts from Computations, Touby Run,
at Mansfield, Ohio. Flood of January 30, 1947

SLOPE-AREA MEASUREMENT OF TOUBY RUN AT MANSFIELD, OHIO
FOR JANUARY 30, 1947

Length of reach, feet	F-E 49	Gage height at gaging station	2.49 feet
Fall in reach (F), feet	0.22	Discharge	452 second-feet
Approximate width of channel, feet	34		

SECTION PROPERTIES

Section	n		A	R	$R^{2/3}$	$K=\frac{1.486}{n}AR^{2/3}$	$\frac{K^2}{A}$	Cm
F	030		81.9	2.33	1.76	7,140		1.00
E	030		80.4	2.30	1.74	6,920		1.00
D								
C								
B								
A								
Average Conveyance $(K_{avg})=\sqrt{K_1K_2}$				F-E 7,030				

FORMULAS

$$Cm=\frac{\Sigma u^2 da}{V^2 A} \doteq \frac{\Sigma(K^2/A)}{K^2_{Total}/A_{Total}}$$

$$\Delta\frac{CmV^2}{2g} = \text{Up-stream}\ \frac{CmV^2}{2g} - \text{Down-stream}\ \frac{CmV^2}{2g}$$

$$h_f = F + \Delta\frac{CmV^2}{2g} - h_i$$

$$V_{Upstream} > V_{Downstream}: h_i = \frac{1}{2}\Delta\frac{CmV^2}{2g}$$

$$V_{Upstream} < V_{Downstream}: h_i = 0$$

$$S=\frac{h_f}{\text{Length of reach}}$$

Fig. 3–16. *Continued. (See next page also.)*

COMPUTATION OF DISCHARGE SECTION F–E

First Approximation, assume $F = h_f = 0.22$

$$S = \frac{0.22}{49} = 0.00448$$

$$S^{1/2} = 0.0670$$

$$Q = K_{avg} S^{1/2} = 7{,}030 \times 0.0670 = 471 \text{ cfs}$$

Second Approximation, assume $Q = 471 cfs$

$$\frac{CmV^2}{2g} \text{ for Section } F = \frac{\left(\frac{471}{81.9}\right)^2}{2g} = 0.514; \text{ for Section } E = 0.532$$

$$\Delta \frac{CmV^2}{2g} = -0.018$$

$h_f = 0.22 - 0.018 = 0.202,$ $S = 0.004125,$ $S^{1/2} = 0.0643,$ $Q = 7{,}030 \times 0.0643 = 452 cfs$

Third Approximation, assume $Q = 452 cfs$

$$\frac{Cm V^2}{2g} \text{ for Section } F = 0.474; \quad \text{for Section } E = 0.491$$

$$\Delta \frac{CmV^2}{2g} = -0.017$$

$h_f = 0.22 - 0.017 = 0.203,$ $S = 0.00415,$ $S^{1/2} = 0.0644,$

$Q = 7{,}030 \times 0.0644 = 453$cfs approximately same as assumed

FIG. 3–16. *Concluded.*

for the several subreaches, weighting them equally or as circumstances may indicate.

The computed discharges for the four subreaches of Fig. 3–16 are 452, 453, 427, and 477 cfs, respectively. A computation of flow by the critical depth method for a drop structure just below the slope area reaches gave 450 cfs. It seems probable that the computed mean flow of 452 cfs is fairly accurate and within 5 per cent of the true discharge. Such accuracy is considered unusual for slope-area computations.

GRAPHICAL PRESENTATION OF RUNOFF DATA

3–14. Hydrographs

The simplest graphical form for presenting runoff data is the hydrograph, a typical example of which is shown in Fig. 3–17. Abscissas of hydrographs are always in units of time; the ordinates may be either gage height or discharge, and the corresponding curves are referred to as "stage hydrographs" and "discharge hydrographs," respectively. A suc-

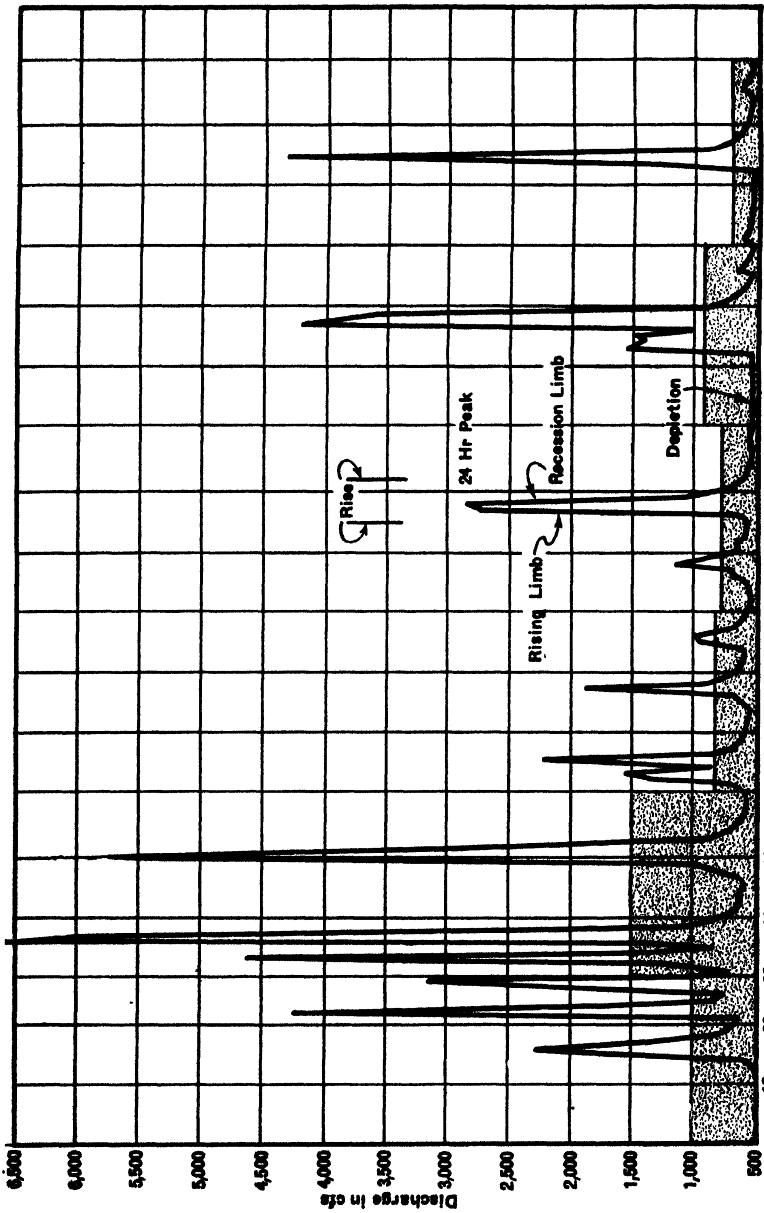


FIG. 3-17. Daily average and monthly average discharges, Big Walnut Creek, at Central College, Ohio, February–July, 1945.

cession of rising and falling stages, as, for example, that which occurred on Big Walnut Creek, between May 15 and May 22 in Fig. 3–17, is known collectively as a "rise"; and the ascending and descending portions of the curve are referred to, respectively, as the "rising limb" and the "recession limb" (or "falling limb"). The highest point on the rise is called the "peak," in the case of a discharge hydrograph, or the "crest," in the case of a stage hydrograph. For clarity, peaks should always be referred to as "instantaneous peaks" or "24-hr peaks," as the case may be. Between rises, streams as small as Big Walnut Creek (drainage area 191 sq mi) may for days at a time derive their flow from ground-water sources alone; such a period is known as a period of "depletion."

The area under a discharge hydrograph between any two points in time, being the product of a time dimension by a rate dimension, is equal to the volume of water that has passed the gage during the period.* It is customary to plot hydrographs of mean daily discharges by points joined by straight lines, as in Fig. 3–17, rather than by a succession of bars, one time unit in width. It is to be noted, however, that this convention in plotting mean daily flows may be misleading, for it makes it appear that every point on the curve truly represents an instantaneous rate of discharge; in particular, it makes the highest daily mean in each rise look like an instantaneous peak, whereas actually the latter may be considerably higher. On March 20, for example, the instantaneous peak flow of Big Walnut Creek was 7250 cfs, as against the plotted average for the day of 5730. For this reason it is good practice to include the words "daily mean" in the title of such a hydrograph. Hydrographs in which every point on the curve does truly represent the instantaneous rate of discharge are in common use for detailed studies; examples appear in Chapters 6 and 8.

3–15. Mass Curves

The "mass curve" is the integral of the hydrograph; the abscissas are in units of time, and the ordinate at any point represents the total volume of flow that has passed the gage from zero time up to that point. A typical mass curve is shown in Fig. 3–18; in this particular case the time unit is the month and the volume unit is the second-foot-day. The slope of the curve at any point measures rate of change of volume with respect to time and is thus a rate of flow; with proper conversion of units the slope

*Sometimes the discharge hydrographs of flashy streams are plotted on semilog paper, in order to accentuate the details of low-flow performance and still keep flood peaks within the limits of the sheet. The area under such a hydrograph is, of course, not a volume.

can thus be expressed in cfs (as has been done on Fig. 3–18) or gpd or any other desired rate terms. The slope of a line joining any two points of the curve represents the uniform rate of discharge that would have yielded the same total incremental volume in the same period.

This last property of the mass curve makes it extremely valuable in studying the possibility of augmenting the dry-period flow of a stream.

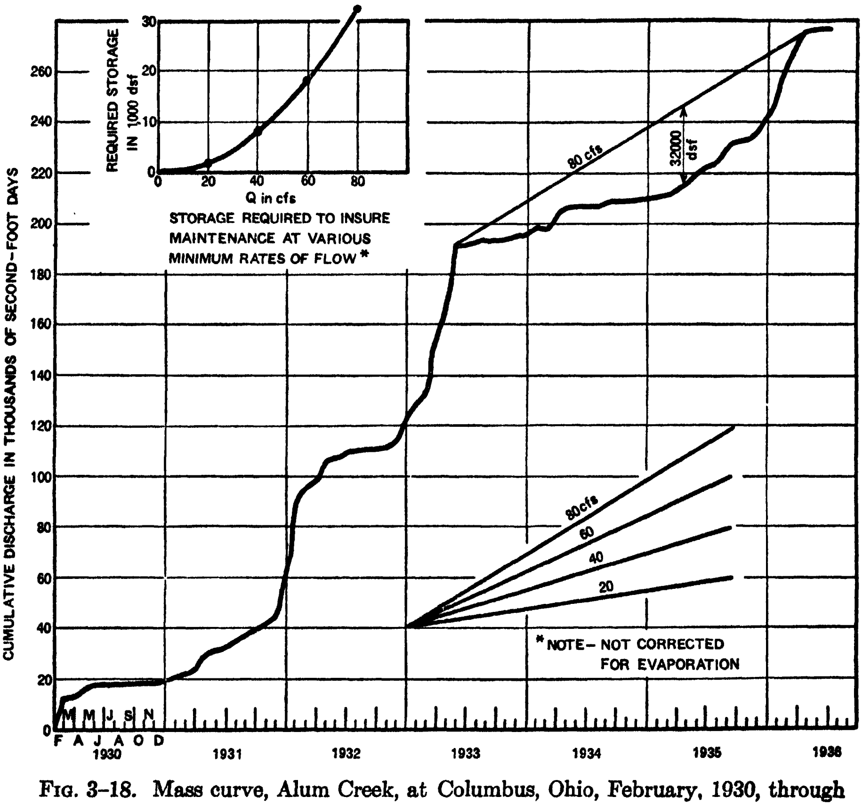


Fig. 3–18. Mass curve, Alum Creek, at Columbus, Ohio, February, 1930, through June, 1936. January, 1936, through June, 1936 is estimated.

In Fig. 3–18 a straight line has been drawn tangent to the nodes at May 30, 1933, and October 31, 1936. This line was found to have a slope of 80 cfs. If adequate reservoir storage had been available and the reservoir had been full on May 30, 1933, discharge could have been maintained at a uniform rate of 80 cfs throughout the period, and the reservoir would once again have been full on October 31, 1936. At any time during this period, the length of ordinate intercepted between the straight line and the mass curve measures directly the draft on the reservoir—that is, the amount by which the reservoir falls short of being full. The maximum

such ordinate, then, measures the total volume of storage that would have been necessary to maintain the flow at 80 cfs throughout the period. On the diagram this maximum can be scaled as 32,000 day-second-feet (which is equivalent to 64,000 acre-ft, or the amount of water that would fill, say, a reservoir with a surface area of 2 sq mi and an average depth of 50 ft). By examining various lesser rates than this, data can be assembled for plotting the curve shown in the upper left corner of Fig. 3–18, from which can be read the storage required to insure maintenance of any desired minimum rate of flow. It must be noted that the mass curve for the entire period of record must be examined to locate the critical period for each rate; in the present case the 1933–1936 period is critical for the larger rates, but the period beginning May 30, 1930, was critical for the 20-cfs rate.

It may be well to digress long enough to point out briefly how a reservoir study might proceed from this point. Topographic maps or field surveys would provide data for plotting a curve of potential reservoir capacity versus depth of water and for making cost estimates of dams and reservoirs of various capacities. By combining the cost-versus-capacity curve with the curve in the upper left corner of Fig. 3–18, corrected for evaporation,* a curve would result showing the cost of providing any given minimum rate of flow. Economic studies of the benefits attributable to various minimum rates would also be made. A comparison of costs and benefits for various rates could then be undertaken, and the most economical size of reservoir arrived at.

3–16. Duration Curves

"Duration curves" of runoff are in some ways the counterpart of the frequency curves discussed in the section on "Presenting Point Rainfall Data" in Chapter 2 (p. 28) and are constructed in the same manner. A typical duration curve, plotted semilogarithmically, is shown in Fig. 3–19. It indicates the percentage of time for the period of record during which the mean daily flow indicated was equaled or exceeded. If the period of record is sufficiently long, the curve may also be taken as representing the percentage of time that various mean daily flows may be expected to be equaled or exceeded in future. Duration curves of mean monthly flow or of mean flow for other periods may also be drawn, and

*As soon as actual reservoir areas are known, a fair estimate can be made of evaporation losses, and the curve in Fig. 3–18 corrected accordingly. This correction will often be appreciable, particularly when the draft on the reservoir continues over long periods of time. For example, assuming 2 sq mi of surface area and an average annual rate of evaporation of 3 ft in depth per year, the dependable rate from 32,000 day-second-feet of storage would be reduced from the 80 cfs shown in Fig. 3–18 to about 75 cfs.

the particular unit period used must always be carefully indicated, for the curves are widely different. For example, a stream that often goes dry for a day or two at a time might have a duration curve of mean daily flow that would drop to zero at, say, the 90-per-cent-of-time point, whereas its duration curve of mean monthly flow might never fall to zero, even at the 100-per-cent-of-time point.

Duration curves have been applied to advantage in some hydroelectric power studies and elsewhere. However, a serious shortcoming of the dura-

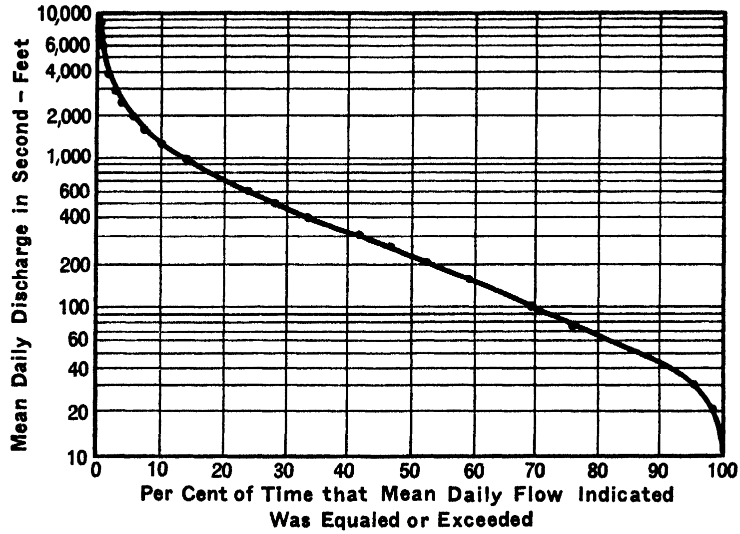


FIG. 3–19. Flow-duration curve, Little Beaver Creek, near East Liverpool, Ohio, October 1, 1922, to September 30, 1939. (From C. H. Wall and C. V. Youngquist, "Ohio Stream Drainage Areas and Flow Duration Tables," *Bull. 111*, Ohio State University, Engineering Experiment Station, 1942.)

tion curve is that it gives no information on the chronological sequence of flows, whereas such information is essential to most studies based on stream-flow data. For example, in studying problems of low flow, a knowledge of the *number of consecutive days* of deficiency is usually of much greater importance than a knowledge of the percentage of time during which a deficiency occurs; hence recourse would be had to the mass curve or the hydrograph rather than to the duration curve. Again, in studying floods, interest centers in the number and time distribution of separate flood occurrences; four floods of 1 day duration each, in four separate years, are likely to be much more damaging than one flood of 4 days' duration. The duration curve, however, makes no distinction

between the two events, since it ignores the chronological sequence of flows; so it is of little use in flood studies. Despite these limitations, duration curves are of value to preliminary studies, for they show in a general way and in one curve the flow characteristics of a stream from floods to droughts. The average slope of the duration curve is indicative of the magnitude of natural storage in the drainage basin and the location and shape of the lower end of the curve is a measure of the average ground-water conditions. Thus flat-sloped duration curves suggest large natural storage, and vice versa; while a well-sustained lower end suggests large ground-water bodies with high specific yield draining slowly into the streams (see Chap. 5, p. 123). Such inference can be drawn with confidence from duration curves of daily or weekly flow.

Agencies and Sources of Data

On June 30, 1947, 5812 gaging stations for measuring the stage and flow of lakes and surface streams were being maintained by the Geological Survey. The records are published annually in the *Water Supply Papers* of the Survey, in fourteen parts, each part covering a major drainage basin or section of the country. Each *Water Supply Paper* contains lists of gaging stations in the area for which unpublished discharge records are collected by other agencies. Advance records may be obtained in the field offices of the Survey, of which there are over one hundred, and at least one in nearly every state. For problems which require more detailed information than the daily discharges published in *Water Supply Papers* (e.g., unitgraph studies), hydrologists may usually obtain copies of the original gage-height records, applicable rating curves, and tables, etc., for the cost of assembling and blueprinting.

Reports have been published that are compilations of records of stream flow for various areas, usually a single state or drainage basin, some as *Water Supply Papers* and some as state reports. These compilations are listed in each *Water Supply Paper* on surface water supply.

Additional hydrological information is contained in *Water Supply Papers* on floods, water utilization studies, ground-water resources, and the chemical quality of water. A list of Geological Survey publications may be obtained by applying to the Director, U.S. Geological Survey, Washington, D.C. Copies of publications may be purchased at nominal cost from the Superintendent of Documents, Government Printing Office, Washington, D.C. Sets of *Water Supply Papers* are available for consultation in the field offices of the Geological Survey, and all government publications are in the principal libraries in the United States.

Other governmental agencies engaged in hydrological investigations, including stream-gaging, are the Corps of Engineers, U.S. Army; the Soil Conservation Service; the Tennessee Valley Authority; and the Inter-

national Boundary Commission. Reports of these organizations are published by the Government Printing Office and may be consulted in the principal libraries. Field offices of these services may be consulted by practicing hydrologists for additional information.

In each of several states there are state water boards, conservancy districts, or similar organizations that act as clearing houses and depositories for hydrological data of all types.

Hydrologic data other than weather, stream-flow, and ground-water records are often difficult to locate. Data collected by federal agencies are listed in *Technical Paper No. 10* of the National Resources Planning Board.*

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CHAPTER 4

ELEMENTARY RELATIONSHIPS BETWEEN PRECIPITATION AND RUNOFF

General Discussion and Definitions

4–1. The Problem

Many of the quantitative problems of hydrology are solved by discovering and applying relationships between precipitation and runoff. In part this approach is made necessary by the paucity of stream-flow data and the (relative) abundance of precipitation data. However, even if stream-gaging stations were a hundred times more numerous than they are, their records could still tell us only what *has happened* at *particular locations*; without recourse to precipitation data we should still be unable to compare the performance of two streams or operate a flood-forecasting service or plan intelligently the operation of a hydropower system or irrigation project. Types of problem involving precipitation-runoff relationships include the following:

(*a*) Estimates of the annual (or seasonal) yield of an ungaged stream

(*b*) Prediction of the total volume of storm runoff from a storm of given magnitude occurring under given conditions

(*c*) Prediction of the time distribution of runoff from a given storm

Problem (*a*) is discussed in the present chapter. Problem (*b*) is somewhat beyond the scope of an elementary text but approaches are indicated

in Chapters 6 and 8. Problem (*c*) is solved by one method in Chapter 6 and by another in Chapter 8, in both cases on the assumption that at least a short gaging record is available for the stream in question; and in Chapter 9 solutions are indicated for ungaged drainage areas.

4–2. Discharge, Runoff, and Yield

Although hydrologists do not always make a precise distinction between "discharge" and "runoff," it is important to do so, at least for the purposes of the present chapter. "Discharge" is a relatively broad term, denoting the total flow past a given cross section of any conduit, without regard to the origin of the flow. Thus we may speak of the discharge of a dredge pipe or a pumped well or an underground aquifer or a stream. "Runoff" has a much narrower connotation; strictly defined, it is that portion of the total surficial outflow from a given drainage area that has its origin in precipitation on that area. "Total surficial outflow" is to be taken as meaning all water moving out of the drainage area in surface streams, regardless of whether it has reached the stream directly from overland flow or indirectly via underground movement.

It will be seen that the *runoff* from a drainage area may be either greater or less than the discharge of the effluent stream. For example, Fig. 4–1 shows two adjacent drainage areas, whose main streams are joined by a canal diverting water from *A* to *C*. The runoff from area *I* is greater than the discharge measured at *B*, and the runoff from *II* is less than the discharge measured at *D*, because of this diversion. In this case, if the discharge of the canal is known and if there are no other complicating factors, the true runoffs for the two areas can be computed. In other cases the determination of runoff is not so simple. For example, there are drainage areas in Florida containing large artesian springs fed by rain that falls in Georgia; the discharge of the stream draining these areas is thus continuously greater than the runoff from these areas, by the amount of flow from the springs—and the latter may be difficult or impossible to measure. It should also be noted that in some localities (particularly in limestone terrains) streams have the habit of disappearing below the surface and reappearing some distance farther on; under such conditions "runoff" has little significance unless the point of measurement is carefully chosen.

Depending on circumstances, "yield" may mean either discharge or discharge plus or minus upstream diversions. Yield, discharge, and runoff may all be expressed as instantaneous rates, average rates over a given period, or total volumes for a given period.

From the above definitions of "discharge," "runoff," and "yield" it is clear that runoff is the only one of the three that can properly be related to precipitation. However, it will also be noted that the three items

become identical, provided that there are no diversions and no substantial cross-boundary inflows of ground water. There are many drainage areas in which these conditions are substantially met; and to simplify the discussion we shall confine ourselves to such areas.

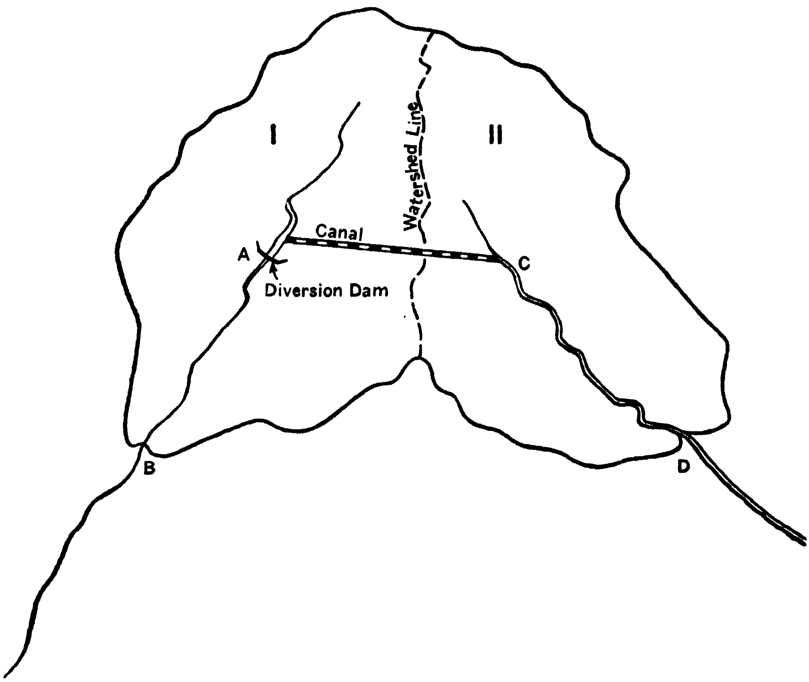

FIG. 4-1. Adjacent drainage areas with transwatershed diversion.

4-3. Runoff from an Isolated Storm

Further to clarify the basic concepts, let us consider a small drainage area on which there has been no precipitation for so long a period that the effluent stream is dry. Now let us assume that a storm occurs, and let P be the equivalent uniform depth of precipitation over the area. In such a case: (*a*) part of this rainfall remains on leaves and grass blades and later evaporates; (*b*) part of it is retained in the upper horizons of the soil, whence it gradually evaporates or is drawn into the root systems of plants and transpired through their leaves; (*c*) part of it soaks down to the zone of saturation, raising the level of the water table; and (*d*) part of it moves overland to rills and gullies and thence to the main stream and ultimately out of the area. Both (*a*) and (*b*) are totally "lost" in so far as runoff is concerned. As for (*c*), the raising of the water table induces ground-water movement toward the stream, and part of this ground water

in course of time may seep into the stream through beds and banks and thus become runoff. Not all the ground water reaches the stream, however, for on its way it is drawn on by deep-rooted vegetation and transpired into the atmosphere. Also, a part of this ground water may move out of the drainage area before it rejoins the stream, and thus it, too, is "lost" to runoff from the upstream area, though it may become surficial flow (and therefore runoff) at some downstream point. As for (*d*), the water flowing overland and in the streams, it, too, is subject to some attrition from evaporation as it flows. That portion of parts (*c*) and (*d*) that reaches the stream gage is the runoff, R, corresponding to the precipitation, P. If after the storm no further rain occurs until the stream has once again gone dry, the total volume of R derived from P is identifiable and measurable.

In a series of isolated occurrences such as this, all identical as to rainfall, one knows from experience that there will be a wide variation in runoff. An early spring storm, falling on moist soil and before the temperature has risen sufficiently to stimulate the growth of plants, will result in high runoff, whereas the same rainfall, breaking a July drought, may produce the merest trickle in the stream. Clearly, the losses are not constant; in other words, R is a function of a number of independent variables in addition to P, and their combined effect is sufficiently important to make impossible a simple correlation between precipitation and runoff, in so far as a single storm is concerned. Noting the seasonal nature of the variation of these losses, however, one is led to inquire whether their total effect on *annual* runoff may be constant, within limits, from year to year. If this proves to be the case, a simple correlation between annual precipitation and annual runoff for a given drainage area will provide a means for estimating the runoff from that drainage area in years for which precipitation records alone are available. More important still, similar studies on a number of streams may furnish a basis for estimating the annual runoff of an ungaged stream for as many years as precipitation records have been kept in the area.

4-4. The Water Year

If our studies are to be of any general usefulness, they must be applicable to *perennial* streams—that is, streams in which flow is continuous throughout the year—as well as to ephemeral streams. But in a stream that has a perennial flow we are never going to be able to identify runoff positively as deriving from any particular rainfall; the discharge of such a stream at any given moment is composed of runoff from quite a number of storms. Thus, in comparing annual precipitation with annual runoff, it is desirable to fix the year's end at a time when stream flow is at its lowest and when the amount of water in temporary storage as ground water is

least and likely to be most nearly the same from year to year. This will insure, as far as possible, that the runoff volume of the preceding 12 months is essentially all chargeable to the precipitation of the same 12 months. Records of stream flow and of ground-water levels show that for a great many drainage areas in many parts of the country this time is late September or early October. Accordingly, October 1 has been fixed as the beginning of the "water year," and the published records of the U.S. Geological Survey and of most other agencies concerned with hydrology are arranged accordingly. By common consent, the water year is designated by the calendar year in which it ends; thus "water year 1947" means the 12-month period from October 1, 1946, through September 30, 1947.

Annual Runoff as a Function of Annual Precipitation

4-5. Example

For a simple study of precipitation-runoff relationships, let us take the record of the South Branch of the Nashua River, at Clinton, Massachusetts, for the 30-yr period 1904–1933 (Table 4-1). In Fig. 4-2, the

TABLE 4-1

Water Year	Precipitation during Year (In.)	Runoff during Year (In.)	Difference or Water Loss (In.)	Water Year	Precipitation during Year (In.)	Runoff during Year (In.)	Difference or Water Loss (In.)
1904.......	47.6	23.6	24.0	1920.......	54.0	33.1	20.9
1905.......	41.7	18.2	23.5	1921.......	45.7	26.6	19.1
1906.......	46.7	21.5	25.2	1922.......	53.9	29.0	24.9
1907.......	40.4	18.1	22.3	1923.......	38.8	22.5	16.3
1908.......	47.4	27.0	20.4	1924.......	49.3	26.0	23.3
1909.......	43.3	18.7	24.6	1925.......	36.6	14.2	22.4
1910.......	37.3	17.7	19.6	1926.......	37.3	19.0	18.3
1911.......	34.2	10.8	23.4	1927.......	50.1	21.5	28.6
1912.......	41.1	21.3	19.8	1928.......	56.5	36.3	20.2
1913.......	41.4	16.8	24.6	1929.......	36.8	22.5	14.3
1914.......	41.1	22.4	18.7	1930.......	34.4	11.6	22.8
1915.......	42.1	17.1	25.0	1931.......	47.0	20.3	26.7
1916.......	47.3	27.9	19.4	1932.......	42.6	18.2	24.4
1917.......	34.4	16.9	17.5	1933.......	56.8	33.1	23.7
1918.......	41.0	17.6	23.4				
1919.......	47.0	23.5	23.5	Mean..	43.8	21.8	22.0

same data are presented graphically, with runoff being plotted as the dependent variable against precipitation. Despite the scatter of the points, it is evident that *annual* runoff is in large measure expressible as a function of precipitation amount. In other words, although some effect of other variables is still present, the trend of the relationship between

runoff and precipitation has become fairly definite. Fitting a straight line by eye to the plotted points, we obtain

$$R = P - L = P - 22.0. \tag{4-1}$$

The student should note that this chart can be used to make estimates of annual runoff of the Nashua River for years not included in the table, provided that the annual precipitation amount is given. He should also form a concept of the amount of error that may be involved in any such estimate for a given year, by subtracting the mean loss ($L = 22.0$) from

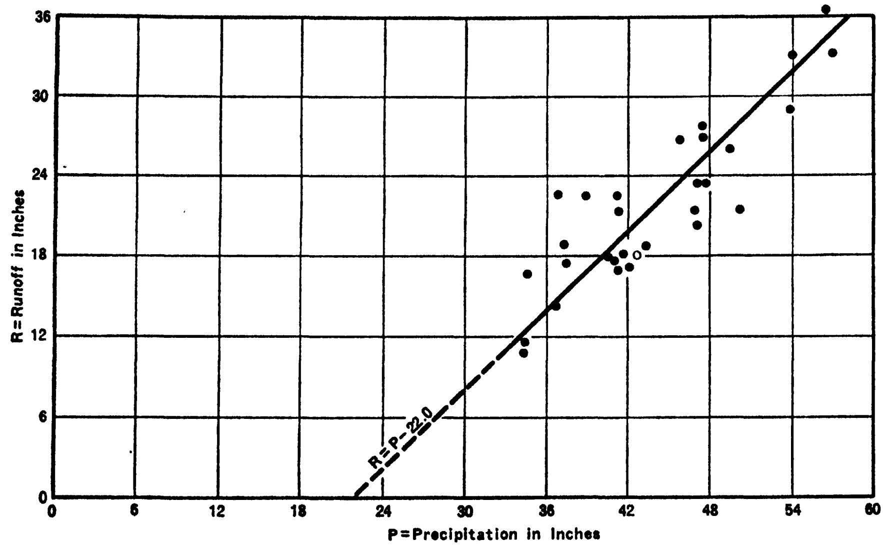

FIG. 4-2. Relation between annual runoff and annual precipitation for South Branch of Nashua River, at Clinton, Massachusetts, for the water years 1904 through 1933.

each annual precipitation in turn and comparing the remainders with the actual observed runoffs. Finally, he should compute the 5-yr moving means of precipitation and runoff, plot one against the other, and note the marked improvement in the correlation.

4-6. Mathematical Form of the Relationship

It is usually unwise to fit a curve to scattered points and define it mathematically as we have done in the preceding paragraph, unless there is some reasonable basis for the form of the equation. Assuming for the present that a straight line is acceptable, there are three forms that might have been adopted:

(a) $$R = pP,$$

where p is a percentage. This is a straight line through origin and would signify that the annual loss, as well as the annual runoff, is a constant percentage of the annual precipitation amount.

(b) $$R = P - L,$$

where L is a constant, in inches. This is a straight line passing to the right of origin and having a slope of 45°. It would signify that a certain minimum amount of rainfall is necessary before any runoff will occur and that for rainfalls greater than that minimum the annual loss is constant.

(c) $$R = qP - C,$$

where q is a percentage and C is a constant, in inches. This is a straight line passing to the right of origin and having a slope of less than 45°. It would signify that a certain minimum amount of rainfall is necessary before any runoff will occur and that for rainfalls greater than that minimum the annual loss increases somewhat with an increase in the precipitation.

As for (a), little support can be offered. It should be clear from section 4-3 that runoff must be considered more in the nature of a residual volume than as a percentage. In any storm it takes a certain amount of rainfall to wet the surface and a certain amount to soak the soil, and, *after* these requirements are satisfied, most of the remainder of the rainfall is rejected*—that is, becomes runoff—be the amount small or great. Thus the curve cannot pass through origin but must lie to the right thereof. As for (b), the next simplest form, more can be said. It satisfies the previous requirement of passing to the right of origin. Above a certain minimum precipitation it implies a constant annual loss, and it can be argued that this is at least approximately true. Over a fairly wide range of annual precipitation amounts in a given area, there should still be a fairly equal opportunity for evaporation, one year with another; moreover, barring protracted drought or extreme variations in average annual temperature, one may expect plants to transpire more or less the same amount each year. Also, barring protracted drought or extremely heavy precipitation, ground-water levels tend to return to more or less the same elevation each autumn. As for (c), the argument may proceed along the same lines as for (b), but with fewer exceptions. By allowing the loss to increase somewhat with the precipitation, it recognizes the tendency of the plant cover to grow more luxuriantly—and therefore to transpire more—in wet years than in dry ones. It also recognizes the tendency toward higher ground-water levels in wet years.

*This is an oversimplification but is adequate for present purposes.

We have chosen an equation of form (*b*) to define the relationship indicated in Fig. 4–2, because it does not appear that the plotted points justify any greater refinement. Elaborate studies using a more refined approach and involving comparisons of a number of drainage basins sometimes make appropriate the use of the more logical form (*c*). The difference is usually slight. Form (*a*), of course, is seldom, if ever, justifiable—*even if it appears to be the best fit,* as it may occasionally where only a short period of record is available.

In the preceding two paragraphs it was assumed that a straight line is an acceptable form of curve to depict the precipitation-runoff relationship. This is not entirely logical, in so far as the lower values of runoff are concerned, and for that reason the portion of the line to the left of $R = 34$ is shown dashed. Supposing that an extremely dry year were to occur on this drainage area—say, one with a rainfall of only 24 in. It is likely that a large part of the plant cover would succumb to such a drought and, therefore, that the portion of the annual loss chargeable to transpiration would be considerably reduced. As a result, the lower portion of the curve should probably bend to the left of its sketched location. This phenomenon is observable in a number of basins, particularly those in semiarid regions. We need not discuss the matter quantitatively here, but the student should note it as one more illustration of the pitfalls of extrapolating an empirical curve.

4–7. Deviations of Individual Years from the Norm

It is instructive to consider some of the possible reasons for the wide variation in water loss between years of approximately equal precipitation. Observe, for example, the difference between 1925 and 1929; in the first of these two years the runoff was 14.2 in. and the loss 22.4 in., while in the latter the values were almost exactly reversed. Where can we seek an explanation? The following may be stated:

(*a*) It is possible that the *seasonal distribution* of the precipitation was widely different in the two years. Noting that the loss in 1925 is near average, we might suspect a fairly normal distribution of rainfall in that year; the lower loss in 1929 suggests a year with a dry growing season and a concentration of precipitation in the winter and early spring, when runoff tends to be high.

(*b*) The *storm occurrences* in the two years may have been widely different in pattern. The lower loss in 1929 suggests that a large part of the precipitation in that year may have come in a few heavy storms rather than in a "normal" number of moderate storms.

(*c*) A *year-end storm* may have occurred in one year. If a heavy rainfall had occurred on September 30, 1928, the precipitation would have been credited to 1928; most of the runoff, however, would have occurred in 1929 and would have reduced the apparent losses for that year.

(*d*) The *conditions in preceding years* may have been different. The low loss in 1929 suggests that that year may have been preceded by one or more years of above normal precipitation and that part of the runoff in 1929 may properly be chargeable to precipitation of a preceding year; in other words, that at the beginning of 1929 the ground-water level may have been abnormally high and may have returned to a near-normal level by the end of that year.

(*e*) The *factors influencing evaporation and transpiration* may have been different. These factors include average annual temperature, wind velocity, humidity, and percentage of sunshine. If 1929 averaged colder, calmer, more humid, and more cloudy than 1925, at least a part of the difference in water losses might be accounted for.

Each of these possible explanations can be investigated, in turn, by referring to the detailed climatological data for the area, though in the case of (*d*) it is desirable to have some information on actual ground-water levels also. If in the aggregate they appear insufficient to account for the difference, there remains the following possibility:

(*f*) The basic data may be inadequate. That is, either the stream-flow data or the precipitation data, or both, may be in error for one or both years.

One should never be in a hurry to blame the data, but it is a source of discrepancies that must not be overlooked. Possibly a portion of the discharge record or of the precipitation record is missing and has been estimated; if so, the annual total can be considerably in error. Further, if the equivalent uniform depth of precipitation over the area has been computed from a small number of rain gages, it is always possible that a quirk of the weather—such as a cloudburst centering over one gage—can produce a figure in error by several inches.

Items (*c*) and (*d*), above, touch on a point that occasionally causes confusion. Strictly speaking, the entries in the right-hand column of the table on page 103 are not annual water losses but simply the differences between the precipitation occurring in a given year and the runoff occurring in the same year; in other words, they are the annual losses *plus or minus the net change in storage during the year*. Although differences in storage at the end of the water year may, in general, be small, they are by no means nil, and they may occasionally be large. A difference of 3 ft in ground-water level, for example, may represent a difference in storage amounting to 3 in. in depth over a drainage area. Again, if a storm occurs on September 30, 90 per cent of the water that will ultimately become runoff may still be "in storage" in the streams of the area at midnight, when the year ends.

4-8. Differences in Water Loss between Adjacent Basins

It has been found that water losses vary rather gradually over extensive regions. Thus the mean value computed for one drainage area may

be applied to a neighboring area with some degree of confidence. This provides a means for estimating the annual runoff of an ungaged area, over the entire period for which precipitation data are available. Such estimates can often be improved, however, by taking into account factors that do produce appreciable differences in the losses of adjacent basins.*

Factors that tend to produce relatively low water losses include

(*a*) Permeable surface soils, which permit rapid percolation to a water table that lies relatively deep below the surface, so that storage space is available for water which later returns to the streams through springs, seeps, and wells;

(*b*) Steep barren slopes that promote rapid runoff and thus decrease the opportunity for evaporation from the surface.

Factors that tend to produce relatively high losses include

(*a*) Groundwater levels within capillary range of the ground surface;

(*b*) Flat slopes or other factors producing poor drainage, which increase the opportunity for evaporation from surface pondage;

(*c*) Underground flow that bypasses the stream gage;

(*d*) Dense plant growth and ground cover which absorbs large amounts of water and permits it to be returned to the atmosphere by evaporation and transpiration by plants.

As an illustration of the effect of some of these factors, consider the 5-yr average water losses on three closely adjacent streams in Tuscarawas County:

Stillwater Creek	24.02 in.
Sugar Creek	25.14 in.
Sandy Creek	23.02 in.

The partially glaciated drainage basin of Sugar Creek, with its flat slopes and relatively impermeable till areas, retains a greater amount of precipitation in surface storage and as soil moisture which is subsequently lost to evaporation and transpiration, than does the more rugged basin of Stillwater Creek, where surface runoff is rapid, or the basin of Sandy Creek, where the soil is relatively loose and more rainfall appears to infiltrate to the water table and then percolate into stream channels.

Water Loss as a Function of Temperature

4-9. Variation in Water Loss over Large Areas

If the losses for the three streams listed in the preceding paragraph are averaged, the result to the nearest inch may be considered as a generalized mean annual water loss for central Ohio. Similar studies of streams

*The remainder of this paragraph is largely a quotation from "The Water Resources of Tuscarawas County, Ohio," *Ohio Water Resources Board Bull. 6*, pp. 44–45.

in all parts of the country, reported in the U.S. Geological Survey's *Water Supply Paper 846*, reveal the following generalized pattern:

Through northern Minnesota, Wisconsin, Michigan, and New York, an average annual loss of about 18 in.

Through Louisiana and southern Mississippi and Alabama, an average annual loss of about 38 in.

Between these extremes, a rather uniform increase from north to south

West of the 95th meridian, a very rapid decrease from east to west, down to 18 in. along a north-south line traversing the Dakotas, Nebraska, western Kansas, and the Panhandle. (Thence westward, average annual loss has little significance, for the rainfall fails by large margins to satisfy the evaporation that would otherwise take place.)

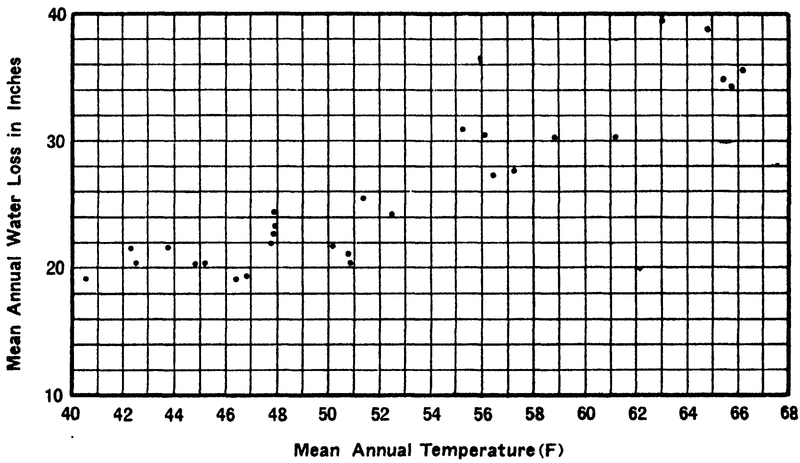

FIG. 4-3. Comparison of mean annual water loss and mean annual temperature for selected basins with mean annual precipitation in excess of 20 inches. (From *Water Supply Paper 846*.)

From this pattern it is obvious that water loss must be largely a function of average annual temperature, so long as there is enough rainfall to satisfy evaporation. The relationship between loss and temperature for a number of basins is shown in Fig. 4-3.

4-10. Importance of the Water Loss-Temperature Relationship

Particularly to the climatologist, the water loss-temperature relationship is highly significant. Rainfall, considered apart from temperature, gives no clue to climate. An annual precipitation adequate to mature crops and support forests at an annual temperature comparable to that of Minnesota would make for semiarid or even desert conditions at the

temperatures prevailing in southern Louisiana. Conversely, the rainfall that is just adequate to produce a good cotton crop in Mississippi would turn northern New York into a vast swampland. Actually, the north-south variation in rainfall in most of the eastern half of the United States is so beautifully balanced with the north-south variation in temperature that one finds "climate" and "crops" and "conditions" much the same over ranges of latitude of 500 mi or more.

The General Hydrologic Bookkeeping Equation

We have seen that differences in storage can measurably affect the computation of water loss for a period as long as a year, even when the end of the year is chosen with a view to minimizing the effect. It is to be expected, then, that the study of relationships between precipitation and runoff for shorter periods, such as a season or a month, must become increasingly complex. Items that balance one another out in the long run may have no opportunity to do so in the shorter periods and thus can no longer be neglected. To be sure that nothing is overlooked that may be of importance, it is helpful to set up a general "hydrologic bookkeeping equation," listing all possible sources of "income" (channel inflow, precipitation, ground-water inflow, etc.); all "cash on hand" (surface storage, ground-water storage, etc.); and all "disbursements" (channel outflow, evapotranspiration, ground-water outflow, etc.). Then, in attacking any particular problem, we can first examine the equation, term by term, and eliminate those terms that appear to be of relatively minor importance. A reasonably complete bookkeeping equation is the following:

$$\underbrace{S_1 + S_{s_1} + S_{g_1} + S_{ss_1}}_{\substack{\text{Cash on hand} \\ \text{at beginning} \\ \text{of period}}} + \underbrace{\int_{t_1}^{t_2} I\,dt + \int_{t_1}^{t_2} I_g\,dt + {}_1P_2}_{\substack{\text{Income} \\ \text{during} \\ \text{period}}}$$

$$= \underbrace{{}_1E_2 + {}_1T_2 + \int_{t_1}^{t_2} D\,dt + \int_{t_1}^{t_2} D_g\,dt}_{\substack{\text{Disbursements} \\ \text{during} \\ \text{period}}} + \underbrace{S_2 + S_{s_2} + S_{g_2} + S_{ss_2},}_{\substack{\text{Cash on hand} \\ \text{at end of} \\ \text{period}}}$$

in which subscript 1 denotes the beginning of period and subscript 2 the end of the period, and

S = Volume of water "in storage" (i.e., present) in the channels and reservoirs of the area under consideration;

S_s = Volume of water in storage on the surface of the ground, on leaves and pavements, etc.;

S_g = Volume of water in storage as ground water;

S_{ss} = Volume of water in storage as soil moisture (i.e., below the surface of the ground but not under full hydrostatic pressure);

I = Instantaneous rate of inflow of water to the area, above the surface, including both channel and overland flow;

I_g = Instantaneous rate of inflow of ground water across the boundaries of the area;

${}_1P_2$ = Total equivalent uniform depth of precipitation over the area between time t_1 and time t_2;

${}_1E_2$ = Total volume of water evaporated during the period;

${}_1T_2$ = Total volume of water transpired during the period;

D = Instantaneous rate of outflow of water from the area above the surface, including both channel and overland flow;

D_g = Instantaneous rate of outflow of ground water across the boundaries of the area.

Terms containing I and D are set up in integral form as a reminder that I and D are, in general, continuous variables. Actually, they can seldom be expressed mathematically, and some form of arithmetic integration by steps is usually necessary.

To illustrate the use of this equation as a check list, consider, first, the example of section 4–5, above. The R of Eq. (4–1) of that section is the term

$$\int_{t_1}^{t_2} D dt$$

of the bookkeeping equation, with $t_2 - t_1 = 1$ yr. P of Eq. (4–1) is, clearly, ${}_1P_2$. Therefore, L must be everything else. By choosing the end of the water year as September 30, a quiescent period in the hydrologic cycle, we hoped to insure that $S_1 + S_{s_1} + S_{g_1} + S_{ss_1}$ would come very near to equaling $S_2 + S_{s_2} + S_{g_2} + S_{ss_2}$; but we recognize that in any given year it may not do so. Since the area under consideration is the entire drainage basin upstream from the Clinton gage,

$$\int_{t_1}^{t_2} I dt$$

does not exist *unless there are diversions into the basin*—a possibility that should always be investigated. It would require a thorough study of the geology of the basin to determine whether or not the term

$$\int_{t_1}^{t_2} I_g dt$$

exists. If the storage terms do balance and if I and I_g do equal zero, then

$$L = {}_1E_2 + {}_1T_2 + \int_{t_1}^{t_2} D_g dt$$

and is truly the water loss. Otherwise, as pointed out on page 107, it is simply the difference between the precipitation for the year and the runoff that occurred in the same year.

The bookkeeping equation is applicable to any sort of area and is useful as a check list in a wide variety of problems. Suppose, for example, that it is desired to determine how much rain fell in an unusually heavy storm in a region where rain gages are few and that a hydrologist goes out in search of "unofficial catches." He locates a farmhouse having a cistern that receives all the runoff from the roof, and the farmer is able to give accurate information on the depth of water in the cistern before and after the storm. As an exercise, let the student define the "area under consideration" and then examine the bookkeeping equation carefully, term by term, to see how many items might affect the accuracy of the computed depth of rainfall. It is important to note how many of these possibly significant items must be evaluated by "guess."

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CHAPTER 5

THE ROLE OF THE LAND

Role of the Land in Periods of Precipitation

In Chapter 4 the multiplicity of variables affecting runoff was touched upon, and the need became apparent for investigating something more than precipitation if we are to develop useful concepts dealing with short-time runoff patterns. In this chapter we shall examine the hydrologic cycle in more detail and investigate some of the intricate paths which a particle of water may follow after it falls as rain or snow and before it returns to cloud. The methods of measurement or estimation used in determining the amounts of water which follow each of these paths are of primary interest to the hydrologist, for many hydrologic problems are solved by determining the disposition of all available water. This chapter, therefore, gives the background essential to an understanding not only of the runoff concepts to be developed in Chapters 6 and 8 but also of many other hydrologic problems.

5–1. Interception

Rain gages measure the amount of precipitation reaching the ground, but only if we define the ground as including trees and other vegetation. Recollection of the appearance of an evergreen tree after a heavy snow storm, with its boughs loaded down with a blanket of snow, will convince the student that not all precipitation reaches the soil. A considerable, greatly varying portion of precipitation is lost through "interception."

Interception losses probably are greatest for areas of coniferous forest in winter, but rain interception may be appreciable for any type of vegetation, including grasses. A light shower may be almost entirely intercepted. As the amount which can be held by leaf surfaces is limited, interception becomes less important with prolonged or heavy rains.

Not all precipitation intercepted by vegetation is lost, as some water drops from the trees and other plants following the precipitation. Appreciable amounts run down tree trunks and plant stalks to reach the soil. "Interception" therefore includes (1) interception *storage* (water that is temporarily held by vegetation but that eventually reaches the soil), as well as (2) interception *loss* (water temporarily held by vegetation and eventually evaporated). Evaporation from leaf and other surfaces of vegetation probably is slight during a storm, because of the high relative humidity, but most of the interception loss is evaporated within a short period after precipitation ends. Practically no moisture is absorbed by plants from wet leaf surfaces, the immediate refreshing effect of a summer shower on vegetation being caused largely by a reduction in transpiration rate.

The difficulties involved in making accurate determinations of interception losses are apparent. Horton* obtained records of interception losses for several types of trees by measuring the catches at various places under tree canopies and correcting for the measured flow down the trunks of the trees. The amount running down the trunks was a relatively small percentage (about 1 to 5 per cent) of the precipitation. Horton's experiments and the results of other investigators (mainly European) show that the interception loss varies with species of trees, density of stand, age of forest, rainfall rates and duration, and other factors. Rate of loss for full-grown field crops may approach that of forests but is less important generally, because of the short period that grown crops stand before harvest.

Horton concluded that interception losses range from nearly 100 per cent for light showers (of 0.1 in. or less) to about 25 per cent for heavy rains (1.0 in. or more). Total annual interception has been estimated to be as great as 40 per cent of the precipitation in mature stands of coniferous forests, and 15 per cent of the annual precipitation has been suggested as a general average for all forests. These figures may apply approximately to mature stands of trees in humid regions; but conditions vary so greatly, even in small areas, that general quantitative statements are of questionable value, especially when expressed on a percentage basis.

5-2. Infiltration

A drop of rain may (1) fall on a water surface, (2) be intercepted by vegetation, or (3) fall directly to the soil. A particle of water reaching

*Robert E. Horton, "Rainfall Interception," *Monthly Weather Review*, September 1919.

the soil may (1) be returned directly to the sky by evaporation, (2) flow overland toward a stream, or (3) enter the ground by infiltration. The amount of evaporation from either land or water surfaces during a storm is negligible because of high relative humidity. Surface runoff does not begin until the soil surface is thoroughly wet and shallow depressions are filled. Infiltration, however, begins with the first drops of rain.

Infiltration often begins at a high rate and decreases to a much lower and more or less constant rate as the rain continues. The maximum rate at which a soil, in a given condition, can absorb water is called its "infiltration capacity." The capacity of a soil to absorb water depends on (1) certain more or less permanent characteristics of the soil, largely structure and composition, and (2) conditions that vary from time to time for any given soil.

Molecular attraction is effective in pulling water into the soil until the upper part of the soil becomes wet. However, after this occurs, gravity is the principal force at work, and infiltration thus becomes largely a function of the permeability of the soil. It follows that, other things being equal, the size and arrangement of soil interstices determines to a large extent the infiltration capacity. Soils composed of large grains will, in general, have pores of greater diameter than soils made up of fine particles only. Thus infiltration of water into sands may be expected to be greater than infiltration into clays. It should be noted that the *porosity* of clays may be higher—that is, the *total pore space* may be relatively greater—but the interstices in clays are so small that the possible rate of water movement is very slow. It is the number of *large* pores in a soil rather than the *total porosity* that is of major significance in determining permeability. Of course, a soil composed largely of clay and silt may on occasion be "aggregated" to such an extent that it has large pore spaces between lumps; and, when this is the case, its infiltration capacity may rate, temporarily at least, along with the capacities of much coarser-grained soils.

Some of the conditions that may cause variations of infiltration capacity for a given soil are: (1) soil moisture content; (2) state of cultivation; (3) perforations of the surface soil and subsoil, such as those left by earthworms and decayed roots; (4) packing of the soil surface and the clogging of the soil pores by fine particles washed down from the surface by rain; (5) temperature changes; (6) shrinking and swelling of surface soils, including sun-checking during dry periods; and (7) depth to less permeable strata.

Of these items, the soil moisture content probably is the most important; this, in turn, depends on antecedent conditions, particularly antecedent rainfall and temperature. Also worthy of special note is item 7, which indicates that infiltration capacity does not depend entirely on the permeability of the surface layer of the soil. For example, a sand underlain

at shallow depths by a relatively impermeable clay may have a high initial infiltration capacity, but, after it becomes saturated, the infiltration capacity may drop nearly to zero because of the effect of the clay layer.

The *actual rate* and *total amount* of infiltration depend not only on the infiltration capacity of the soil but also on the time distribution of the rainfall. A rain of long duration and low intensity results in more infiltration than does a short rain of high intensity. Surface conditions, as reflected in the amount of water stored in shallow depressions, and topography, which determines the length of time that the water is in contact with the soil in overland flow, are also important factors.

Since the infiltration capacity is affected by several soil qualities that cannot be duplicated in the laboratory, it is usually necessary to measure it on soils in place. Among the methods are the following: (1) measurement of the rate of intake of water on very small areas bounded by metal rings or tubes; (2) measurement of the rate of intake of water in areas artificially flooded by irrigation; (3) measurement of runoff of water applied to small areas by rainfall simulators; (4) comparisons of measured precipitation with measured surface runoff; (5) lysimeter studies; and (6) detailed measurements of soil moisture content at various horizons. Items (3) and (4) above will be studied in detail in Chapter 8.

The range of infiltration capacities is quite large, and quantitative measurements are applicable only to the soils and conditions at the time of measurement. Initial infiltration capacities may exceed 10 in./hr, and the ultimate or nearly constant capacity reached toward the end of a rain may approach or reach zero. Initial capacities are difficult to estimate, as the capacity usually drops rapidly after rain starts. The ultimate capacities may be measured more accurately but may vary considerably, even for the same soil. For example, experiments on Marshall silt loam* showed that the infiltration capacities became approximately constant at points ranging from 0.21 to 1.90 in./hr.

5–3. Surface Runoff

As long as the rate at which rainfall reaches the soil surface is less than the infiltration capacity, all the available supply of water sinks into the soil. As a rain continues, plant surfaces become saturated, the interception-loss rate declines, and the infiltration capacity also decreases. The rain supply rate may exceed the infiltration capacity initially or at any time during a storm or not at all; in the latter case there is no surface runoff. Whenever the supply rate exceeds the infiltration capacity, shallow depressions begin to fill with water, and, when these depressions are

*F. L. Duley and L. L. Kelly, "Surface Condition of Soil and Time of Application as Related to Intake of Water," *Circular No. 608*, U.S. Department of Agriculture, August 1941.

filled to overflow level, water begins to move by overland flow toward streams.

The water required to fill depressions prior to the beginning of surface runoff is called the "initial detention" or "depression storage." It is often classified as an initial abstraction or initial loss, together with interception losses; but "initial" must be interpreted as meaning "prior to each period of surface runoff," rather than being limited to the beginning of the storm period; and the word "loss" is also misleading, for much of the depression storage is available for infiltration after surface runoff ceases.

The amount of water on the land surface in transit toward stream channels is called "detention storage" or "surface detention." It is available for either infiltration or overland runoff; and whether or not it reaches a stream channel depends on the infiltration capacity and retentive capacity of the surface over which it flows. Some of it may, of course, be lost by evaporation, particularly after the rain ceases. Overland flow usually reaches a tiny rivulet or channel within a short distance. The time of travel from raindrop to stream channel depends on distance, slopes, and surface conditions and may also be affected by depth of flow. Generally, the time of travel is of the order of minutes.

"Overland runoff" is defined as the water flowing over the land surface before it reaches a stream and thus is equivalent to "detention storage." "Surface runoff" is the water which reaches a stream by overland runoff. Surface runoff is a residual equal to precipitation *minus* the total evapotranspiration losses and the total ground-water flow.* It can be directly measured on small sample plats and on ephemeral streams, but on larger streams the hydrograph of stream flow is complicated by ground-water inflow and channel storage. The analysis and synthesis of the hydrograph is one of the fundamental problems of river hydrology, and all phases of this subject will be discussed in later chapters.

5-4. Subsurface Storm Flow

One might assume that a clear-cut division takes place at the soil surface between soil moisture accretion, on the one hand, and surface runoff, on the other; but this is an erroneous oversimplification. As pointed out above, not all overland flow reaches a stream channel. Conversely, some of the water that infiltrates into the soil may be diverted to become stream flow without reaching the water table. This water is called "subsurface storm flow." Under some conditions, when a relatively impermeable layer retards or prevents the percolation of water downward and diverts it back to the surface or into stream channels, sub-

*Note the distinction between "surface runoff," as here defined, and "runoff," as defined in Chap. 4, p. 100.

surface storm flow may reach stream channels almost as rapidly as does overland runoff. If a soil horizon above the water table becomes saturated, it constitutes a temporary or perched water table and may contribute a substantial amount of water to stream flow during or shortly after a storm. Longitudinal flow along a soil horizon may be substantial. A sharp division between surface runoff and ground-water seepage flow is therefore an impossibility.

5-5. The Effect of Snow

In all the above discussion the precipitation has been considered to be rain falling on unfrozen ground. The presence of snow, ice, or frost may have a great effect on the disposition of water in the hydrologic cycle. Interception losses may be much greater if the precipitation is snow. Evaporation losses from snow surfaces may also be very rapid. Much water may be stored temporarily on the ground until released by melting or evaporation. Frost in the ground may either increase or decrease the infiltration capacity, depending on the moisture content of the soil; a frozen soil with high moisture content may approach impermeability. A thick blanket of snow will prevent freezing in the ground, and snow-melt at the soil surface may keep the soil near saturation.

Freshly fallen snow is relatively "dry"—that is, of low density. The absorption of heat, even at temperatures below freezing, increases the density. Snow-melt is insignificant until maximum density is reached, at which time the snow is said to be "ripe." Additional heat then causes relatively rapid melting. Considerable rainfall can be absorbed by dry snow, but even a small amount of rain when the snow is ripe or nearly so may result in complete snow-melt and in runoff exceeding the precipitation. The rate of melting is proportional to the heat absorbed and, for this reason, is a gradual process unless hastened by rain. The ultimate disposition of snow-melt is similar to that for rain, but it should be noted that the initial losses of rainstorms, consisting of interception, wetting of surface, and depression storage, are not so important in snow-melting, one priming of the soil being sufficient, compared to the repriming losses of intermittent showers.

Role of the Land in Dry Weather

After rain ceases, infiltration continues as long as there is depression storage. Overland flow continues for a short interval. Evaporation from land and water surfaces continues, and at an increased rate. Transpiration rates also increase.

5-6. Evaporation from Water Surfaces

The measurement of evaporation from free water surfaces by so-called "direct means" requires the measurement of all inflow, outflow, and

storage, with the difference in the storage equation assigned to evaporation. There are few natural water surfaces for which evaporation losses can be adequately measured by this method. All the errors involved are thrown into the answer, and, if the evaporation losses are relatively small, minor errors in the main items result in a large error in the computed evaporation.

There are several approaches to the indirect measurement of evaporation, and there is much difference of opinion concerning this subject. The process of evaporation is complex, and various factors affecting it are difficult to estimate and to correlate with the evaporation. Factors which affect evaporation are temperature of the air and water, differences in vapor pressure, humidity of the air, solar radiation, wind movement, barometric pressure, and chemical quality of the water. Some of the approaches to this subject are largely theoretical, such as the study of the disposal of solar radiation received by a water surface or the study of rates of diffusion and character of air movement in contact with water surfaces. Others have correlated measured evaporation with other meteorological observations, so that evaporation can be estimated where certain weather records are available but direct evaporation records are not. But most work on evaporation has been by measurements on small areas, such as pans, tanks, or atmometers. These results are more or less affected by the artificial conditions prevailing, and there are great differences of opinion concerning the proper means of correcting evaporation-pan measurements to equivalent evaporation from natural water surfaces. The recommended* coefficient for reducing "standard evaporation-pan" records to natural water surface evaporation is 0.70, but the probable errors for short-period estimates with this coefficient are undoubtedly large.

Much remains to be learned about the measurement of evaporation, and, until a satisfactory direct method is discovered, some one of the indirect or empirical methods mentioned must be used.

Evaporation-pan records converted to the equivalent evaporation from natural water surfaces have been tabulated.† These records indicate that the Great Lakes region has the lowest evaporation in the United States—from 15 to 20 in./yr. In the eastern United States the range is from about 20 in. in Maine to 43 in. in Birmingham, Alabama. Along the Gulf Coast there is a decrease because of higher humidity. West of the Great Lakes there is an increase in evaporation to about 40 in. in the upper Missouri River basin. In Texas and New Mexico the evaporation is as much as 70 in., and at Salton Sea in California an extreme of 97.10 in. has been recorded. In the mountainous regions of the West, temperature is a governing factor, and it, in turn, depends on elevation. In the

*"Evaporation from Water Surfaces: A Symposium," *ASCE Transactions*, Vol. 99 (1934), pp. 671–747.

†*Ibid.*

arid part of the Northwest, evaporation is from 40 to 50 in., and in the humid areas from 20 to 40 in.

5-7. Soil Moisture

If no forces other than gravity were effective within the soil, then there would be a sharp line dividing dry soil from saturated soil. Molecular and capillary forces, however, counteract in part the effect of gravity, and thus there is no sharp line of demarcation. The upper surface of the zone of saturation is the level at which water stands in unpumped wells and is called the "water table." Above it is the zone of aeration, in which the moisture content may vary from near-dryness to near-saturation. As shown in Fig. 5-1, the zone of aeration is divided into three belts: the

FIG. 5-1. Section showing the three belts of the zone of aeration. (From O. E. Meinzer, *Water Supply Paper 494.*)

"capillary fringe" just above the water table, the "intermediate belt," and the "belt of soil water." The "belt of soil water" is defined as that part of the soil, immediately below the surface, from which water is discharged by evaporation or transpiration of plants. It may extend down to the capillary fringe, in which case there is no intermediate belt.

Before we continue our discussion of soil moisture, it would be well to define a few of the more commonly used terms.* A "capillary interstice" (with respect to water) is an interstice small enough for water to be held in it above the hydrostatic pressure level but large enough to preclude the attraction of the molecules of its walls from extending across the entire span that it occupies. A "supercapillary interstice" is larger than a capillary interstice, and a "subcapillary interstice" is one small

*Largely taken from O. E. Meinzer, *Water Supply Paper 494*, U.S. Geological Survey.

enough to hold water by adhesion, a force greatly exceeding any other usually found in subsurface water. "Hygroscopic water" is the water in the soil which molecular attraction and surface tension can hold against evaporation. The "wilting coefficient" of a soil is the moisture content (in percentage of dry weight) at the moment when (with gradual reduction in the supply of soil water) the leaves of the plants growing in the soil first undergo a permanent reduction in their water content as a result of a deficiency in the supply of soil water. The "moisture equivalent" of a soil is the moisture content which the soil, initially saturated, will retain against a centrifugal force one thousand times the force of gravity. The "field moisture capacity" is defined as the amount of water held in the soil after the excess gravitational water has drained away and after the rate of downward movement of water has materially decreased. The "specific retention," a closely allied term, is the ratio of the volume of water which a soil, initially saturated, will retain against the pull of gravity to the soil's own volume. The "specific yield" is the ratio of the volume of water which, after being saturated, a soil or rock will yield by gravity, to the soil's own volume. "Percolation" is the movement of water under hydrostatic pressure through a rock or soil, excluding turbulent flow through large openings. "Seepage" is the percolation of water through the ground surface and is essentially the same as "infiltration" (influent seepage), on the one hand, or ground-water discharge (effluent seepage), on the other. "Permeability" of a rock or soil is the property of transmitting water. A permeable soil has interconnecting interstices of capillary or supercapillary size.

Since *gravity water* (i.e., water in excess of the field moisture capacity) percolates downward more or less rapidly, depending on the permeability, the water usually available to plants is that in the range from field moisture capacity to wilting coefficient. Measurements of moisture equivalent are often used in estimating field moisture capacity. Other things being equal, a soil which has the greatest range from field moisture capacity to wilting coefficient—that is, one with a large number of interconnected interstices—will have the most water available for plant use.

It should be noted that there is no sharp demarcation between gravity water, capillary water, and hygroscopic water. Studies of the energy relations involved in removing water from a soil suggest that there is no distinct break in the nature of the factors that cause the retention or release of water by a soil. For this reason some of the above definitions are somewhat empirical.

Soil moisture has been the subject of many extensive and complex investigations, and the number of methods used approaches the number of investigators. These experiments have added much to the knowledge of the theory of capillary flow, the mechanics of percolation and soil

moisture movement, the relation of soil moisture to transpiration and evaporation, and other phases of this involved subject. No one method of investigation is applicable to all soil moisture problems, and the limitations of space will not permit more than a brief mention of a few of the many ingenious devices used in such studies.

There is but one direct method of measuring soil moisture—that of soil sampling. In this method a sample of soil is obtained, usually by means of a cylindrical soil sampler, driven into the soil at the horizon desired, the core being removed from the ground, weighed, and re-weighed after drying in an oven. By this means the actual content of moisture is determined; but, unfortunately, the soil is destroyed in the process, so that another determination at a later date under different moisture conditions cannot be made at exactly the same spot. Nevertheless, this method is often used in calibrating apparatus for indirect determinations and is the only method of checking such determinations or estimates. Indirect methods are affected by varying degrees of artificiality, as the soil column is disturbed to some extent in all cases. This is particularly true of experiments in which the soil structure is rearranged, for no amount of tamping will reproduce the original arrangement of particles.

Soil lysimeters have been used extensively in soil moisture work, and, if the soil arrangement is not too much disturbed, appear to give good results. A lysimeter is a watertight box or cylinder of soil, so arranged that moisture content can be determined by weighing the entire box or by collecting the percolation at the bottom or both. Lysimeter techniques vary, but generally the conditions are made to approximate as nearly as possible those of an undisturbed soil block. Early lysimeter experiments were made with graded or rearranged soils and, although useful for comparative purposes, are of limited application to natural conditions.

Another group of methods utilizes the relation between moisture content and electrical conductivity or resistance. To illustrate these general methods, the determination of soil moisture content by the gypsum block and the electrical bridge will be described. It is not to be inferred that this method is the best, although it is a satisfactory one for some purposes and soils. In this method a standard block of gypsum, equipped with leads extending to the ground surface, is buried in the soil at the desired horizon, the soil being disturbed as little as possible in the process. The conductivity of the block is measured by means of a bridge connected in a circuit through the block. The relation between the measured conductivity and the moisture content of the soil is determined by calibration in the laboratory as follows: A sample of soil at the horizon measured is taken, another gypsum block is inserted in it, and the soil is saturated and allowed to dry in the air. A series of conductivity measurements is made as the soil dries, and the sample is weighed at the time of each

measurement. Usually these calibrations are made in replicate. Another carefully taken volume sample is oven-dried and weighed to determine the volume-weight relationship. The cores on which the conductivity measurements were made are also oven-dried and weighed. From these data a calibration curve of conductivity against water content for the soil in question may be drawn and used for converting the conductivities measured in the field into water content.

As might be expected, the moisture content of soils, the rates of movement, and the height of capillary movements vary between wide limits. Some soils have the capillary power to draw water from a depth of as much as 100 ft; in others the capillary fringe is nearly nonexistent. Flow of water in the zone of aeration, as well as in the zone of saturation, is usually laminar, and there is a linear relation between velocity and hydraulic head. For movement in saturated soils this relationship is expressed by Darcy's law:

$$V = K\frac{H}{l},$$

in which V = velocity, H = hydraulic head, l = length of the soil column or distance betwen measurements of head, and K is the coefficient of permeability, which varies with soil characteristics and also, to some degree, with temperature.

5-8. Ground Water

Ground-water hydrology is a complex subject related closely to geology. It is equal in importance to river hydrology, and in many parts of the world is of greater importance. Any discussion of ground water, other than the bare essentials necessary to the study of ground-water flow to streams, is beyond the scope of this text.

Ground water, as defined here, is water in the zone of saturation. It may occur either under "water table conditions" or under artesian conditions. Under artesian conditions there may be no water table, the water flowing under pressure as in a pipe (see Fig. 5-2). At the upper end of an artesian aquifer or "pipe," water-table conditions usually exist. Artesian flow may occur at any break in the "pipe," in the form of artesian springs or seepage into a water-table aquifer above the artesian aquifer. Artesian flow may be for a great distance, far beyond the topographic limits of the drainage area on which the originating rain fell, and the discharge to streams from artesian aquifers (water-bearing beds) may be perennial and relatively constant. In other words, storage in artesian aquifers may be quite large and the outflow not subject to great fluctuations. For example, the flow of Silver Springs, in north-central Florida, is about 800 cfs and is relatively constant, indicating large storage in

the ground-water body. The flow is from the Ocala limestone formation, with an outcrop and recharge area extending as far as Georgia. Additions and subtractions of water from a drainage area by artesian flow complicate the hydrologic inventory.

With the exception of accretion from artesian sources, the origin of all water in the zone of saturation in a drainage basin is infiltration or percolation from rain or from the beds of streams and lakes within the limits of the basin. After entering the soil, a water particle moves downward, fairly rapidly as long as there is an excess of water but more slowly as the soil moisture lessens. The particle may be held in the belt of soil moisture for weeks or months and ultimately be returned from that belt

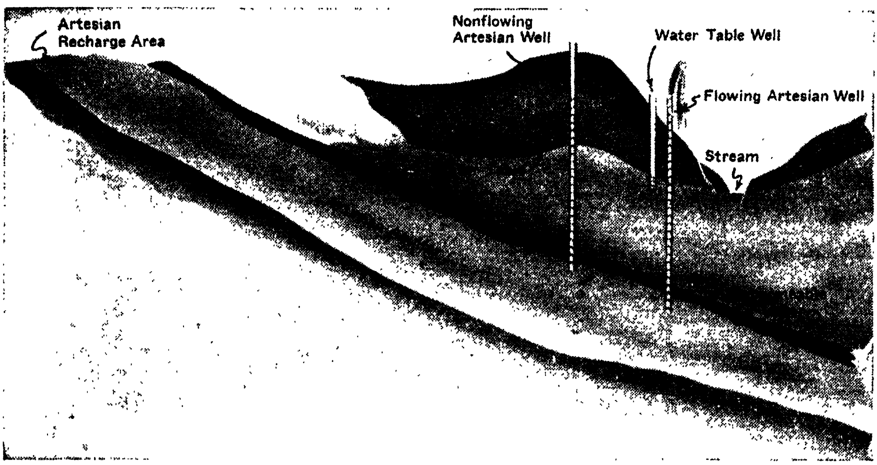

FIG. 5–2. Artesian and water table conditions.

to the atmosphere. Assuming that it continues on into the intermediate belt, it is no longer likely to be utilized by plant growth or to be evaporated, but it may be held indefinitely in the soil by capillary forces. Again on the assumption that the particle is part of an excess of water from a heavy rain, it continues down to the capillary fringe, where it serves to saturate the already moist soil just above the saturated zone, thereby raising the water table.* From here the direction of flow is nearly horizontal rather than vertical, and it probably is much slower. Velocities of ground

*It must be remembered that the drop of water may reach a perched water table or be diverted in its downward course by relatively impermeable soils and that it may pass to the stream as subsurface storm runoff, without reaching the water table. Also on its way to the stream, it may again become evapotranspiration, as soon as a point is reached where there is no intermediate belt in the zone of aeration. There is no point in the hydrologic cycle, except possibly in the intermediate belt, where water is not subject to evaporation and transpiration.

water vary greatly, but generally a rate of flow greater than a few feet a day would be exceptional. A range of from 5 ft a day to 5 ft a year has been suggested.*

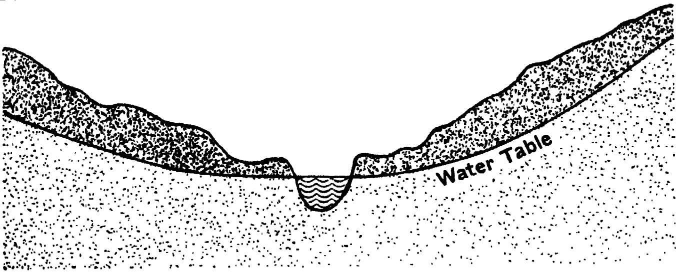

FIG. 5-3A. A "normal" water table.

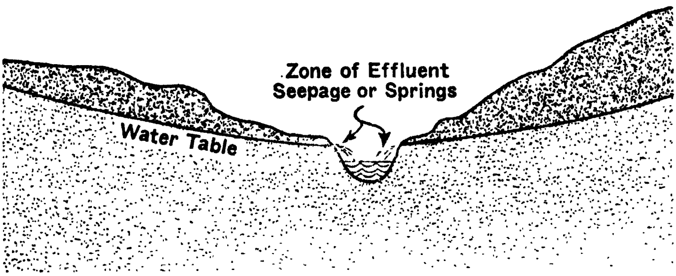

FIG. 5-3B. Water table following a period of rain.

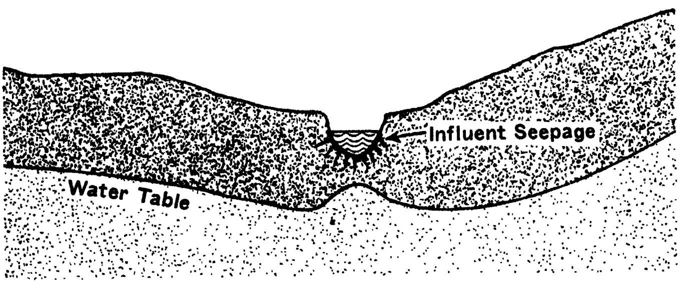

FIG. 5-3C. Water table typical of a mountain-fed stream traversing an arid, permeable region.

The water-table elevation is more or less continuously changing. Often it is as shown in Fig. 5-3A, following the contour of the land but with lesser slopes and subdued "topography" and intersecting the ground at stream level. Sometimes, however, the situation is as shown in Fig. 5-3B, and sometimes as shown in Fig. 5-3C. The amount of rise of the

*O. E. Meinzer (ed.), *Physics of the Earth*, Vol. IX: *Hydrology* (New York: McGraw-Hill Book Co., Inc., 1942), p. 449.

water table following a rain depends on the amount of water percolating to the water table and the specific yield of the saturated material. The rate of flow into the streams depends on the slope or head available and on the permeability of the aquifer, as discussed in the preceding section. The permeable soil or rock beneath the water table acts as a reservoir; percolation (or "influent seepage" from the zone of aeration) can be considered as inflow to this reservoir, and effluent seepage to streams as outflow from it. There are usually losses from the ground-water reservoir by evaporation and transpiration, either directly or by capillary movement. The capacity of the reservoir depends on the specific yield and the volume of the soil and rock zone which may become saturated. That water which will not drain from the soil by gravity (measured by specific retention) may be considered dead storage.

There are several means at hand for determining specific yields and permeabilities. In some cases laboratory tests on undisturbed soil samples give satisfactory results; but, as in many other hydrologic determinations, the best results are obtained by measurements in place, particularly if the depth to the zone of saturation is great or if the water table is in consolidated material. Pumping tests on wells which are under either water-table or artesian conditions usually give very satisfactory and accurate results, and such tests are of value to the study of river hydrology, particularly if they can be made on several wells scattered over the drainage area. (The technique of pumping tests will not be discussed here; the student will find a complete description in *Water Supply Paper 887.*) Measurements of the rate of movement of the ground water may also be made by the use of dyes or chemicals.

Records of water-level elevations in wells and the fluctuations in level are particularly pertinent to a study of ground-water storage and flow to streams. Such records preferably are continuous rather than intermittent, and wells should be equipped with water-stage recorders similar to those used in stream-gaging. It should be recognized that soils and ground-water conditions are seldom, if ever, homogeneous and that a record of water levels applies to one particular point only. In planning a program of investigation of ground-water levels, the specific objectives should be kept in mind; a few wells on a large area are sufficient for general studies of average ground-water conditions, while a large number of wells on a small area are required for detailed studies that may involve the preparation of periodic contour maps of the water table.

Stream-flow records are usually of great value to a study of ground water under water-table conditions. Advantage is taken of the fact that dry-weather flow is entirely from ground-water sources. If measurements of total flow in a stream at both ends of a reach are available, the influent

or effluent seepage in the reach during dry weather can be computed by subtraction. If, in addition, the thickness of the water-bearing beds is known and contour maps of the water table can be drawn with reasonable accuracy, these seepage measurements can be used to compute permeability and specific yields.

During the period of overland flow into a stream, it is extremely difficult to estimate the rate of effluent seepage. If the stream is above the level of the water table, there may be influent seepage into the water table or what amounts to backwater on the ground-water flow. After the cessation of overland flow, all the water passing a cross section of a stream is from channel storage plus or minus ground-water flow. This suggests the "channel-storage" method of determining ground-water flow. In this method the amount of storage in the stream channel is computed from surveys of the widths of water at various stages for the entire channel. The fundamental "bookkeeping equation" can then be applied, and the ground-water flow computed by subtracting the change in channel storage over a given time interval from the total measured flow over the same time interval at a gaging station.

At some time after a storm the rate of change of channel storage becomes negligible, and the total flow is, for all practical purposes, ground-water flow. As flow continues, the ground-water storage is depleted, the water table is lowered, and the rate of ground-water flow gradually decreases. The hydrograph of flow gradually tapers off, in what is called a "depletion curve," until another rain brings an increase in flow or a change in the rate of draft on the ground-water reservoir.

Depletion Curves. Dry-weather periods are usually short, and, for this reason, depletion curves are plotted as a combination of several arcs of the hydrograph, the arcs coinciding or overlapping in their lower parts. Fig. 5-4 shows this method of plotting for a portion of a curve for Home Creek, near New Philadelphia, Ohio.

The first step is to plot a hydrograph of daily flows, preferably on semilog paper (discharges on logarithmic scale and days on linear scale). Then on tracing paper, using the same horizontal and vertical scale, trace the lowest arc of the hydrograph (that is, ground-water flow), working backward in time from the lowest discharge to a period of surface-water runoff. Then slide the tracing paper *horizontally* until another arc of the hydrograph coincides in its lowest part with the arc already traced. Plot the second arc on top of the first. Continue this process until all the available arcs are plotted on top of one another, as shown in Fig. 5-4*A*. Disregard the upcurving portions of the individual arcs, which presumably are affected by channel storage or surface runoff or both. The remaining *continuous* arc is a "mean" or "normal" depletion curve, which

presumably represents the hydrograph that would result from ground-water flow alone over a protracted dry period.

If all losses from the ground-water reservoir were to stream flow, then one depletion curve would govern throughout the year. However, the

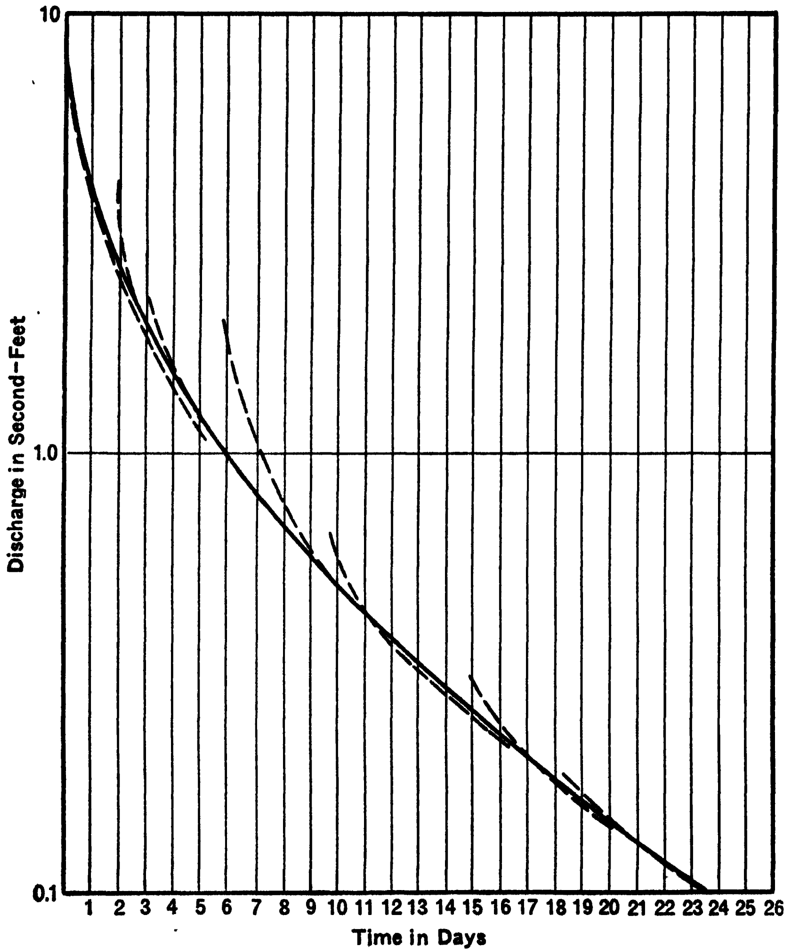

FIG. 5-4A. Constructing a depletion curve.

other losses that normally take place from the water table—evaporation, transpiration, etc.—vary widely from season to season; and, as a result, there may be appreciable differences in the shape of the depletion curve for different times of the year. In such cases a family of depletion curves may be desirable. The depletion curves for Home Creek, near New

Philadelphia, Ohio, are shown in Fig. 5-4*B*. Curve *I* applies during the winter and presumably is applicable to periods of little or no evapo-

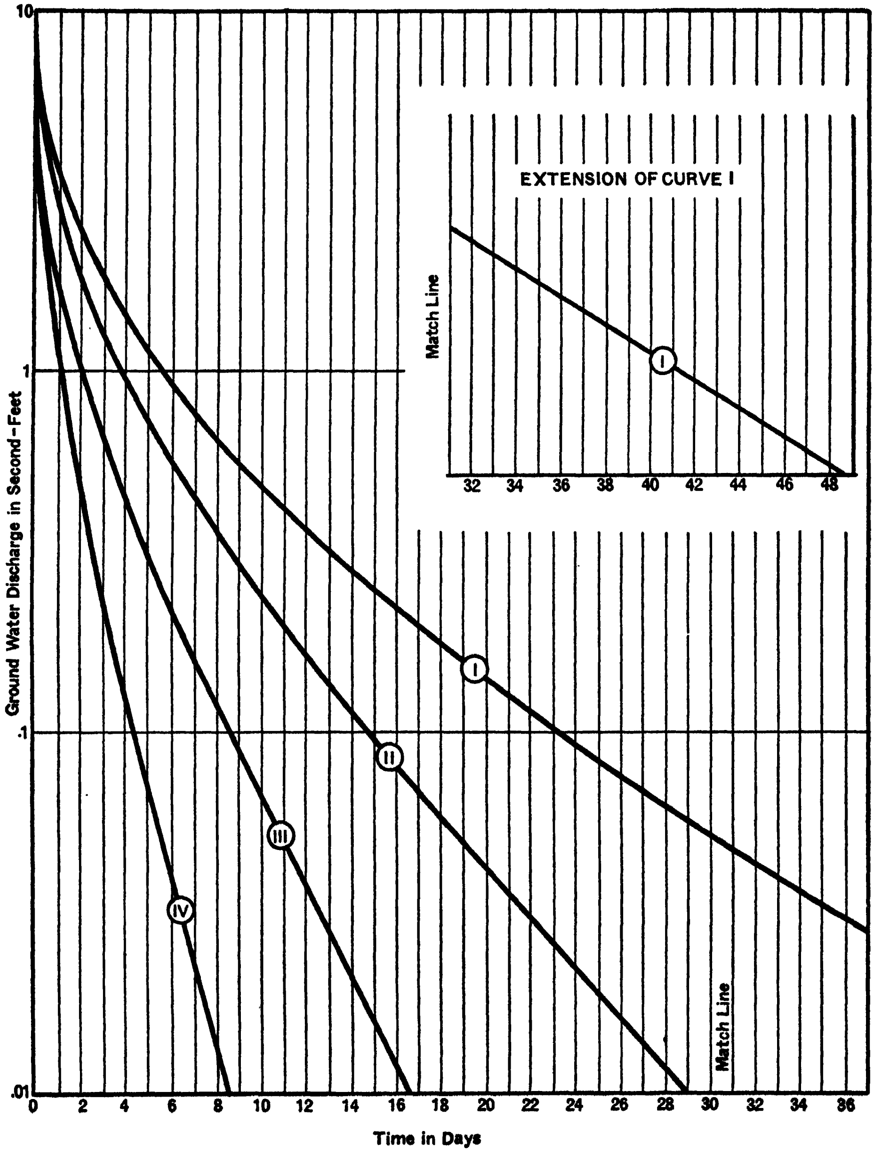

Fig. 5-4*B*. Ground-water depletion curves, Home Creek, near New Philadelphia, Ohio.

transpiration. Curve *IV* is for the hot period of late summer, and the other two curves are intermediate.

Depletion curves are the basis for estimating the rate of base or ground-water flow during periods of surface runoff or flow from channel storage. (Their use in this connection is explained in Chaps. 6 and 8.)

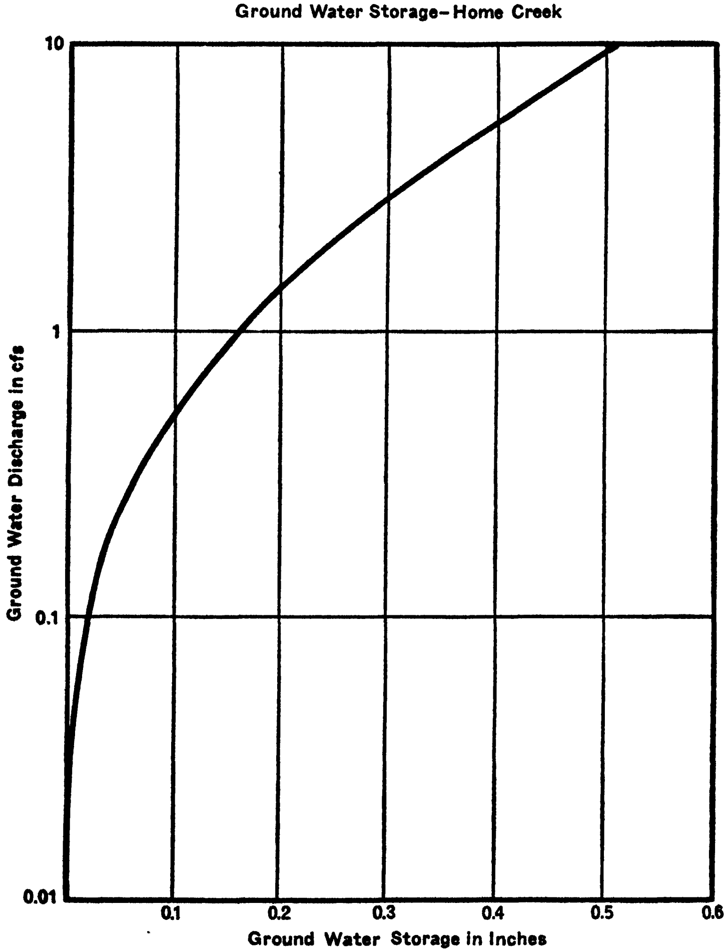

FIG. 5–4C. Storage-discharge relationship. Integration of depletion curve.

They also shed a great deal of light on the characteristics of the ground-water reservoir. For example, they may be integrated above some selected base flow, to show the relation between the rate of ground-water flow and the volume of water in storage above the base in the ground-water reservoir. Fig. 5–4C shows such a relationship computed for curve *I* of Fig. 5–4B. If the winter curve (curve *I*) can be assumed to be for conditions

of no evapotranspiration loss from the water table, then the storage curve is total storage; otherwise, the storage is that available with minimum draft by evapotranspiration. It should be noted that the curve for Home Creek is a special case, in that the flow reaches zero, so that the storage may be summed above zero rather than above some selected flow, as in the usual case. By means of the storage curve and ground-water level records plus stream-flow records, the approximate average specific yield may be computed. The storage curve also makes it possible to determine the amounts of accretion to the water table during a storm period and to approximate the amounts of evapotranspiration losses from the water table during dry-weather periods.

The slope of the depletion curve (and of the related storage curve) depends on: (1) constant controlling factors—i.e., geologic and topographic features—which include the size, shape, thickness, and slope of the water-bearing beds and their specific yield and permeability; and (2) controlling factors with seasonal variation—principally evaporation and transpiration, which are, in turn, related to temperature.

5-9. Transpiration

Here we come in contact with the biological sciences—botany, ecology, and plant physiology. Water is as necessary to plant life as it is to animal life. Without a continual water supply, a plant dies. A plant absorbs water, principally through the roots; uses the water in several involved ways in its physiologic processes; and loses the water to the atmosphere, largely through water-vapor diffusion through the pores or stomata in the process called "transpiration." The amount of water held in storage by a plant is less than 1 per cent of that lost by it during the growing season. From the hydrologic standpoint, therefore, plants are pumps which remove water from the ground and raise it to the atmosphere.

Plants vary as regards their water requirements and transpiration rates. One type of plant, the hydrophyte, lives wholly or partially submerged in water or with roots in a saturated soil (examples: water lilies, cattails). Another type, the mesophyte, grows where usually there is neither an excess nor a deficiency of water (examples: grasses, trees of humid regions). The third type, the xerophyte, is adapted to high evaporation rates and deficient water supply (examples: cacti, sagebrush). The last two types live with stems and foliage in air and with roots in aerated soil, and their roots require oxygen, so that they drown when soils are saturated for long intervals. The range of soil moisture favorable for plant growth for these two types of plants is between wilting coefficient and field moisture capacity. These factors depend principally upon the character of the soil, though wilting coefficient in a given soil may be somewhat different for different plants.

The transpiration rate depends upon physiologic factors to some extent; but, since transpiration is essentially a process of evaporation, it depends largely on the same factors that influence evaporation from land and water surfaces. These factors are solar radiation, temperature, wind, and relative humidity.

Numerous experimental methods of determining transpiration rates have been developed by research workers in the fields of botany, agriculture, silviculture, and hydrology. Their general applicability to hydrologic studies is varied, as they were developed for the study of specific problems. A few of these methods will be discussed briefly.

One method consists of weighing freshly cut parts of plants, immediately after cutting and periodically thereafter until wilting starts. It is based on the assumption that transpiration continues at the normal rate immediately after cutting. Another method is by potometer measurements. A potometer is a vessel containing water into which the cut end of a plant or leaf is inserted. After sealing, measurements are made of the amount of water removed from the vessel. A third method is by phytometers. These differ from potometers, in that they contain soil in which the whole plant is grown, thus approaching natural conditions. The closed phytometer is used extensively.

5–10. Evaporation from Land Surfaces

Evaporation from soil surfaces varies roughly in the same manner as does transpiration and usually, in nature, cannot be separated from transpiration losses. However, it has been measured as a separate item in lysimeter and tank experiments.* These experiments show that the rate of loss from saturated sands approaches or exceeds the evaporation rate from a free-water surface. For loamy or clayey soils it may be as low as 75 per cent of that value. These figures are for saturated soils, such as exist where the water table is at the soil surface. Evaporation from nonsaturated soils, such as takes place where the capillary fringe reaches the soil surface, is less than from saturated soils. Evaporation from the soil of water derived from the water table ceases when the water table falls to a greater depth than the limit of capillary rise for the soil. It will thus be seen that evaporation losses from the soil depend not only on the same factors that influence evaporation from water surfaces but also on "evaporation opportunity"—that is, on the amount of water available for evaporation. Soil evaporation rates, therefore, vary within wide limits, from approximately the maximum rate for free-water surfaces to zero. Vegetation shades the soil and reduces the soil evaporation, but transpiration usually exceeds this reduction, so that plants increase the total losses.

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Because of the nearly insurmountable difficulties of making separate measurements of the two phenomena, evaporation and transpiration are generally lumped as "evapotranspiration" and, more often than not, are calculated from the fundamental storage equation after all other factors are measured or estimated. If we neglect storage in the soil moisture and in ground water or make use of a long record, so that the effect of change in storage is negligible, then the difference between total precipitation and total stream flow is the total evapotranspiration. This has already been discussed quantitatively in Chapter 4.

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CHAPTER 6

ANALYSIS AND SYNTHESIS OF THE HYDROGRAPH BY UNITGRAPH METHODS

THE PROBLEM

Knowledge of the flood regimen of a stream is essential to the intelligent design of a wide variety of hydraulic structures, including bridge and culvert openings, reservoir spillways, and flood control works of all sorts. In some cases (e.g., the design of bridge openings) the item of information principally needed is the peak flow; in others, particularly those involving reservoir storage, the time distribution of the entire volume of the flood may be a primary factor. In either type of problem the concept of frequency may or may not be involved; that is to say, the designer may

wish to plan a structure adequate for all floods up to those with a frequency of once in 20 or 25 yr, or, on the other hand, he may wish to know the flood that may be expected as the result of some specific storm, without regard to the possible frequency of such an occurrence. It is the purpose of the present chapter to present one of the standard methods of predicting flood-period behavior. It is assumed here that flow records covering two or three floods of at least moderate intensity are available for the stream in question; later (in Chap. 9), the method will be extended to the case of streams without records. Also deferred for later consideration (in Chap. 10) is the question of frequency. Thus limited, the problem to be considered here may be stated as follows:

Given. Flood hydrographs for a number of at least moderate flood occurrences, together with adequate data on the rainfall producing the floods.

To Find. The probable hydrographs resulting from other time distributions and rates of rainfall on the same drainage basin.

ELEMENTARY UNITGRAPH THEORY

6-1. Separation of Surface Runoff and Base Flow

From the preceding chapter it is clear that any flood-period hydrograph may be considered as a hydrograph of surface runoff superposed on a hydrograph of ground-water discharge. It is also clear that such fluctuations as may exist in ground-water discharge are of a much lower order of magnitude than are the fluctuations in surface runoff and that they are subject to different laws. It is thus logical to attempt a separation of a flood-period hydrograph into two parts, so that the phenomena of surface runoff may be analyzed independently.

In the ideal case of an isolated flood on a stream whose depletion curve is well defined, this separation may sometimes be accomplished with a fair degree of accuracy. In Fig. 6-1*A*, *ab* depicts a period in which the flow of the stream is entirely ground water; that is to say, *ab* is a segment of the depletion curve. In the absence of rain, flow would continue to follow the depletion regimen indicated by the dashed line *bm*. Departure of the hydrograph from the depletion curve at *b* indicates precipitation, and the sharpness of the departure suggests that at least a part of this precipitation is appearing as surface runoff. Turning for a moment to a later portion of the hydrograph, we observe that *de* is also a segment of a depletion curve, which if extended backward in time would lie along the dashed line *dn*. Clearly, at points to the left of *d*, the hydrograph consists partially of surface runoff; clearly, also, surface

runoff terminates at *d*. Between *b* and *d*, the arcs *bm* and *nd* define, respectively, the lower and upper limits between which a line may reasonably be drawn separating ground-water flow from surface flow. Two such lines are shown as *bc′d* and *bc′ ′d*. A straight line joining *b* and *d* would also be justifiable, if little is known of the rate at which accretions to ground-water discharge take place as a result of infiltration. As will be shown later, however, one is on the side of safety in assigning too much, rather than too little, flow to the ground-water portion of the hydrograph. By "side of safety," we mean that the unitgraph will have a peak higher than the theoretically correct value. Let us then accept *bc′ ′d* as representing a reasonable division of the hydrograph and consider the portion above it as consisting entirely of surface flow. This portion is reproduced separately as Fig. 6–1*B*. The ordinate at any time, *t*, represents the rate of surface runoff at that time; the area under the curve represents the total volume of surface runoff; and *bd*, the "base length," represents the duration of surface runoff.

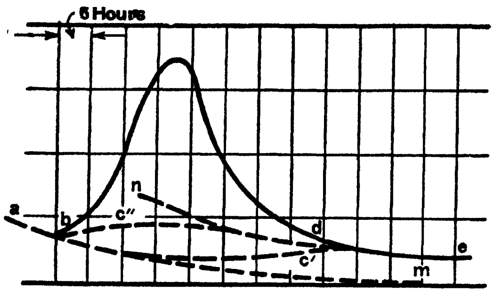

FIG. 6–1*A*. Hydrograph resulting from a 6-hr storm.

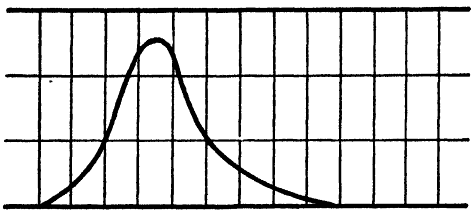

FIG. 6–1*B*. Surface runoff from Fig. 6–1*A*.

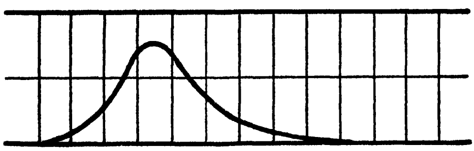

FIG. 6–1*C*. 0.6 × surface runoff from Fig. 6–1*B*.

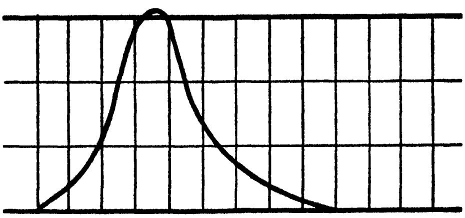

FIG. 6–1*D*. 1.2 × surface runoff from Fig. 6–1*B*.

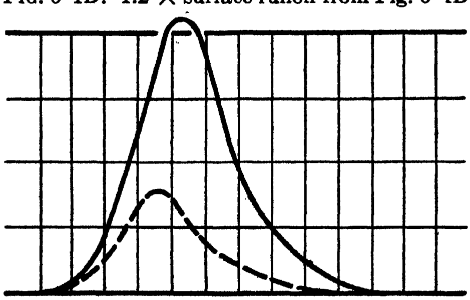

FIG. 6–1*E*. Sum of Figs. 6–1*C* and 6–1*D*, with Fig. 6–1*D* offset 6 hr to right.

6–2. The Three Basic Propositions

We are now in a position to state the three basic proposi-

tions of unitgraph theory,* all of which refer solely to the surface-runoff hydrograph:

I. For a given drainage basin, the duration of surface runoff is essentially constant for all uniform-intensity storms of the same length, regardless of differences in the total volume of surface runoff.

II. For a given drainage basin, if two uniform-intensity storms of the same length produce different total volumes of surface runoff, then the rates of surface runoff at corresponding times t, after the beginning of two storms, are in the same proportion to each other as the total volumes of surface runoff.

III. The time distribution of surface runoff from a given storm period is independent of concurrent runoff from antecedent storm periods.

In Propositions I and II the phrase "uniform-intensity storm" is to be taken as meaning a storm which produces a reasonably uniform depth of rainfall over the entire drainage basin and in which the rate of rainfall is, within rather broad limits, constant.

All these propositions are empirical. It is not possible to prove them mathematically. In fact, it is a rather simple matter to demonstrate by rational hydraulic analysis that not a single one of them is mathematically accurate. Fortunately, Nature is not aware of this. We shall have occasion later to discuss the limitations of the theory; at present, more will be gained by seeing how it works in practice.

If we assume the validity of the foregoing propositions, it follows from propositions I and II that a hydrograph can be constructed for *any* given total volume of surface runoff resulting from a uniform-intensity storm of given length, provided that there is available a hydrograph of surface runoff from *one* storm of that length. For example, if Fig. 6–1*B* represents a total surface-runoff volume of 0.5 in. over the drainage basin and if the hydrograph is desired corresponding to a surface-runoff volume of 2 in. over the drainage basin, it is necessary only to multiply all ordinates of Fig. 6–1*B* by 4—for by Proposition I the base lengths of two hydro-

*The concept of the unitgraph is due to L. K. Sherman, who first presented it in *Engineering News-Record* for April 7, 1932. Proposition I stems from earlier work by the Boston Society of Civil Engineers. It may be noted that all the essential elements of unitgraph theory were presented in 1929 in a book by J. A. Folse, entitled *A New Method of Estimating Stream Flow Based upon a New Evaporation Formula* (Carnegie Institute of Washington). This imposing volume is a monument to painstaking research; unfortunately, it is also an example of what happens when the researcher neglects to present his findings in simple, usable form. In all the papers that have been examined in the course of preparing this chapter, the present writers have found no mention of this independent development by Folse. Those interested are referred to pp. 139 ff. of the Folse book, wherein are presented the ideas of separation of base flow, reduction of gross rainfall by a variable infiltration loss, and derivation of a series of "physical constants" which are, in effect, the successive ordinates of a 24-hr unitgraph. The derivation is by least squares and exceedingly complex.

graphs are equal and by Proposition II the corresponding ordinates are proportional to the total volumes.

Moreover, from Proposition III it follows that a composite hydrograph can be constructed for any given sequence of total volumes of surface runoff resulting from a sequence of uniform-intensity storm periods of unit length, provided that there is available a hydrograph of surface runoff for one storm of unit length. For example, assume that the hydrograph of Fig. 6–1*B* represents a total surface-runoff volume of 0.5 in. over the drainage basin, resulting from a uniform-intensity storm, 6 hr in length, and let it be desired to construct the hydrograph of surface runoff resulting from a 12-hr storm, if the first 6-hr period produces a total surface-runoff volume of 0.3 in. and the second 6-hr period produces 0.6 in. In this case the ordinates of Fig. 6–1*B* are to be multiplied, first, by $0.3/0.5 = 0.6$, to produce the curve of Fig. 6–1*C* and then by $0.6/0.5 = 1.2$, to produce the curve of Fig. 6–1*D*; and, finally, the two curves are to be combined by superposition, offsetting the second curve 6 hr to the right, as shown in Fig. 6–1*E*.

It is to be noted that the hydrographs of both the preceding examples must be combined with a suitable hydrograph of ground-water discharge to give total flow. In the absence of better information, it should ordinarily be satisfactory to use for this purpose a constant value of ground-water discharge chosen from a segment of the highest depletion curve characteristic of the given stream. Even sizable errors in the base-flow quantities that are added to obtain total flow will ordinarily result in only a negligible percentage error in total flow, in the case of storms of sufficient magnitude to be of interest for flood studies.

6–3. The Unitgraph

For convenience, it is customary to develop a "unitgraph" for use in the operations just described. A unitgraph may be defined as the hydrograph of a unit volume of surface runoff that is produced by a uniform-intensity storm of unit length. The unit volume is almost invariably taken as the volume corresponding to a depth of 1 in. over the entire drainage basin. The unit length of storm chosen depends in any given case on the size of drainage area, the type of records available, and the desired precision of the study. Typical unit lengths are 1 day, 6 hr, and 1 hr; criteria for selection of unit length are discussed hereinafter. Since unit length may vary, it must always be stated if there is possibility of confusion—thus: "A 6-hr unitgraph" or "a 1-day unitgraph." From the principle of superposition (proposition III) it is obvious that a 1-day unitgraph is not identical with a 6-hr unitgraph but is, rather, one-fourth the sum of four such graphs, each of which is offset 6 hr to the right of its predecessor.

One of the principal advantages of reducing a derived surface-runoff hydrograph to unitgraph form is that it permits ready comparison with other similarly derived hydrographs. Because of inaccuracies in the basic data, nonuniform distribution of storms, and departures of drainage basin performance from unitgraph theory, it is not to be expected that unitgraphs derived from a number of isolated flood periods on a given stream will be identical. It is common practice to derive a number of such graphs—say, five or six—and plot them on a single set of coordinates, shifting individual graphs slightly to right or left as may be necessary to make their peaks coincide in time. The mean of the peaks may then be taken as the best value for the peak of a "composite" unitgraph,* and the remainder of the composite graph may be sketched in by eye, adjustments being made as necessary to insure that the area under it totals unity. If the peak values of all the individual graphs fall within ± 10 per cent of the mean peak, results would be considered excellent.

The hypothetical example of Fig. 6–1*A*,*B*, should suffice to illustrate the process of deriving a unitgraph from an isolated storm. One point, however, is worthy of special mention—namely, the selection of base length. Application of the depletion curve to the hydrograph, as described on page 136, does not always give a clear-cut indication of the point of termination of surface runoff, both because of irregularities in the actual hydrograph and because of the slow rate of convergence of the two curves. In such cases, if hydrographs of several isolated storm periods are available, the base lengths for all unitgraphs should be taken as the average of the various lengths indicated by application of the depletion curve to the individual flood hydrographs. Whatever his final choice of base length, the student should remember (1) that even experts will not always agree on a selection, (2) that a difference of one or two periods is not usually of much importance, and (3) that in case of doubt, choosing the *shorter* length will make the results conservatively high, which in most design problems is not objectionable.

Unitgraphs from Multiperiod Storms

In practice, one soon finds that the "ideal" case of the isolated storm is all too rarely encountered; a complete year of record may not yield a single such occurrence. Methods must therefore be available for deriving acceptable unitgraphs from portions of the hydrograph that contain surface runoff from more than one storm period. The procedure will be illustrated by a study of the storm of June 10–11, 1940, on the Olentangy River, Ohio (drainage area 387 sq mi).

*This is not always good practice. It may be desirable to give more weight to the unitgraphs derived from the larger storms or to the storms that distributed themselves most nearly uniformly over the drainage area.

Data available included the record of a continuous recording stream gage, a rating curve for the gaging station, and the hourly precipitation records for four recording rainfall gages located within or adjacent to the drainage area. These data are summarized graphically in Fig. 6–2, which

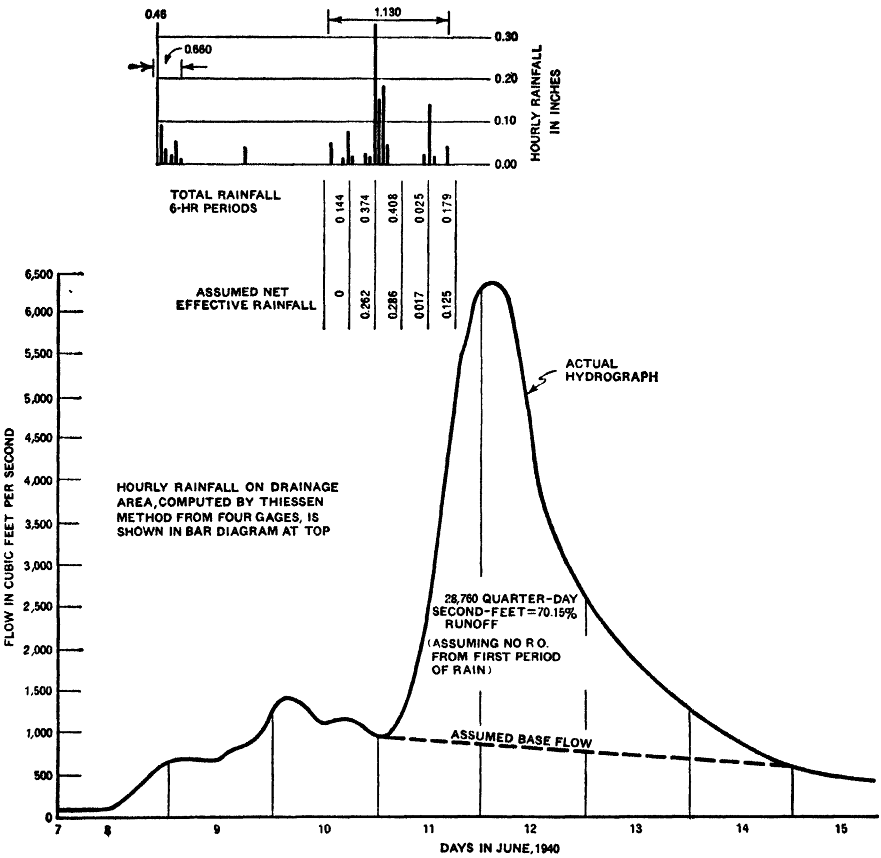

FIG. 6–2. Hydrology of Olentangy River, at Stratford, Ohio, June 8–15, 1940.

shows the discharge hydrograph for June 8–15 and the Thiessen-computed equivalent uniform depths of precipitation over the drainage area by hours for the same period of time.*

*The precipitation is actually rather unevenly distributed over the drainage area, being centered over the most rapidly concentrating portion. This is, of course, not evident in Fig. 6–2 but must be borne in mind in evaluating the resulting unitgraph.

6-4. Choosing the Unit Period

The first question to be answered is: What period shall be chosen for the unitgraph? To reduce the labor of the unitgraph computations, the longest period consistent with accuracy should be chosen. Also, for fairly obvious reasons, the unit should be either 24 hr or some convenient fraction thereof, such as 12, 6, or 3. Clearly, ordinates spaced 24 hr apart would not define the hydrograph of Fig. 6-2 with any degree of accuracy whatever. Twelve-hour ordinates would define it with moderate accuracy, but only provided that the location of the first one could be specified. On the other hand, ordinates at 6-hr intervals would define the curve with good accuracy, regardless of the hour chosen for the location of the first one. It thus appears that a 6-hr period should be adopted.

Next, and purely arbitrarily, we select the four quarters of the calendar day as the dividing points between periods. The rainfall in each 6-hr period is then totaled and entered on the graph for reference.

6-5. Base Length and Base Flow

The second question is: What is the base length of the unitgraph? The hydrograph does not offer a very well-defined break point, and we shall assume that no depletion curve is available. Under such conditions the following scheme for locating the approximate end-point is sometimes helpful:

By unitgraph theory the surface runoff contained in the natural hydrograph at the beginning of the last 6-hr period in which surface runoff occurs will be derived wholly from the last period of rainfall, while the surface runoff, 12 hr earlier, will be derived partly from the last period of rainfall and partly from each of the two immediately preceding periods. Now, although we do not yet know the shape of the unitgraph in its last three periods, we can be reasonably sure that it is sloping downward to the right and that the ordinate 18 hr from its end-point is at least twice as great as the ordinate 6 hr from its end-point, with the 12-hr ordinate somewhere in between—say, one and a half times as great. Applying these proportions to the total rainfall amounts in the last three periods of rainfall, we have:

(*a*) Surface runoff in the natural hydrograph 18 hr before end of surface runoff is proportional to $2 \times 0.179 + 1.5 \times 0.025 + 1 \times 0.408 = 0.803$.

(*b*) Same item 6 hr before end of surface runoff is proportional to $1 \times 0.179 = 0.179$,

and (*a*) is to (*b*) as 4.5 is to 1, approximately.

We can now test various points in the vicinity of the probable end of surface runoff of the natural hydrograph, setting up the work in tabular

form as shown in Table 6–1. Clearly, the residuals for assumed base flows of less than 585 are not behaving like surface runoff; it follows that the end-point is not later than hour 24 on June 14. On the other hand, there is no special reason yet apparent for setting it earlier. The base length of the unitgraph is thus initially fixed at fourteen 6-hr periods—that is, one period longer than the time from the cessation of surface runoff.

TABLE 6–1

Day	Hour	Total Instantaneous Discharge	Surface Runoff, Assuming Base Flow To Be			
			660	585	510	450
June 14	0	1205	545	...	...	...
	6	940	280	355	...	...
	12	770	110	185	260	...
	18	660	0	75	150	210
	24	585	...	0	75	135
June 15	6	510	...	...	0	60
	12	450	...	...	...	0
Ratio (a) to (b)	...	...	4.95	4.7	3.47	3.5

The third problem is to set off the base flow on the natural hydrograph. All things considered, it appears that in this case a straight line joining the beginning of the rise with the end-point of surface runoff is a reasonable assumption. This is apparently high enough to include any surface runoff remaining from the storm that occurred on the night of June 8, which we are not planning to take into account otherwise.

6–6. Net Effective Rainfall by Periods

The fourth preliminary step is the computation of the net effective rainfall for each period. The total surface runoff (area under the natural hydrograph and above the base flow line) is 28,760 quarter-day-second-feet, or 0.69 in. over the drainage area. This is 61 per cent of the total rainfall, and it might be acceptable to consider 61 per cent of the rainfall in each of the five periods (beginning at noon on June 10) as effective. However, the hydrograph gives little indication that the initial period of rain did much more than wet the ground; so it seems wiser to consider the net effective rainfall for that period as zero. The net effective rainfall in each of the remaining four periods then becomes 70 per cent of the actual.*

*The student may wonder why the rainfall from hour 6 to hour 12 on June 11 is considered at all. The answer lies in the fact that 0.025 is an equivalent uniform depth over the entire drainage area, whereas the evidence of the individual gages shows that this particular rainfall was concentrated on about one-eighth of the drainage area and fell in less than an hour. Thus it is quite reasonable to assume that it actually did produce runoff.

(The accuracy of the final unitgraph is dependent to some extent on the accuracy with which the period-by-period effective rainfall is evaluated, so careful attention must be given to this step.)

6–7. Unitgraph by Successive Approximations

From this point on, the process of deriving the unitgraph is purely mechanical. Several methods are available, one of the simplest being the one described by W. T. Collins in *Civil Engineering* for September 1939. It is a method of successive approximation, the procedure being as follows:

(*a*) *Assume* a unitgraph, and apply it to all effective rainfalls except the largest.

(*b*) Subtract the resulting hydrograph from the actual hydrograph of surface runoff, and reduce the residual to unitgraph terms.

(*c*) Compute a weighted average of the assumed unitgraph and the residual unitgraph and use it as the revised approximation for the next trial.

(*d*) Repeat steps (*a*), (*b*), and (*c*) until the residual unitgraph does not differ by more than a permissible amount from the assumed unitgraph.

For these operations it is convenient (though not essential) to introduce the concept of the "distribution graph," which is a unitgraph presented in histogram form, with the ordinate for each period representing the percentage of total surface runoff that occurs during that period.* Simultaneously, we replace the actual hydrograph with a histogram giving the equivalent uniform discharge for each period, as shown in Fig. 6–3.

The work is set down in convenient form in Table 6–2. The first seven columns are self-explanatory. The figures at the top of the next thirteen columns ("First Trial Coefficients") are the percentages of runoff in the thirteen significant periods of the initially assumed distribution graph†; they total 100 per cent. One may follow any sort of whim or fancy in selecting these percentages, though, of course, the closer one comes in the first instance to guessing the proper values, the more quickly will the approximation converge.

To begin the computation, the net rainfall for the first period (0.262 in.) is converted to quarter-day-second-feet on the drainage area (by multiplying by 41,600) and is then multiplied successively by each of the assumed percentages, yielding the values 0, 218, 654, . . . , which are recorded on the top diagonal. The significance of these quantities should be appreciated. Each of them represents the average flow, in cfs, that would result

*The distribution graph was first introduced by Merrill Bernard in "An Approach to Determinate Stream Flow," *ASCE Transactions*, 1935, p. 347.

†The fourteenth period of the distribution graph appears from Fig. 6–3 to have so small a percentage of runoff that it may be considered as zero.

in a given period from a net rainfall of 0.262 in., provided that the assumed distribution is correct. Thus, for example, from hour 12 to hour 18 on June 11, the average discharge due to the first period of effective rainfall would be 1854 cfs.

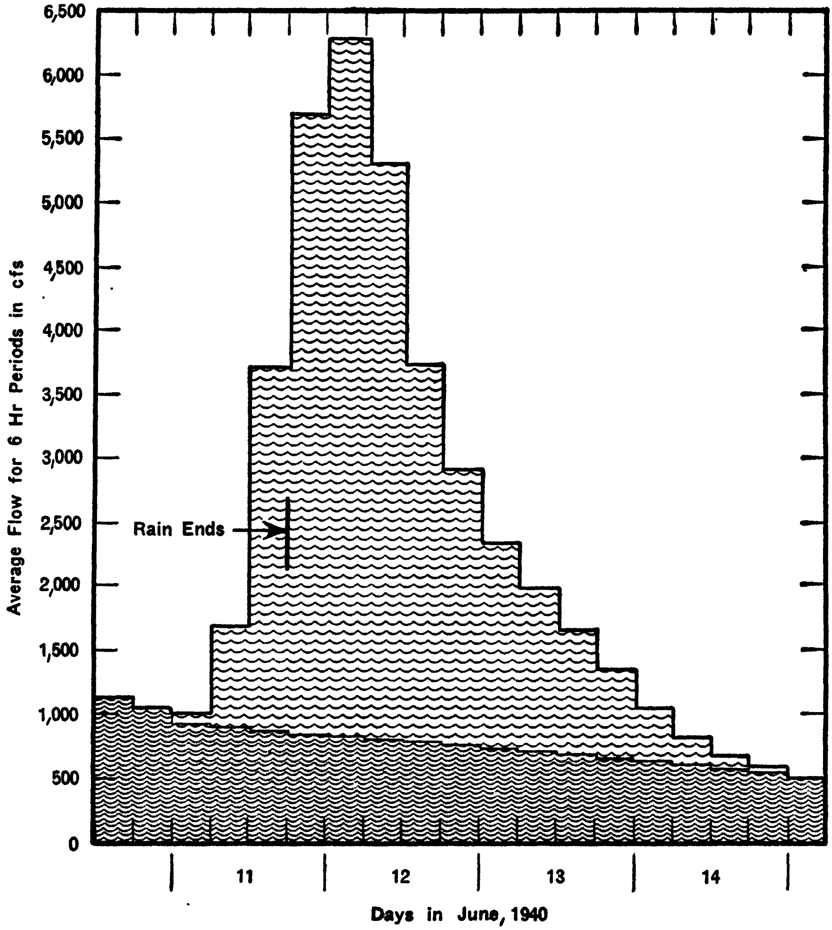

Fig. 6–3. Histograms for critical period of Fig. 6–2.

In accordance with step (*a*), the same operation is performed on the 0.025-in. rainfall and the 0.179-in. rainfall, but *not* on the 0.408-in. rainfall. There now appear on each horizontal line three discharge values, the sum of which is entered in the summation column and represents the surface runoff that would have occurred during that period, provided that the assumed distribution is correct and further provided that the

TABLE 6-2

DISTRIBUTION GRAPH DERIVATION, OLENTANGY RIVER

Period	Rain	% R.O.	Net Rain	Ave. Q (cfs)	Base	Net Q	First Trial 0	2	6	17	15	13	11	9	8	7	5	4	3	Σ	Resid-ual	Coef.
10-12-18	0.144	0	0	1130	1130	0														0	0	
18-24	0.373	70.2	0.262	1050	1050	0	0													0	0	
11- 0- 6	0.408	70.2	0.286	1010	940	70	0	218												218	−148	−0.9
6-12	0.025	70.2	0.017	1700	917	783	0	—	654											654	+129	+0.8
12-18	0.179	70.2	0.125	3715	894	2821	0	14	—	1854										1868	+953	+7.9
18-24				5680	871	4809		104	43	—	1636									1783	+3026	+25.1
12- 0- 6				6290	848	5442			313	123	—	1418								1854	+3586	+29.7
6-12				5815	825	4490				886	109	—	1199							2194	+2296	+19.0
12-18				3732	802	2930					782	94	—	981						1857	+1073	+8.9
18-24				2925	779	2146						677	80	—	872					1629	+517	+4.3
13- 0- 6				2362	756	1606							573	65	—	763				1401	+205	+1.7
6-12				2005	733	1272								469	58	—	545			1072	+200	+1.7
12-18				1678	710	968									417	51	—	436		904	+64	+0.5
18-24				1374	687	687										365	36	—	327	728	−41	−0.3
14- 0- 6				1066	664	402											261	29	—	290	+112	+0.9
6-12				852	641	211												208	22	230	−19	−0.2
12-18				712	618	94													156	156	−62	−0.5
18-24				624	595	29																
15- 0- 6				545	545	0																
						28,760																

Second Trial

	0	1	7	21	22	16	10	7	5	4	3	2	2
[Computations omitted here]													
Coefficients from 2d trial residuals	0	0.2	4.8	12	29.3	19.8	8.3	6.4	5.5	5	4.2	2.2	2.1

Third Trial

	0	0.7	5.9	16.5	25.7	17.9	9.1	6.7	5.2	4.5	3.6	2.1	2.0
Coefficients from 3d trial residuals	0	1.2	8.6	16.3	25.9	20.7	6.2	4.9	5.0	4.1	3.7	1.8	1.7
Adopted values	0	1	7.2	16.4	25.8	19.3	7.6	5.8	5.1	4.3	3.6	2.0	1.9

0.408-in. rainfall had not occurred. The entries in the summation column, subtracted from the corresponding entries in the net-flow column, yield the "residuals," which represent the rates of flow that would have had to be provided by the omitted rainfall to balance the initial hydrograph. In the final column the residuals have been converted to percentages by dividing by $0.408 \times 41{,}600$; if the arithmetic is correct, this column should total approximately 100 per cent.

Application of step (*c*) supplies the percentages for the second trial. There is no point in being extremely precise in computing the individual averages; it is important, however, to adjust them until they total 100 per cent.

In the present case, three trials were sufficient to define the distribution graph, the criterion being that the residual percentage in the peak period should be within 1 per cent of the corresponding trial percentage. It will be noted that, except in the vicinity of the peak, the distribution had converged reasonably well on the second trial.

The derived distribution graph can now be converted to unitgraph form, as shown in Fig. 6–4, by drawing a smooth curve through the histogram and reinstating the cfs scale for the ordinates.*

The student should appreciate that many thoroughly acceptable unitgraphs are derived with much less attention to detail than has been given in the preceding paragraphs. Nevertheless, experience is the only proper justification for short cuts. Until one has become thoroughly familiar with the effects of various approximations, it is not wise to ignore detail.

Applications of the Unitgraph

6–8. Introductory—Estimating Net Effective Rainfall

Among the principal uses of the unitgraph are (1) the estimation of hypothetical major floods, (2) short-term flood forecasting, and (3) the filling-in of missing discharge records. These are all discussed below. First, however, it is important to note that, in connection with any application of the unitgraph, there must be available some method of estimating the net effective rainfall, for unitgraph theory of itself does not provide any clue to this factor. One common method is to apply a percentage factor to the actual rainfall. The choice of percentage should take into account (*a*) the general characteristics of the drainage area, (*b*) the season of the year, (*c*) the actual depth of rainfall for the period in question, (*d*) the amount of rain that has fallen in the several days preceding that

*The peak flow of this unitgraph is about 45 per cent higher than the peaks of other Olentangy unitgraphs derived from storms in which the rainfall is distributed nearly uniformly over the drainage area. Not all streams are so sensitive to nonuniform distribution as is the Olentangy, which has two principal branches, one of them steep and quick-cresting and the other one flat and relatively slow to rise.

period, and (*e*) the purpose of the study. In the present state of the art, it is probably wise for every hydrologist to compile for himself data on

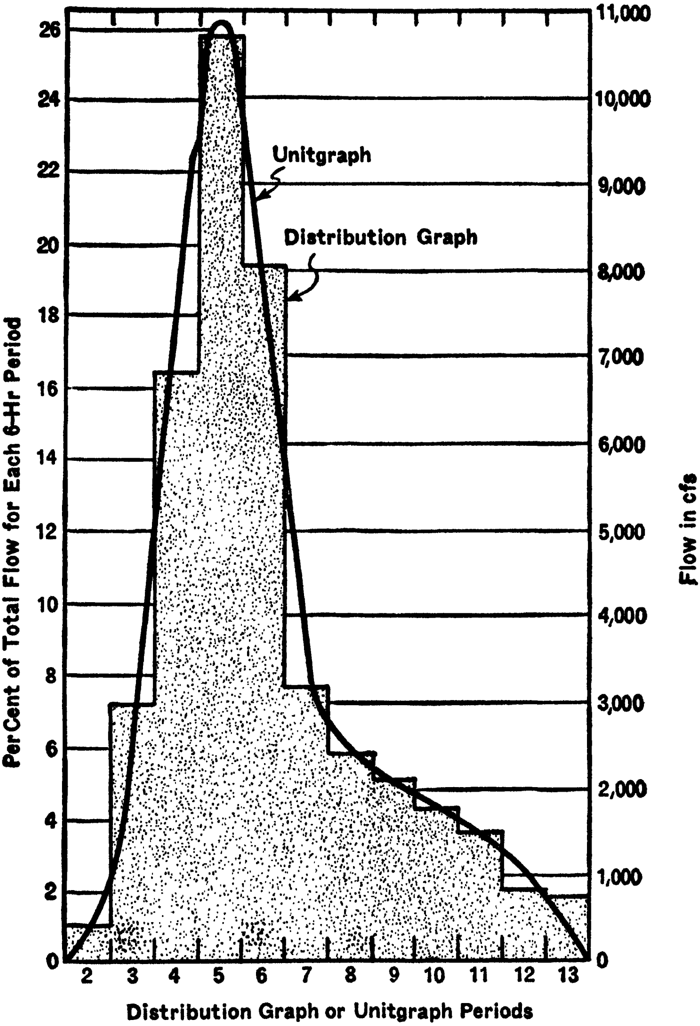

FIG. 6-4. 6-hr distribution graph and unitgraph from hydrograph of Fig. 6-2.

the streams with which he is especially concerned, as a supplement to the general information available in handbooks and government publications. In computing these percentages, it is often not necessary to introduce such refinements as a Thiessen determination of equivalent uniform

rainfall depth or a curved line of separation between surface runoff and base flow on the actual hydrograph. On the Olentangy River, for example, useful coefficients can be computed by the following simple method:

(*a*) For a well-defined flood peak like that of Fig. 6–2, make an approximate separation of base flow by drawing a straight line on the hydrograph from the beginning of the rise to a point about 3 days beyond the end of the rainfall producing the rise. (Three days is the length of the unitgraph to the nearest whole day.) Compute the surface-runoff volume as the area bounded by this line and the hydrograph.

(*b*) Compute the total rainfall depth for the storm as the unweighted mean of the depths reported by gages within or adjacent to the drainage area.

(*c*) Compute the runoff coefficient by dividing the surface-runoff volume from (*a*) by the rainfall depth from (*b*).

Along with the computed data it is well to record the rainfall that may have occurred on each of the 9 or 10 days just preceding the storm period.

6–9. Hypothetical Major Floods

To estimate a hypothetical major flood, it is customary to begin by selecting some great storm which has occurred somewhere in the vicinity of the drainage area in question and to shift it until it is centered in a critical position over the drainage area. Shifts of several hundred miles may be permissible in such regions as the Ohio or upper Mississippi River valleys; in mountainous regions, on the other hand, shifts may have to be more limited in extent. A storm centering on the windward slope of a mountain range, for example, should not be shifted to a drainage area lying on the other side of the divide. In important studies it is desirable to have the advice of qualified meteorologists on the reasonableness of a storm transposition; competent advice is also needed on the reasonableness of rotating the axis of a storm to coincide with the axis of the basin.

Once the storm has been selected and transposed to a critical position, the next matter is the choice of runoff coefficient. Since the objective of the study is to estimate a hypothetical major flood, it is logical to choose the largest runoff coefficient that may be expected of the given drainage area during the season of the year in which the selected storm could have occurred. The following data from *Water Supply Paper 772* are of interest as illustrating the possible range:

(*a*) On the French Broad River in Tennessee, drainage area 4450 sq mi, a coefficient as high as 88 per cent has been observed in the winter months, whereas autumn storms have never produced more than about 41 per cent runoff.

(*b*) On the Red River in Texas, drainage area 39,400 sq mi, the maximum observed coefficient was 20 per cent and occurred in May; autumn storms have not produced more than about 12 per cent runoff.

With such a range, it is obvious that an intelligent selection of runoff coefficient is the crux of the whole matter. The student must guard against the tendency simply to "take 100 per cent runoff" and let it go at that. On a basin like that of the Red River, such a procedure would yield an unjustifiably exaggerated estimate of flood possibilities; on the other hand, in a region where great storms may fall on snow-covered, frozen ground, a value of 100 per cent may easily be exceeded.*

6-10. Short-Term Flood Forecasting

For short-term flood forecasting, the objective is to predict, a day or possibly several days in advance, the stages that will actually be reached by a given stream. Those who depend on flood forecasts want the predictions to be as accurate as possible; an overestimate of the coming flood may cause them to take expensive protection measures—for example, evacuation of designated areas—that a correct prediction would have shown to be unnecessary. Here, then, the problem is not the selection of a *maximum* runoff coefficient but, rather, the selection of the most probable value of the actually existing coefficient. In this connection the use of "index areas" may be a valuable aid.

"Index areas" are small drainage areas within the larger basin for which the prediction is to be made. They are presumably representative of the larger basin from the standpoint of infiltration capacity; presumably, also, they receive approximately the same average depth of rainfall during the storm as does the larger basin. Ideally, they are of such size that all surface runoff is accounted for within a few hours after occurrence of the rain that produces it. During large storms, observers in the index areas submit reports of total rainfall and river stage at predetermined intervals, by telephone or radio, to the central office of the flood-predicting agency. Almost as soon as the first data are received, they can be converted into a preliminary estimate of the runoff coefficient for the basin as a whole. Meteorologists, meanwhile, are making their first rough predictions of the total amount of rainfall that may be expected from the storm; and this amount, modified by the runoff coefficient and applied to the proper unitgraph, provides an initial "guess" as to what may be expected of the main stream in the next several days. As the storm continues, index-area coefficients become more accurate, and errors in the prediction of rain yet to come become less significant, with the result that predictions of main-stream behavior can be made with more confidence. By the time the storm is over, predictions of time and height of peak ("in the absence of additional rain") may often be made with high accuracy.

*Special procedures, beyond the scope of this text, are employed in estimates of winter flood possibilities. It is desired here simply to point out that 100 per cent runoff is not necessarily the upper limit.

Space does not permit any further discussion of the complex procedures of flood forecasting. In conclusion, it is desired simply to emphasize that index areas are neither the sole nor necessarily the best method of arriving at runoff coefficients. There is great difficulty in selecting suitable areas that, even in the aggregate, are "typical" of the basin as a whole; moreover, there is no assurance that the rainfall on the index areas, in any given storm, will be of even the same order of magnitude as the rainfall in other parts of the larger basin. Another method of selecting runoff coefficients makes use of a chart developed from the past performance of the basin and featuring a family of curves showing the relation of runoff coefficient to "equivalent inches of 24-hr rainfall." There may be a separate curve for each season of the year, or even for each month. "Equivalent inches of 24-hr rainfall" is the actual rainfall for the day in question, plus decreasing percentages of the rainfall that has occurred on each of the 9 or 10 preceding days. Currently, still another method of selecting runoff coefficients is being investigated and shows promise; it is based on the assumption that the soil moisture content at the beginning of the storm period is the major determining factor.

6–11. Filling in Missing Discharge Records

The unitgraph is sometimes used to fill in missing portions of a stream-discharge record. Rainfall data for the missing periods must, of course, be available, and, in addition, there must be some basis for selection of runoff coefficients. A chart like that described in the preceding paragraph is sometimes used. Another method is to compute the coefficients that actually occurred on a neighboring stream during the same period and apply them to corresponding storms on the stream in question. This, obviously, is an adaptation of the index-area concept.

Limitations of Unitgraph Theory and Practice

Like any other empirical procedure, the unitgraph should be employed only with a thorough knowledge of its limitations. It is proposed to examine some of these in the following paragraphs, giving particular attention to the following points:

(*a*) The assumption of equal base lengths (proposition I)
(*b*) The assumption of proportional ordinates (proposition II)
(*c*) The effect of incorrect separation of base flow
(*d*) The effect of nonuniformity of storms
(*e*) The special limitations that arise when only daily average discharge records and daily total rainfall records are available.

6–12. The Assumption of Equal Base Lengths

By about the time that the crest of a flood has been reached, essentially all the water that will ultimately become surface runoff has already

reached the stream or one of its tributaries and hence may be said to be "in storage" in the stream system. Thereafter, the flow of the stream is essentially discharge from storage. Quite clearly, it takes longer to empty a full reservoir than to empty the same reservoir when it is only half-full. This observation has caused a number of engineers to criticize the unitgraph theory as being founded on a fallacy—namely, equal duration of surface runoff, regardless of total volume. This criticism overlooks the fact that proposition I is empirical and is based on the observed performance of a wide variety of streams. It is valid, however, to this extent: that, before a unitgraph analysis is made of any given stream, the hydrograph of that stream should be examined in detail to determine whether it conforms reasonably well to theory. The examination may consist of noting the length of time from cessation of rainfall to apparent cessation of surface runoff for a number of floods *of various magnitudes*, and comparing these lengths. If they increase consistently and notably with the size of the flood, then it may be suspected that that particular stream does not behave in accordance with unitgraph theory.

The student may be puzzled as to why proposition I can ever be even approximately true. Consider, then, a stylized example, consisting of a 3-mi reach of trapezoidal channel 8 ft in bottom width, with 1½:1 side slopes; let $s = 0.0016$ and $n = 0.040$. To begin with, let the water be flowing at a depth of 7 ft, so that $Q = 477$ cfs, and the total volume of water "in storage" in the 3-mi reach is 2,050,000 cu ft. Now assume that inflow ceases, except for a "base flow" of 10 cfs. Immediately, withdrawals from storage begin—initially at a rate of 467 cfs. As the water surface in the channel lowers, the discharge also decreases; by the time the water surface has lowered 1 ft, Q will have been reduced to 345 cfs, of which 335 cfs is being supplied from storage. The average rate of withdrawal from the 435,000 cu ft of storage in the top foot has thus been approximately 400 cfs, so that the time required for the discharge to change from 477 to 345 cfs is about 1080 sec. Similar computations at succeeding 1-ft intervals yield the "recession curve" marked *a* in Fig. 6–5. Suppose, now, that the flow had initially been only 239 cfs, instead of 477. The recession curve for this case (obtained by shifting *a* to the left and designated *b* in Fig. 6–5) is only 2460 sec, or about 10 per cent, shorter than *a*, whereas the initial Q was 50 per cent smaller than in the first case. When one considers, further, that the two recession curves are practically indistinguishable beyond 20,000 sec, it becomes still clearer that large differences in total volume of runoff may have essentially no effect on the duration of runoff.

6–13. The Assumption of Proportional Ordinates

Proposition II, concerning the proportionality of ordinates, is likewise empirical. However, a thoroughly independent test of it on a given

stream is rarely possible, even when we confine ourselves to an examination of peak ordinates. One reason for this will be clear when it is remembered that, before comparing peak ordinates of two hydrographs, we must first subtract an *assumed* base flow from each; likewise, before comparing volumes of surface runoff, we must first subtract an *assumed* volume of base flow from each. If, after performing these operations, we find the ratio of the peak flows to be the same as the ratio of surface runoff volumes, we cannot say definitely whether unitgraph theory is confirmed or whether the equality is due to the effects of a chance error

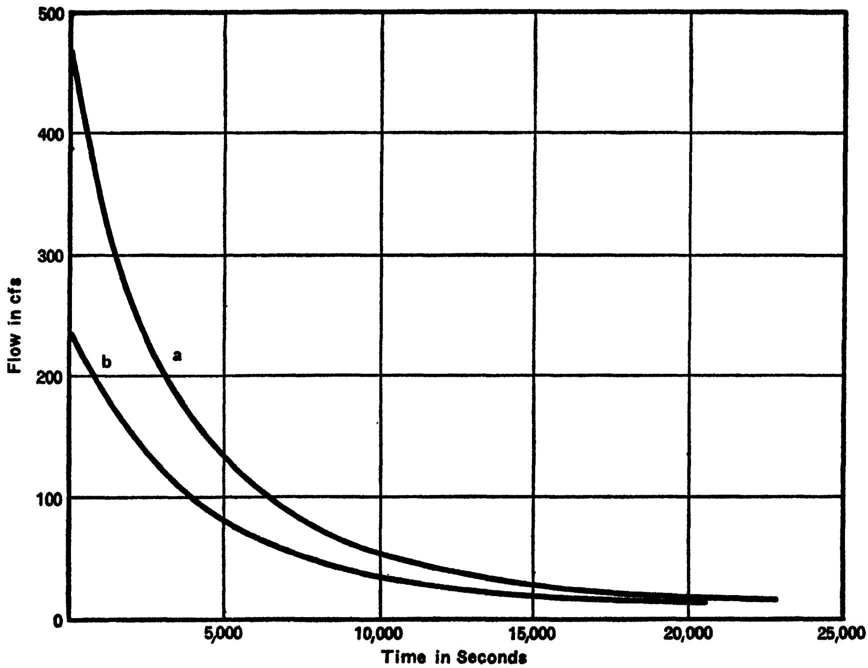

FIG. 6–5. Theoretical recession curves for trapezoidal channel.

in the selection of one or both base flows. Conversely, a discrepancy in the ratios does not necessarily indicate departure from unitgraph theory. Another difficulty that stands in the way of an independent test is the difference in the distribution of rainfall in the two flood occurrences. Unitgraph theory presupposes uniform distribution of rainfall—or, if not uniform distribution, then at least the *same* distribution in both storms. However, the probability that two *actual* storms, chosen at random from among those occurring on a given drainage area, have the same distribution is extremely remote. If, for example, one of them were moving streamdownward and the other streamupward, then the resulting hydrographs might be of quite different shapes (see p. 157, below). Again, therefore,

if we compare two floods and find peak flows to be in the same ratio as total surface runoff volumes, we cannot say whether unitgraph theory is confirmed or whether the equality is due to the effects of a chance difference in storm distribution; and, conversely, a discrepancy in the ratios is not necessarily chargeable to departure from unitgraph theory.

The significance of the preceding comments is this: Although there is adequate statistical support for proposition II as a *general* concept, one must be extremely slow either to affirm or to deny, on the basis of examination of actual flood hydrographs, that it applies *to a given stream.* The question of what constitutes adequate confirmation in a particular case would involve statistical investigations far beyond the scope of this text. Fortunately, by using the proper precautions, we can still make good use of unitgraph procedure without pushing the verification of this point too strenuously.

It is of interest to consider whether there are any particular types of streams that may be especially likely to produce peak flows that are not in a constant ratio to surface runoff volumes.* To begin with, let us assume a stream which at some point A does behave exactly in accordance with unitgraph theory, and let us further assume a reservoir to exist just downstream from A. In Fig. 6-6, let *1* be the unitgraph at A, and *2* be the corresponding outflow hydrograph from the reservoir. Construct curve *3*, with ordinates twice as great as those of the unitgraph, so that it represents the inflow hydrograph for a net rainfall of 2 in. in unit time. Now if outflow also follows unitgraph theory, the outflow hydrograph for the 2-in. runoff will have ordinates twice as great as those of curve *2* at all points. Construct curve *4* accordingly. From the construction it is obvious not only that the maximum outflow from the reservoir is twice as great in the second case as in the first but also that the storage in the reservoir, at the time of maximum outflow, is likewise twice as great. But storage in a reservoir is uniquely a function of discharge. Thus, for the outflow to follow unitgraph theory, a necessary condition is that storage be directly proportional to discharge. However, storage in a reservoir is, in general, a function of some power of the discharge other than 1. Suppose, for example, that the reservoir assumed in the preceding discussion has a square mile of surface area and discharges over an ogee crest. Unless the banks of the reservoir are exceedingly flat-sloped, storage above crest level will be approximately proportional to depth on the crest; on the other hand, flow over the ogee spillway will vary as the three-halves power of the depth. Thus, in this case, $S = kQ^{0.67}$, and the stream below the reservoir will not follow unitgraph theory; instead, the peak discharges will increase faster than the total runoff

*In what follows, it will be necessary to introduce a number of concepts about storage that are based on the material in Chap. 7. The rest of this section may, accordingly, require restudy after the flood routing discussion has been covered.

volumes. Or, to put it differently, unitgraphs derived from large floods will be characterized by higher peaks than will unitgraphs derived from small floods.

Again, suppose that in place of the reservoir there is a detention basin discharging through an outlet conduit. Storage in such a basin will vary with some power of the depth at least as great as unity, while discharge will vary with the square root of the depth, whence $S = kQ^2$. In

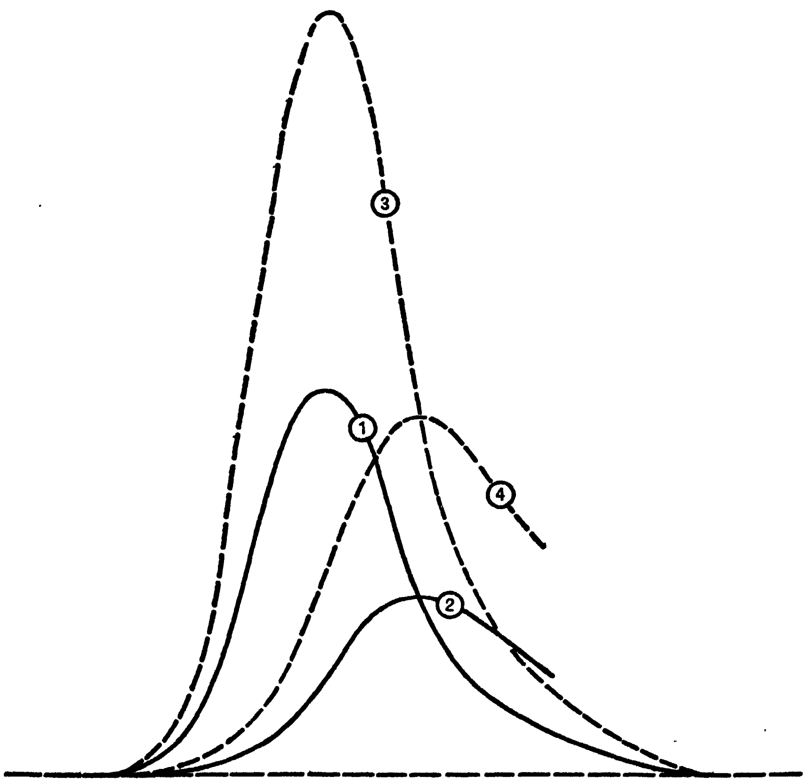

FIG. 6–6. Hydrographs modified by reservoir storage.

this case, peak discharges below the reservoir will increase much more slowly than will total runoff volumes; or, what is the same thing, unitgraphs derived from large floods will be characterized by lower peaks than will unitgraphs derived from small floods.

It follows that drainage areas containing reservoirs or their natural equivalent—lakes—are especially likely to depart from unitgraph theory. It may be noted that natural lakes are more likely to behave like reservoirs with overflow spillways than like detention basins. (Conversely, flooded overbank areas generally have detention-basin effect.)

In streams with flat slopes, a phenomenon may take place in the channel itself that has somewhat the same effect as a detention basin. To illustrate: A moderate rain may result in a rise so gradual that surface slopes are not appreciably affected, and the flood may move downstream with little or no attenuation. On the other hand, a heavy rain lasting the same length of time may produce a very rapid rise and correspondingly large changes in surface slopes, so that the attenuation of the flood as it moves downstream is pronounced. Thus, on very flat-sloped streams, unitgraphs derived from large floods may possibly tend to have somewhat lower peaks than those derived from small floods.

It appears that on a very large percentage of natural streams the conditions tending in one direction are in fair balance with those tending in the other, so that the net result is an acceptable degree of conformance with unitgraph theory. The careful hydrologist, however, will always be on the alert for the exceptions.

6-14. Incorrect Separation of Base Flow

Criteria for separating base flow are mentioned on pages 136, 141, and 142, and there is further discussion of the same subject in Chapter 8. Whatever method may be employed, there is always a question as to the accuracy of the division; about the best that can be said in any given case is that "the base flow is probably not less than about ____ nor more than about ___." It is therefore important to investigate the effect on the unitgraph of an error in base-flow selection.

In Fig. 6-7 let *1* be a natural hydrograph resulting from a one-period rain on a drainage area of 500 sq mi, and let *2* be the true base flow. The total volume of surface runoff is 5250 second-foot-days, whereas the total volume of a unitgraph would be 13,420 second-foot-days. Hence the maximum ordinate of a true unitgraph would be 4250 × 13,420/5250 or 10,900 cfs. Now suppose that, in place of the true base flow, a lower value, *3*, has been assumed. The corresponding apparent value of surface runoff is 6750 second-foot-days. Hence the maximum ordinate of the computed unitgraph would be 4500 × 13,420/6750, or 8960 cfs, or almost 18 per cent lower than the correct value.

This specific example illustrates the general proposition (to which there are some exceptions) that an error in the selection of base flow produces an error *in the same direction* in the maximum ordinate of the computed unitgraph.* If the unitgraph is to be used principally for such purposes as the derivation of spillway design floods, it follows that in

*It will be noted also that the larger the flood from which the unitgraph is derived, the smaller is the error resulting from an incorrect choice of base flow. Thus if one consistently chose base flows too low, he might find (other things being equal) that the peaks of his unitgraphs increased with the size of the flood from which they were derived.

deriving the graph one errs on the side of danger in choosing a base flow lower than the true value, and on the side of safety in choosing one higher than the true value. Until one has developed the sixth sense that comes with experience, it is worth while in deriving unitgraphs to mark off both a "maximum probable" and a "minimum probable" base flow on the

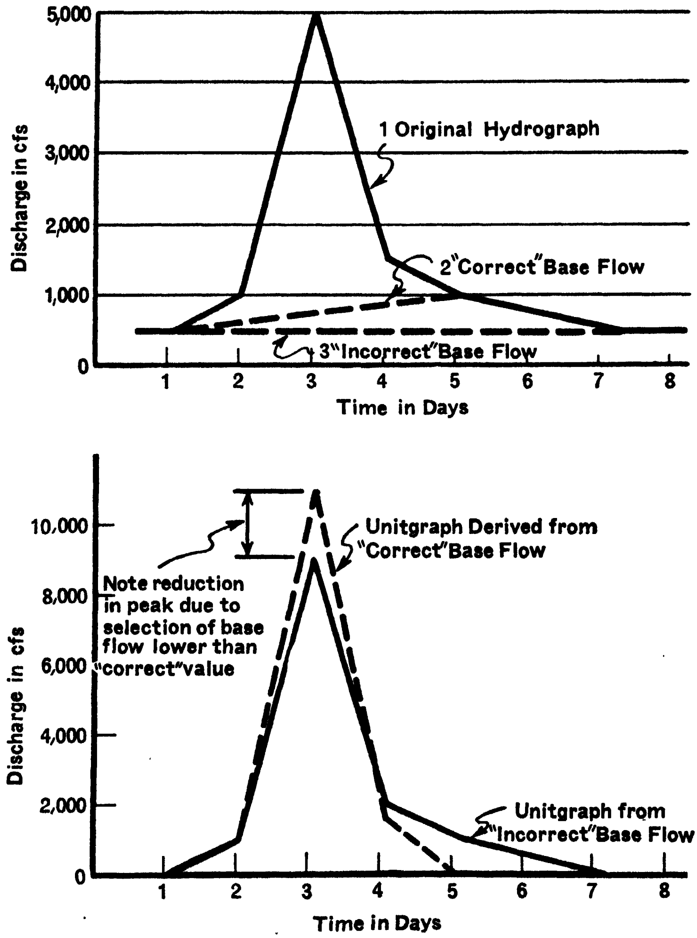

FIG. 6–7. Effect on unitgraph of error in base flow selection.

natural hydrograph, compute the corresponding two unitgraphs, and compare their peak values to get an indication of the possible amount of the error resulting from a poor selection.

6–15. Nonuniformity of Storms

Unitgraph theory presupposes a fairly uniform distribution of rainfall over the drainage area. Clearly, this imposes a maximum limit on the

size of drainage area to which the method is applicable. However, just what this limit may be is a moot question. Certainly, the so-called "great storms" are the only ones that are even approximately uniform over areas of as much as 10,000 sq mi; hence for drainage areas of that size it would be but rarely that the data necessary for deriving a unitgraph would be available. Such areas are more susceptible to analysis if broken up into subdivisions, for each of which a unitgraph may be derived. The performance of the main stream in response to a "great storm" can be then estimated by applying the proper rainfall and coefficient to each of the unitgraphs and combining the resulting hydrographs in accordance with the principles of flood routing (see Chap. 7).

The authors incline to the opinion that, where adequate stream-gaging stations are available, the same procedure should be followed on even much smaller drainage areas.* Perhaps 4000 or 5000 sq mi is about the largest area that can be handled with any degree of confidence by unitgraph methods. Even here, a difference in the location of the center of the storm or in the direction or rate of its travel may have a great effect on the shape of the hydrograph. A storm moving streamdownward will "bunch" the flow, producing an excessively high peak discharge for a given total volume of surface runoff, whereas a storm moving in the opposite direction will lengthen the base of the hydrograph and flatten the peak considerably. It is customarily considered that if unitgraphs are derived from a number of different storms, variously centered and moving in various directions, then the mean of these unitgraphs will represent the probable performance of the stream in response to a great storm that is fairly uniformly distributed.† This may or may not be true.

Unitgraph theory also presupposes, at least within broad limits, a fairly uniform rate of rainfall throughout the unit period. This does not mean, if the unit period is 24 hr, that one twenty-fourth of the total rain, more or less, must take place in each hour; but it does mean that the rainfall should be distributed over the greater part of the day rather than concentrated in an hour or less. On large drainage areas the effect of differences in time distribution of rainfall has an opportunity to be smoothed out and hence may be of relatively minor importance; but on small areas a difference in the actual duration of two storms treated as of equal length may result in very appreciable differences in the unitgraphs derived from them. Variations in rainfall rates within periods as short as 10 min may be important in drainage areas with unitgraph base lengths of 1 or 2 hr.

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*On studies of the Scioto River, Ohio, the 6500-sq mi drainage area was broken down into areas of 200 sq mi and less for purposes of accurate unitgraph analysis.

†If the unitgraphs are to be used for day-by-day flood forecasting, the various graphs may be left separate, so that in any given storm the forecaster may base his work on a graph derived from a somewhat similar storm, instead of on a "mean" graph.

6–16. Working from Daily Records

In practice, the hydrologist must often content himself with published records of rainfall and of stream discharge. This means, in general, that his data are limited to the record of daily total rainfall at each rain gage and the daily average discharge of the stream. In such cases, 1000 sq mi is about the lower limit of size of basin for which unitgraphs or distribution graphs *of local significance* can be derived. "Of local significance" implies "suitable for use in predicting the flood performance of the stream at the gaging station." If the purpose of the unitgraph is simply to provide data on the daily average contribution of a tributary, for use in flood-routing studies on a larger stream, then possibly useful graphs can be developed from daily records for basins as small as 400 or 500 sq mi.

Some idea of the inaccuracies resulting from the use of daily records on a small watershed may be gained from the following notes on a study of the Olentangy River, Ohio (drainage area, 387 sq mi). A 6-hr distribution graph was carefully prepared from the continuous discharge record of a recording stream gage and the hourly rainfall record of four recording rainfall gages. Then a series of 24-hr distribution graphs was computed from this 6-hr graph, assuming various durations of rainfall (6, 12, 18, and 24 hr) and various times of beginning of rainfall with respect to the beginning of the "discharge day." The 24-hr graphs showed peak-day percentages varying from 43 to 59 per cent of the total surface runoff, with peaks occurring in some cases on the first day and in others on the second day of the runoff period. These values indicate the possible variation in 24-hr graphs derived from published daily average discharge records and 24-hr total precipitation records. Another observation made in the course of this study was that the momentary peak produced by a 6-hr storm on this stream can be expected to be more than 20 per cent greater than the peak produced by a storm of the same magnitude but lasting 24 hr.

6–17. Minimizing the Limitations

In conclusion, the following list of suggestions on operating technique is offered, with a view to minimizing the effects of the various limitations of unitgraph procedure.

1. If at all possible, obtain the actual record of a continuous recording stream gage and use it, rather than published daily records, for constructing the hydrograph.
2. Make every effort to obtain the hourly rainfall record for as many rainfall gages as possible, in order to permit use of as short a unit period as may be required by No. 3 below.

3. For convenience, the unit period should be a simple fractional part of 24 hr. For accurate results, it should be short enough that ordinates spaced one unit apart will define the natural hydrograph with good accuracy, regardless of the location of the first ordinate.
4. In laying off the base flow and in establishing the end-point of surface runoff, make use of a depletion curve if possible; however, to be on the "side of safety," choose short base lengths and high base flows in preference to long base lengths and low base flows.
5. Despite the extra labor involved, include in every important study at least one or two unitgraphs derived from storms extending over more than one unit of time. Derivation from a long storm reduces the effect of whatever error may be made in the selection of base flow; it also provides unitgraphs based on storms of more nearly the same order of magnitude as those to which the unitgraph is later to be applied.
6. Derive as many unitgraphs as possible, from storms of various magnitudes, to verify whether it is likely that the stream in question behaves in accordance with unitgraph theory.
7. If the variation between unitgraphs appears to be random, prepare a master-unitgraph as follows: Plot the individual unitgraphs on a single set of coordinates; shift individual graphs left or right as may be necessary to make their peaks coincide in time; take the mean of the individual peaks for the peak of the master-graph; and sketch the remainder of the master-graph by eye, adjusting as necessary to insure that the area under it is unity.
8. If the variation between unitgraphs is not random but appears rather to be a function of the size of the storm from which they were derived, it may be better to prepare a set of two or three master-graphs, for use with storms of various sizes.
9. If the unitgraphs are to be used for day-by-day flood forecasting, it may be desirable to prepare a set of unitgraphs, one based on "storms heavy upstream," another on "storms heavy near middle of basin," and so forth, so that in any given storm a graph will be available that should produce better results than a mean graph.

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CHAPTER 7

FLOOD ROUTING*

The Problem

The unitgraph, discussed in the preceding chapters, provides a means for predicting the time distribution of flow during flood periods at points on the stream for which it has been derived. In general, however, the unitgraph is inadequate for the complete solution of a flood control problem or for the operation of a satisfactory flood prediction service. Specifically, unitgraphs (*a*) are strictly applicable only when precipitation is concurrently uniformly distributed over the drainage area; (*b*) are strictly applicable only to the points on the stream for which they were derived; (*c*) are applicable only as long as channel conditions remain unchanged; (*d*) are not applicable to drainage areas having a great amount of storage in and adjacent to the stream channels. On the other hand, for the solution of a flood control problem or for the operation of a flood prediction service, we must be able to (*a*) predict the behavior of a river during periods when the tributaries are discharging different quantities;

*This chapter was revised by John S. Shapland, Lieutenant Colonel, Corps of Engineers, U.S. Army, and John C. Guillou, Research Assistant Professor of Hydraulic Engineering, University of Illinois.

(b) predict the behavior of a river at any desired point, including points for which data for construction of unitgraphs is not available; (c) predict the behavior of a river after a change in channel conditions, such as the installation of a reservoir or a levee system. Such predicting of the hydrograph at one point on a stream from the known hydrograph at another point is termed "flood routing."

The Simplified Bookkeeping Equation

Consider a reach of river along which no tributaries enter. Assuming (1) that accretions from and losses to ground water are negligible or equal, (2) that no rain falls during the passage of the flood, and (3) neglecting evaporation, the general hydrologic bookkeeping equation reduces to

$$S_1 + \int_{t_1}^{t_2} I\, dt = S_2 + \int_{t_1}^{t_2} D\, dt\,; \tag{7-1}$$

and, transposing, we obtain

$$\int_{t_1}^{t_2} I\, dt - \int_{t_1}^{t_2} D\, dt = S_2 - S_1\,. \tag{7-2}$$

In the latter form the equation says simply: "The total quantity of inflow into the reach during a given period of time *minus* the total quantity of outflow from the reach during the same period equals the change in the volume of water stored in the reach."

Since neither I nor D can be expressed mathematically in terms of t for any but the simplest cases, arithmetic integration is necessary for the solution of Eq. (7-2). Selecting a time interval short enough that both I and D may be considered linear functions of t within each time interval, Eq. (7-2) may be rewritten as

$$(t_2 - t_1)\left(\frac{I_2 + I_1}{2} - \frac{D_2 + D_1}{2}\right) = S_2 - S_1. \tag{7-3}$$

By the terms of the problem, I_1 and I_2 are known, and assuming that D_1 and S_1 are known, Eq. (7-3) has two unknowns—D_2 and S_2. For a solution of the flood-routing problem it is therefore necessary that another relation involving D and S be found. This relation will be obtained from the physical characteristics of the reach.

Flood Routing through Reservoirs and Retarding Basins

7-1. Storage as a Function of Discharge Alone

Fig. 7-1 represents a retarding-basin type of reservoir with an unvalved orifice at stream level, to restrict discharge to the capacity of the channel below the dam, and an uncontrolled overflow spillway. A

storage reservoir or detention basin would be similar in profile but would have means of controlling the discharge from the orifice, and perhaps from the spillway as well. If the width and depth of the pool are great in comparison to that of the inflowing stream, the pool surface will approximate the level line *dce* instead of the backwater curve *abce*, and velocity in the reservoir will be negligible. In what follows we shall assume this to be the case; except in long, narrow reservoirs, it is very nearly true. In addition, storage in the stream channel between the upstream gaging station (point *a*, Fig. 7–1) and the transitory surface of the pool will be

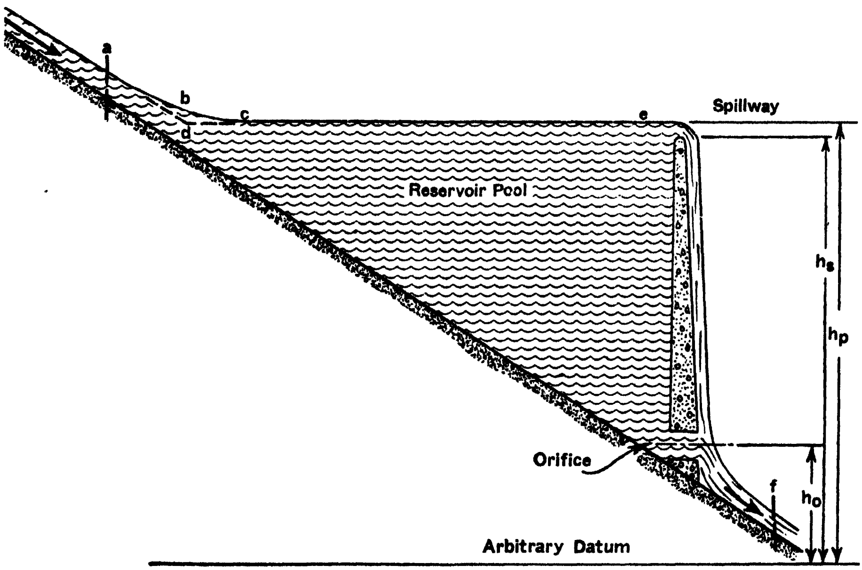

FIG. 7–1. Retarding basin or reservoir.

ignored. This neglect may not be justified if channel storage is large in proportion to the total reservoir volume. Its effect may be estimated in individual cases by the procedure outlined in the section on "Flood Routing in a Stream" (pp. 171 ff.).

If a level water surface is assumed, storage is a function of only the height of water in the pool (h_p). Both the orifice discharge and the spillway discharge can be expressed as functions of the head.* These relationships between S and D will permit solution of Eq. (7–3) for reservoirs.

*Specifically, the orifice discharge can usually be expressed as $D_0 = C_1 A_0 \sqrt{2g\,(h_p - h_0)}$, where C_1 is an empirical coefficient, A_0 the area of the orifice, and $h_p - h_0$ the head on the orifice, as indicated in Fig. 7–1. The spillway discharge expression takes the form $D_s = C_2 L_s (h_p - h_s)^{3/2}$, where C_2 is an empirical coefficient, L_s the length of the spillway, and $h_p - h_s$ the head on the spillway, as indicated in Fig. 7–1.

7-2. Example of Flood Routing, Using a Mass Diagram

For this example assume a storage reservoir which is full at the beginning of the flood (0930, June 4), and assume there is no discharge from the reservoir except that which takes place over the spillway. The spillway crest is at elevation 5110.0. In addition, the problem has the following data given:

(*a*) The inflow hydrograph above the reservoir (point *a*, Fig. 7-1)
(*b*) The physical characteristics of the reservoir area in the form of a contour map or cross sections
(*c*) Data on the characteristics of the spillway from which discharge at various pool elevations may be computed.

The problem is to find:

(*a*) The discharge hydrograph at the dam (point *f*, Fig. 7-1)
(*b*) The time-stage relationship for the reservoir pool

The mass diagram is a convenient device for solving a problem of this type. A different method which is also applicable is given in the section which follows.*

The procedure is as follows:

(*a*) Compute the storage capacity (volume) of the reservoir at various pool elevations. For preliminary studies of large reservoirs, capacity data can frequently be obtained with sufficient accuracy by planimetering standard topographic maps and converting the areas to volumes. For small reservoirs and for final studies of large ones, special topographic surveys may be necessary.

(*b*) Compute the discharge over the spillway at various pool elevations.

(*c*) Plot the storage and discharge curves (Fig. 7-2) from the data computed in (*a*) and (*b*).

(*d*) Compute and plot the ordinates of the mass curve from the inflow hydrograph (Table 7-1 and Fig. 7-3). Ordinates are plotted at one-half the time interval of the hydrograph after the time of the hydrograph reading. Since it is assumed, for example, that the 0800 reading represents an average rate from 0730 to 0830, the ordinate representing cumulative inflow at the end of the period is plotted at 0830. Note that ordinates should be based on *net* inflow; evaporation, seepage, or similar losses should be deducted before plotting the mass curve. Note also that the ordinate of the mass curve at any time represents the *total volume* of

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*Another method, outlined by N. S. Haines in *Engineering News-Record*, June 26, 1947, is based on the time rate of change of pool levels. It is particularly adapted to continuously predicting pool levels in a reservoir or to routing many hydrographs through a reservoir after its dimensions are finally determined.

inflow up to that time and that the slope of the mass curve at any time represents a *rate* of inflow.

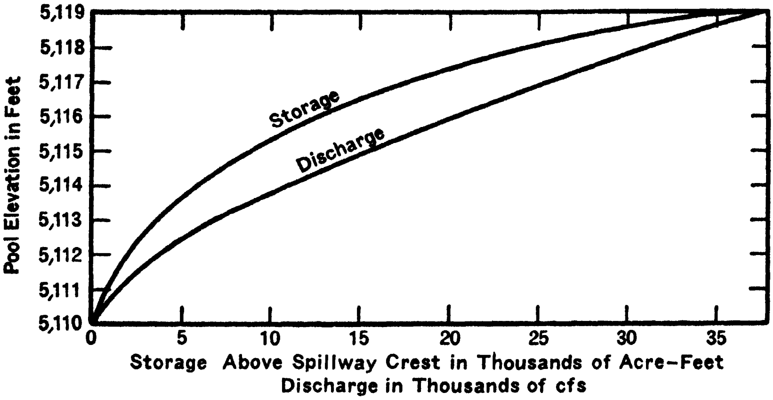

FIG. 7-2. Storage and discharge curves for reservoir.

TABLE 7-1

DATA FOR MASS CURVE FOR RESERVOIR

(1) Date	(2) Time	(3) Net Inflow (cfs × 1000)	(4) Net Inflow (Acre-Ft)	(5) Cumulative Net Inflow (Acre-Ft)*	(1) Date	(2) Time	(3) Net Inflow (cfs × 1000)	(4) Net Inflow (Acre-Ft)	(5) Cumulative Net Inflow (Acre-Ft)*
June 4..	10	.52	43	43	June 5..	07	15.15	1263	37,510
	11	.62	52	95		08	13.28	1107	38,617
	12	.84	70	165	(*Cont.*)	09	11.62	968	39,585
	13	1.00	83	248		10	10.05	838	40,423
	14	2.20	183	431		11	8.97	748	41,171
	15	5.60	467	898		12	8.00	667	41,838
	16	8.26	688	1,586		13	7.20	600	42,438
	17	15.42	1285	2,871		14	6.73	561	42,999
	18	23.40	1950	4,821		15	6.14	512	43,511
	19	28.62	2385	7,206		16	5.67	473	43,984
	20	34.56	2880	10,086		17	5.20	433	44,417
	21	40.65	3388	13,474		18	5.12	427	44,844
	22	41.42	3452	16,926		19	5.02	418	45,262
	23	42.20	3517	20,443		20	4.82	402	45,664
	24	39.00	3250	23,693		21	4.60	383	46,047
June 5..	01	35.10	2925	26,618		22	4.35	363	46,410
	02	31.20	2600	29,218		23	4.20	350	46,760
	03	25.80	2150	31,368		24	4.10	342	47,102
	04	22.30	1858	33,226	June 6..	01	3.98	332	47,434
	05	18.90	1575	34,801		02	3.80	317	47,751
	06	17.35	1446	36,247		03	3.57	298	48,049

*Ordinates of mass curve.

(*e*) Select an increment of storage volume such that the mass curve for that increment will approximate a straight line. Erect a vertical line with a height representing the volume of the storage increment at a

point on the mass curve representing the start of the period (in this case at 0930, when the pool surface is at spillway crest elevation) (Fig. 7–3).

(*f*) From the storage and discharge curves (Fig. 7–2) determine (1) pool surface elevation at the end of the selected storage increment and (2) the rate of discharge over the spillway at this pool elevation.

(*g*) Construct a table recording this data (Table 7–2).

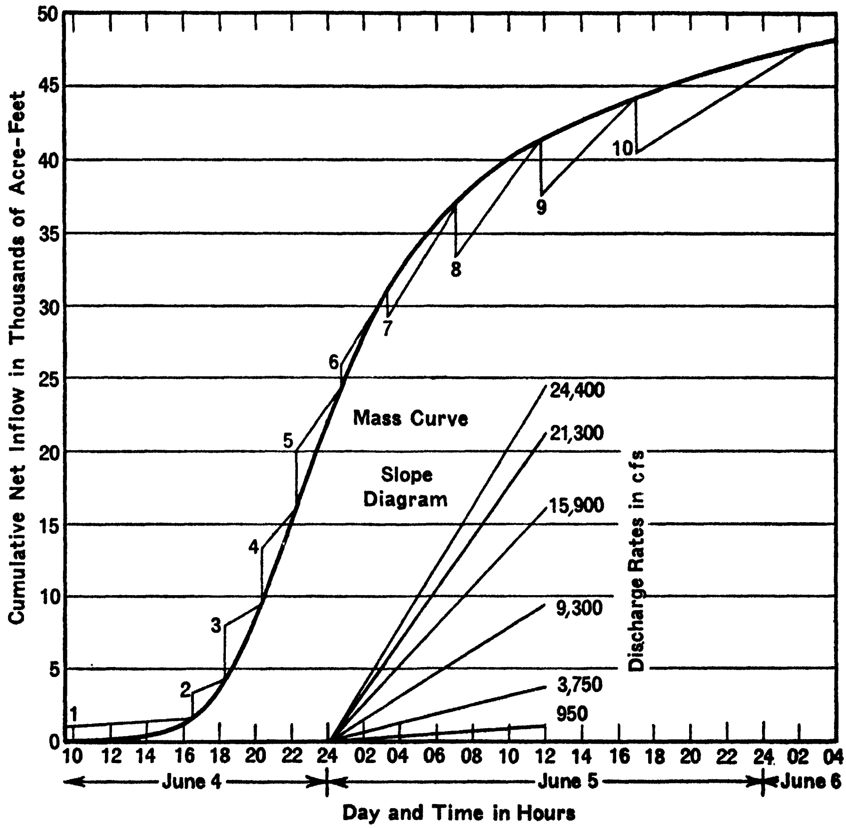

FIG. 7–3. Mass diagram for reservoir.

(*h*) Determine the average rate of discharge for the period of the increment and construct a slope diagram near the mass curve representing this average rate of discharge.

(*i*) From the top of the vertical line representing the storage increment on the mass diagram lay off a line parallel to the slope representing the average rate of discharge for the period. The point where this line meets the mass curve represents (1) the time when the storage will be equal to the assumed amount and (2) the total amount of water which

must flow into the reservoir in the elapsed time in order to place the assumed amount in storage.

TABLE 7-2

ROUTING COMPUTATIONS FOR RESERVOIR

(1) Period	(2) Storage Increment: Amount (Acre-Ft)	(3) Storage Increment: Total (Acre-Ft)	(4) Pool Elevation at End of Period (Ft)	(5) Discharge at End of Period (cfs)	(6) Average Discharge During Period (cfs)	(7) Date	(8) Time at End of Period
1.....	+1000	1,000	5111.2	1,900	950	June 4	1630
2.....	+2000	3,000	5112.6	5,600	3,750		1812
3.....	+4000	7,000	5114.4	13,000	9,300		2020
4.....	+4000	11,000	5115.6	18,800	15,900		2215
5.....	+4000	15,000	5116.6	23,800	21,300	June 5	0045
6.....	+1500	16,500	5116.8	25,000	24,400		0320
7.....	−1500	15,000	5116.6	23,800	24,400		0715
8.....	−4000	11,000	5115.6	18,800	21,300		1148
9.....	−4000	7,000	5114.4	13,000	15,900		1700
10.....	−4000	3,000	5112.6	5,600	9,300	June 6	0248

(*j*) Continue steps (*e*) through (*i*) until the slope representing the outflow rate is tangent to the mass curve. (If necessary the last increment

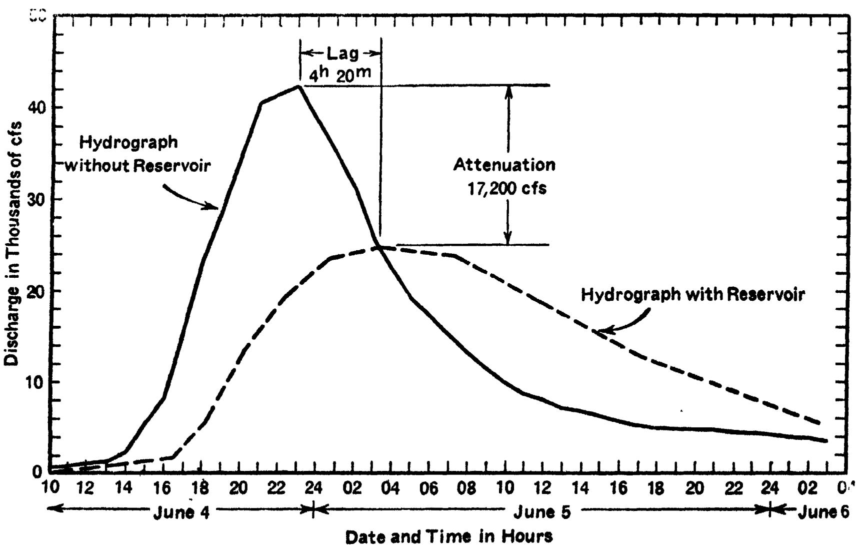

FIG. 7-4. Inflow and discharge hydrographs for reservoir.

of storage volume should be adjusted to make the slope tangent to the mass curve.) The point of tangency represents the time of maximum discharge over the spillway.

(*k*) Continue steps (*e*) through (*i*), only lay off storage increments *below* the mass curve to represent depletion of stored water.

(*l*) Plot the inflow and discharge hydrographs from the data in columns (2) and (3) of Table 7-1 and columns (5) and (8) of Table 7-2. The time-stage relationship may be plotted from columns (4) and (8) of Table 7-2.

Considering the inflow and outflow hydrographs for the reservoir (Fig. 7-4), the area between the hydrographs when the outflow is increasing represents additions to storage, while the area between the hydrographs when the outflow is decreasing represents withdrawals from storage. The lowering of the peak flow is called "attenuation"; the time interval between occurrence of the peaks may be designated as "lag." In reservoirs in which discharge is not controlled and the pool surface is assumed to be level, the peak outflow must occur where the discharge hydrograph intersects the inflow hydrograph, since the maximum discharge must occur at maximum storage. In natural streams, peak discharge usually occurs later than the time when the hydrographs intersect.

7-3. Example of Flood Routing, Using a (2S/t) +D Curve

Another method of routing floods when storage is a function of discharge alone employs a modification of Eq. (7-3). Let $t_2 - t_1 = t$; then

$$t\left(\frac{I_2 + I_1}{2} - \frac{D_2 + D_1}{2}\right) = S_2 - S_1,$$

$$I_2 + I_1 - D_2 - D_1 = \frac{2S_2}{t} - \frac{2S_1}{t}.$$

Transposing all known quantities to the left side of the equation, we obtain

$$(I_1 + I_2) + \left(\frac{2S_1}{t} - D_1\right) = \left(\frac{2S_2}{t} + D_2\right). \qquad (7\text{-}4)^*$$

---

*Note that the inflows and discharges I_1, I_2, D_1, and D_2 are in cfs; the storage S_1 and S_2 in cu ft; and the time t in sec. This equation is sometimes used in the approximate form

$$(I_2 + I_1 - D_1)t + S_1 = S_2 + D_2t, \qquad (7\text{-}5)$$

where S_1 and S_2 are in acre-feet, t is in days, I and D in acre-feet per day. If t is 24 hr, (7-4) may be further reduced to

$$(I_2 + I_1) + (S_1 - D_1) = S_2 + D_2, \qquad (7\text{-}6)$$

where all units are acre-feet. Procedure for graphical solution is practically the same in any case, Eq. (7-5) requiring S and $S + Dt$ curves and Eq. (7-6) an $S + D$ curve, where Eq. (7-4) uses a $(2S/t) + D$ curve.

Eq. (7–4) can be solved for values of $(2S/t) + D$, and a plot of D versus $(2S/t) + D$ can be used to determine D.

For an example of this method, assume a retarding basin with the storage and discharge characteristics given in Table 7–3. The dam has an

TABLE 7–3

COMPUTATIONS FOR $\frac{2S}{t} + D$ CURVE FOR RETARDING BASIN

(t = 2 Hr)

(1) Elevation (Ft)	(2) S (Cu Ft × 1,000,000)	(3) $\frac{2S}{t}$ (Cfs × 1000)	(4) D (Cfs × 1000)	(5) $\frac{2S}{t}+D$ (Cfs × 1000)	(1) Elevation (Ft)	(2) S (Cu Ft × 1,000,000)	(3) $\frac{2S}{t}$ (Cfs × 1000)	(4) D (Cfs × 1000)	(5) $\frac{2S}{t}+D$ (Cfs × 1000)
525...	0.03	0.008	0.138	0.146	680...	105.0	29.17	3.40	32.57
530...	0.13	0.036	0.610	0.646	700...	141.1	39.19	3.61	42.80
540...	0.47	0.131	1.056	1.187	720...	183.6	51.00	3.81	54.81
560...	2.68	0.74	1.61	2.35	740...	231.0	64.2	4.01	68.2
580...	7.49	2.08	2.02	4.10	750...	257.0	71.4	4.09	75.5
600...	15.45	4.29	2.36	6.65	755...	270.0	75.0	4.13	79.1
620...	28.4	7.89	2.65	10.54	758...	278.0	77.2	6.97	84.2
640...	47.8	13.28	2.93	16.21	760...	284.0	78.9	10.22	89.1
660...	73.6	20.44	3.17	23.61					

unvalved orifice 5 ft high, with its center at elevation 525, and a spillway crest elevation of 755. Discharges given in Table 7–3 for pool elevations above 755 ft include both orifice and spillway discharge. The problem of routing a flood hydrograph through a retarding basin has given the following data:

(*a*) The inflow hydrograph at the upper end of the reach (point *a*, Fig. 7–1).

(*b*) The physical characteristics of the retarding basin area—usually in the form of a contour map or cross sections.

(*c*) Data on the characteristics of the orifice and spillway from which discharge at various pool levels may be computed.

The problem is to find:

(*a*) The outflow hydrograph at the dam (point *f*, Fig. 7–1).

(*b*) The time-stage relationship for the retarding basin pool.

The procedure is as follows:

(*a*) Compute the storage capacity of the retarding basin at various pool levels (col. (2), Table 7–3).

(*b*) Compute and plot the discharge from both the orifice and the spillway of the retarding basin at various pool levels (col. (4), Table 7–3, and Fig. 7–5).

(*c*) From (*a*) and (*b*) compute the values of $(2S/t) + D$ for various pool levels (col. (5), Table 7–3).

(*d*) Plot a curve of D versus $(2S/t) + D$ (Fig. 7–6).

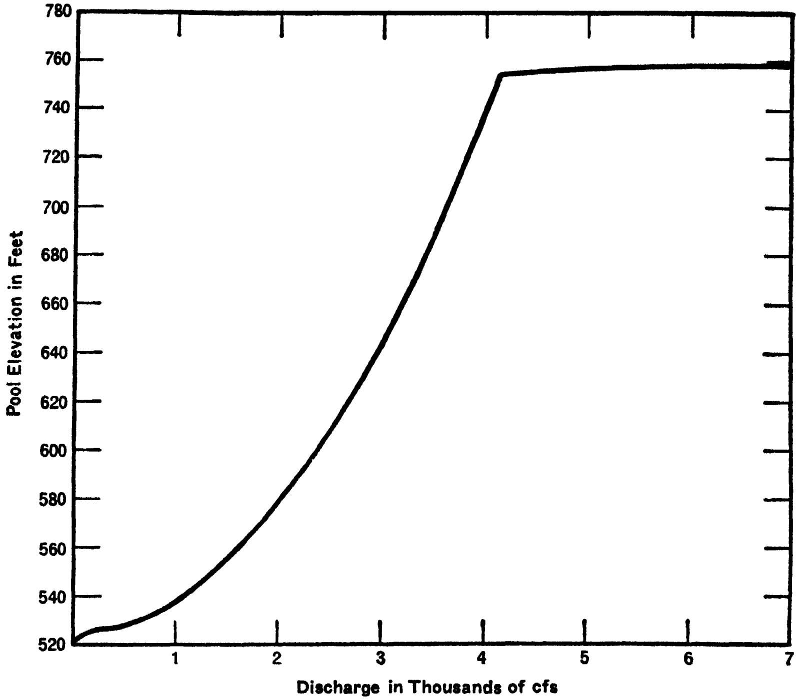

FIG. 7-5. Discharge curve for retarding basin.

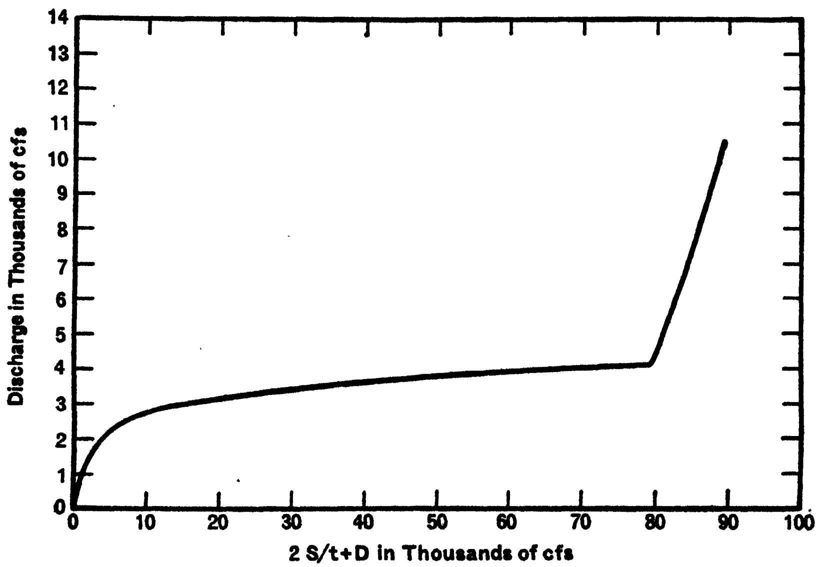

FIG. 7-6. $2S/t + D$ curve for retarding basin ($t = 2$ hr).

(*e*) Compute the outflow hydrograph from the known inflow hydrograph using Fig. 7–6 (Table 7–4). At the start, the first value in column (6)

TABLE 7–4

ROUTING COMPUTATIONS FOR RETARDING BASIN

(1) Date	(2) Time	(3) I (Cfs)	(4) I_1+I_2 (Cfs)	(5) $\frac{2S}{t}+D$ (Cfs)	(6) D (Cfs)	(7) $\frac{2S}{t}-D$ (Cfs)	(8) Pool Elevation (Ft)
Mar. 8....	00	210	....	200	210*	− 220	526
	02	600	810	590	600	− 610	530
	04	2150	2,750	2,140	1520	− 900	556
	06	3450	5,600	4,700	2110	+ 480	585
	08	3800	7,250	7,730	2470	+ 2,790	607
	10	4300	8,100	10,890	2680	+ 5,530	622
	12	4750	9,050	14,580	2870	+ 8,840	635
	14	5250	10,000	18,840	3020	+12,800	647
	16	6050	11,300	24,100	3180	+17,740	661
	18	6650	12,700	30,440	3360	+23,720	676
	20	7300	13,950	37,670	3520	+30,630	692
	22	7800	15,100	45,730	3680	+38,370	706
	24	8700	16,500	54,870	3830	+47,210	723
Mar. 9....	02	7700	16,400	63,610	3990	+55,630	738
	04	7000	14,700	70,330	4020	+62,290	742
	06	6650	13,650	75,940	4090	+67,760	749
	08	6300	12,950	80,710	4940	+70,830	756
	10	5500	11,800	82,630	6000	+70,630	757
	12	4900	10,400	81,030	5250	+70,530	756
	14	3600	8,500	79,030	4130	+70,770	755
	16	3300	6,900	77,670	4100	+69,470	751
	18	2750	6,050	75,520	4090	+67,340	750
	20	1950	4,700	72,040	4020	+64,000	742
	22	1600	3,550	67,550	4000	+59,550	739
	24	1300	2,900	62,450	3960	+54,530	735
Mar. 10...	02	1050	2,350	56,880	3880	+49,120	728
	04	880	1,930	51,050	3770	+43,510	716
	06	640	1,520	45,030	3660	+37,710	704
	08	470	1,110	38,820	3640	+31,540	703
	10	300	770	32,310	3400	+25,510	680
	12	210	510	26,020	3220	+19,580	664
	14	200	410	19,990	3040	+13,910	649
	16	200	400	14,310	2850	+ 8,610	634
	18	190	390	9,000	2550	+ 3,900	613
	20	190	380	4,280	2050	+ 180	582
	22	190	380	560	620	− 680	530
	24	190	380	....	190	....	526
Total.	....	118,270	....	....	118,510	....	....

*Known at start of computation.

of Table 7–4 is known. The first value in column (5) is the value of $(2S/t) + D$ corresponding to the known value of D. Column (7) is column (5) *minus* twice column (6) in each case. In subsequent lines, column (5) is column (4) *plus* column (7) from the line above. The discrepancy between total inflow and total discharge is about two-tenths of 1 per cent, which is well within the accuracy of the original data.

(*f*) Plot inflow and discharge hydrographs from columns (3) and (6) of Table 7-4. Pool elevations may be determined from column (6) of Table 7-4 and Fig. 7-5. They are given in column (8) of Table 7-4.

Fig. 7-7 shows the inflow and discharge hydrographs for this example. The peak in the discharge hydrograph between 0600 and 1400, March 9,

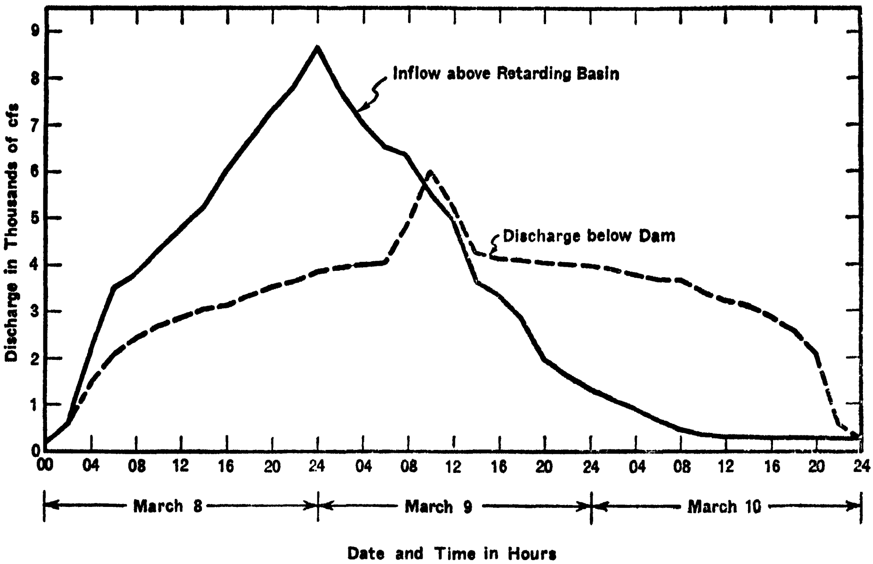

FIG. 7-7. Inflow and discharge hydrographs for retarding basin.

is caused by the spillway discharging during this time in addition to flow through the orifice.

Flood Routing in a Stream

In routing floods through a stream, the stream is divided into convenient lengths called "reaches," each of which is treated in much the same way as a reservoir. In this discussion we shall consider only a reach which has no accretions from precipitation, ground water, or tributaries; all flow will be considered as entering the reach at its upstream limit and progressing to the downstream end of the reach. Flow is considered to be at less than critical velocity and unaffected by backwater from lower reaches.

7-4. Storage as a Function of Inflow and Discharge

Flood routing in a stream differs from routing through a reservoir because the storage in a reach of a stream is not a function of stage or

discharge only. Fig. 7–8 shows schematically some of the possible variations in the longitudinal profile of the water surface in the reach *AB*

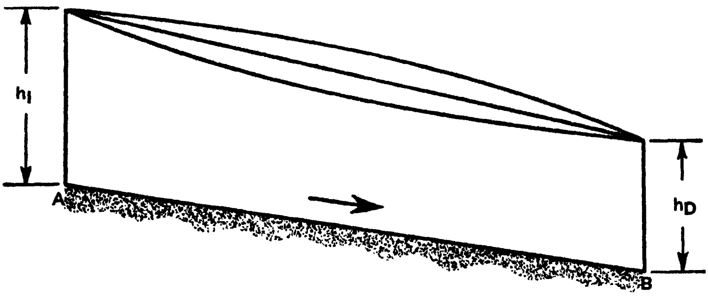

FIG. 7–8. Some possible longitudinal water surface profiles in a reach corresponding to a single pair of stages at the ends.

when inflow into the reach and discharge from the reach are both fixed. When inflow varies and discharge remains constant, there are a large number of possible profiles, a few of which are indicated in Fig. 7–9.

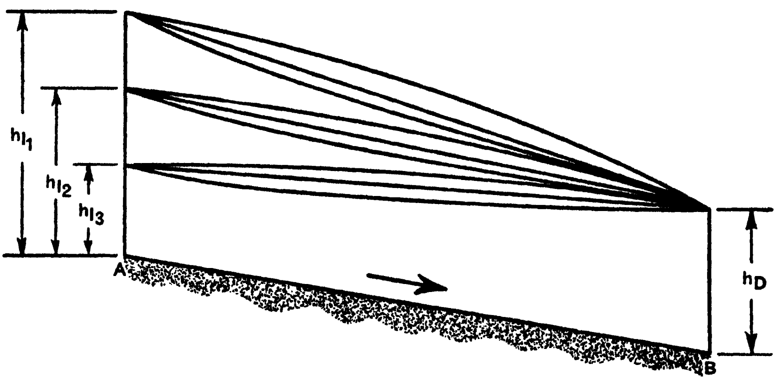

FIG. 7–9. Some possible longitudinal water surface profiles in a reach corresponding to different inflows, the discharge from the reach being constant.

Reference to Fig. 7–10 will assist in visualizing the effect of variation of the longitudinal water surface profile on storage in the reach. Consideration of Fig. 7–8 and 7–9 indicates that storage in a reach cannot always be considered even as a function of the instantaneous inflow and discharge values jointly.*

*Reference to Chap. 3 will convince the student that discharge is not a function of stage alone but also depends upon other factors. Discharges should be corrected for slope of the water surface before routing; the other factors are usually negligible for unsteady flow of the type usually found in streams.

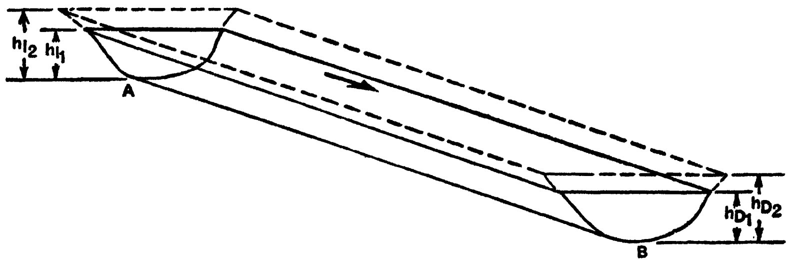

FIG. 7-10. Storage in a reach.

In Fig. 7-11, lines *abf*, *cdf*, *ef*, *gh*, and *ij* represent schematically successive profiles of the reach *AB* resulting from a uniform increase in

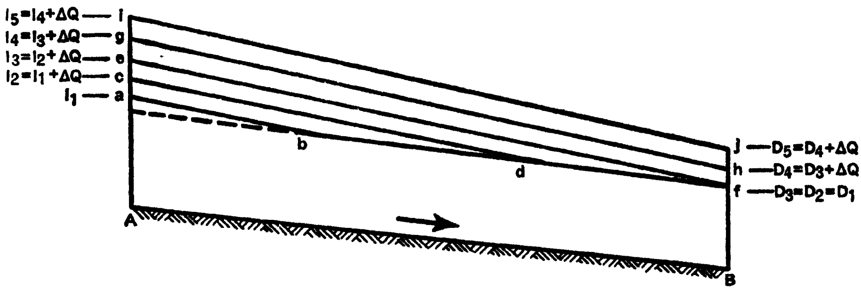

FIG. 7-11. Schematic representation of successive water surface profiles in a reach during a uniform increase in inflow.

inflow (ΔQ) at the upstream end. Then, assuming the rate of increase to be constant, the time required for a variation in the rate of inflow to appear as a change in the rate of outflow is equal to the time of travel in the reach. (Actually, because of the increased velocities due to the water surface slope, in a natural stream this time may be equal to, or slightly less than, the average time of travel through the reach.) After the discharge rate begins to increase, still assuming a constant rate of increase in inflow, the time of flow will be less important, since the increases in storage between *ef* and *gh* and *ij* are more nearly equal than are the increases in storage between *abf* and *cdf* and between *cdf* and *ef*. This rather simple analysis indicates that, if storage is to be considered even approximately a unique function of inflow and discharge, (1) the time interval selected for arithmetic integration should be nearly equal to, or greater than, the length of the reach divided by the average velocity, and (2) the time rate of change of inflow should be nearly constant during the time interval chosen. The length of the reach, then, should not exceed *tv*, the product of the time interval chosen for arithmetic integration (t) and the average velocity of the stream in the reach (v).

Now consult Fig. 7–12. If the length of the reach AB does not exceed tv and the rate of increase in inflow is constant for the period t, storage in the reach is uniquely determined by D_2, I_2, and I_1 or $I_2 - I_1$ (or some

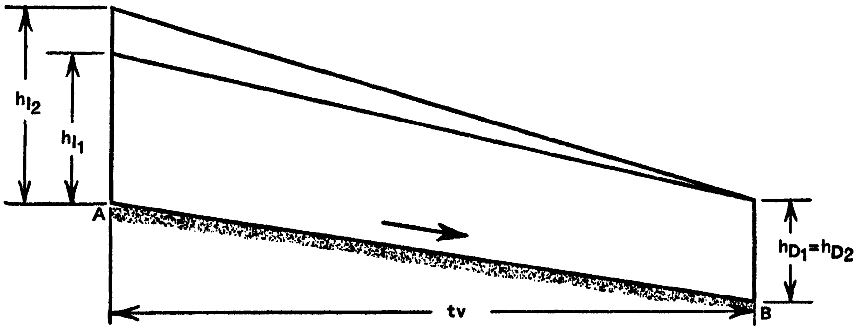

FIG. 7–12. Schematic longitudinal water surface profiles for a reach during a uniform change in inflow.

other quantity which measures the rate of *change* of inflow). If for a given stream this rate of change of inflow at any specific inflow is constant, I and D may be used as a measure of storage in the reach. Uncontrolled natural streams often exhibit this property, as is evidenced by a similarity in hydrographs; and for them I and D may be used as an approximate measure of storage in a reach. Consider, now, the effect of a reach longer than tv—say, $3tv$ in length—as shown in Fig. 7–13. If the rate of change

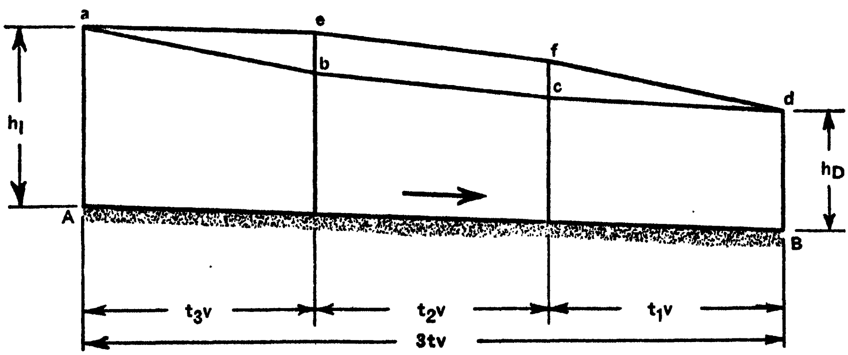

FIG. 7–13. Schematic longitudinal water surface profiles for a reach during a variable change in inflow.

in flow for the equal periods t_1, t_2, and t_3 are all different but all increasing, a profile similar to *abcd* would result. Again, if the rate of change of inflow at any specific inflow is constant, I and D may be used as a measure

of storage in the reach. However, when an abrupt change in slope of the hydrograph, such as occurs at its peak, passes through the reach, a profile similar to *aefd* results, and it is apparent that I and D no longer uniquely determine storage in the reach. The effect of this discrepancy at the peak of a hydrograph is to give computed discharges in excess of actual discharges, since storage during passage of the peak is greater than it is considered to be in the computations. In practice it has been found that in many cases reaches considerably longer than tv may be permissible, and inflow and discharge may be used as a measure of storage in a reach.*

7-5. Deriving the Storage Relationship for a Reach

To define the relationship between inflow, discharge, and storage for a reach, two methods are available. In practice the two are combined whenever possible; however, they will be considered separately here.

(1) *Deriving the Storage Relationship from Hydrographs.* Considering the conditions listed on page 171, Eq. 7-4 may be re-written in the form:

$$(I_1 + I_2) - (D_1 + D_2) + \frac{2S_1}{t} = \frac{2S_2}{t}. \tag{7-7}$$

If, for at least one reasonably large flood, hydrographs of both inflow into and discharge from the reach are available, successive solutions of Eq. (7-7) at intervals throughout the flood period are possible, and there results a set of values for $2S/t$, each value being related to one or more specific pairs of values of I and D. Given enough data of this type, a set of D versus $(2S/t) + D$ curves can be constructed, with discharge *minus* inflow $(D - I)$ as parameter; and this set of curves is then available for the routing of other floods through the reach.

To illustrate this method, let us consider the reach of the Licking River between Toboso and Dillon, Ohio. This reach is 15 mi in length. Inflow and discharge hydrographs for the flood of June 3-5, 1944, are plotted in Fig. 7-14. The average velocity of the stream is known to be about 2 mph. For a reach of 15 mi this suggests a minimum integration time interval of 8 hr, if storage is to be considered a function of I and D alone. On the other hand, the hydrographs indicate that a 2-hr interval (the minimum time interval of record) is about the maximum for which I and D can be considered linear functions of time. Evidently, 15 mi is too long a reach on this particular stream for S to be considered a unique function of I and D unless the hydrograph to be routed through the reach has slopes similar to the hydrograph used to establish the storage rela-

*This discussion is necessarily rather limited in scope. The student will find a more detailed presentation in the book by H. A. Thomas, *The Hydraulics of Flood Movements in Rivers* (Carnegie Institute of Technology, 1934).

tionship. Since the hydrographs (Figs. 7–14 and 7–16) are similar, we shall take an integration interval of 2 hr.

Rates of flow (inflow and discharge from Fig. 7–14) are tabulated by 2-hr intervals in columns (3) and (5) of Table 7–5. The fact that the total discharge for the period is slightly less than the total inflow indicates that the intermediate drainage area is contributing little or

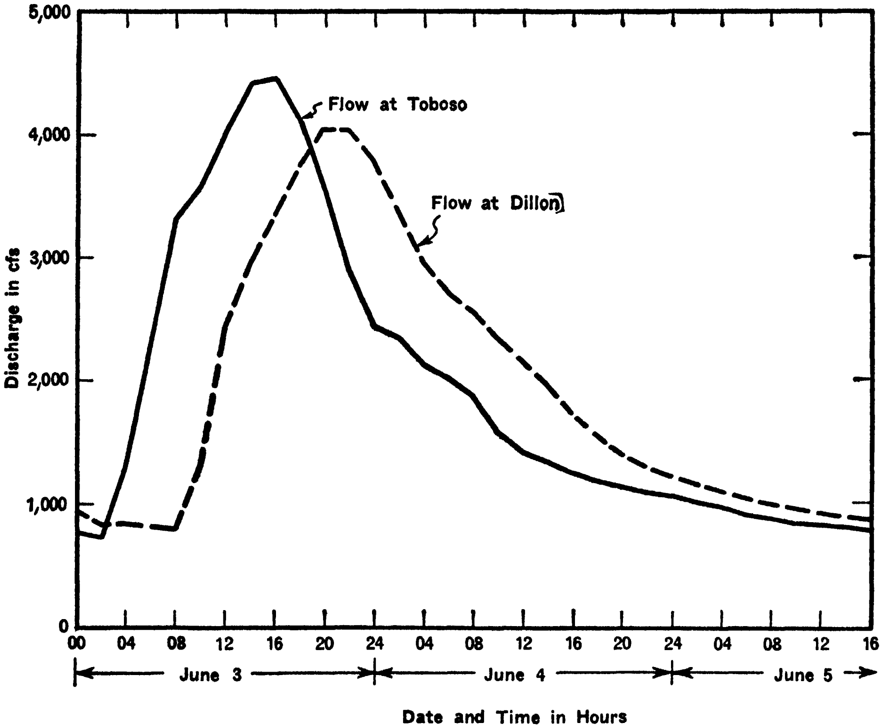

FIG. 7–14. Inflow and discharge hydrographs, June 3–5, 1944, Toboso-Dillon Reach, Licking River, Ohio.

nothing to the flood. Columns (4) and (6) through (9) show all the work necessary for the successive solutions of Eq. (7–7) at 2-hr intervals. The initial value of 2000 in column (9) is arbitrary, since we are interested in change of storage and not in absolute values.

A word about how the points of beginning and ending were chosen. By hypothesis, a specific value of S corresponds to each pair of simultaneous values of I and D when both are either increasing or decreasing at the same rate. Whenever a pair repeats, S should return to the same value it had on the first occurrence of that pair. Such repetitions provide the only checks on whether the river is performing in accordance with

our hypothesis; they also provide "balance points" between which minor discrepancies can be distributed more or less logically. As a first step, then,

TABLE 7-5

COMPUTATIONS FOR $\frac{2S}{t} + D$ CURVES, TOBOSO-DILLON REACH, LICKING RIVER, OHIO

$$(I_1 + I_2) - (D_1 + D_2) + \frac{2S_1}{t} = \frac{2S_2}{t}$$

(1) Date	(2) Time	(3) I (Cfs)	(4) I_1+I_2 (Cfs)	(5) D (Cfs)	(6) D_1+D_2 (Cfs)	(7) $D-I$ (Cfs)	(8) (4)—(6) (Cfs)	(9) $\frac{2S_2}{t}$ = (8) + $\frac{2S_1}{t}$ (Cfs)	(10) Adjustment* (Cfs)	(11) Adjusted $\frac{2S_2}{t}$ = (9)—(10) (Cfs)	(12) $\frac{2S_2}{t}+D_2$ (Cfs)
		I_1		D_1				$\frac{2S_1}{t}$			
		I_2	I_1+I_2	D_2	D_1+D_2	D_2-I_2	$(I_1+I_2)-(D_1+D_2)$	$\frac{2S_2}{t}$			$\frac{2S_2}{t}+D_2$
June 3	00	789	.....	941	......	....	....	2000†	.....	....	....
	02	766	1555	856	1797	+ 90	— 242	1758	.....	1758	2614
	04	1320	2086	856	1712	— 464	+ 374	2132	46	2086	2940
	06	2415	3735	825	1681	—1590	+2054	4186	91	4095	4920
	08	3300	5715	802	1627	—2498	+4088	8274	137	8137	8939
	10	3590	6890	1320	2122	—2270	+4768	13,042	183	12,859	14,179
	12	3995	7585	2420	3740	—1575	+3845	16,887	229	16,658	19,078
	14	4415	8410	2980	5406	—1435	+3010	19,897	274	19,623	22,603
	16	4465	8880	3350	6330	—1115	+2550	22,447	320	22,127	25,477
	18	4100	8565	3725	7075	— 375	+1490	23,937	366	23,571	27,296
	20	3500	7600	4040	7765	+ 540	— 165	23,772	411	23,361	27,401
	22	2890	6390	4040	8080	+1150	—1690	22,082	457	21,625	25,665
	24	2455	5345	3767	7807	+1312	—2462	19,620	503	19,117	22,884
June 4	02	2340	4795	3350	7117	+1010	—2322	17,298	549	16,749	20,099
	04	2120	4460	2980	6330	+ 860	—1870	15,428	594	14,834	17,814
	06	2020	4140	2700	5680	+ 680	—1540	13,888	640	13,248	15,948
	08	1900	3920	2560	5260	+ 660	—1340	12,548	686	11,862	14,422
	10	1595	3495	2350	4910	+ 755	—1415	11,133	731	10,402	12,752
	12	1440	3035	2140	4490	+ 700	—1455	9678	777	8901	11,041
	14	1340	2780	1960	4100	+ 620	—1320	8358	823	7535	9495
	16	1250	2590	1740	3700	+ 490	—1110	7248	868	6380	8120
	18	1180	2430	1550	3290	+ 370	— 860	6288	914	5374	6924
	20	1128	2308	1400	2950	+ 272	— 642	5746	960	4786	6186
	22	1080	2208	1290	2690	+ 210	— 482	5264	1006	4258	5548
	24	1050	2130	1220	2510	+ 170	— 380	4884	1051	3833	5053
June 5	02	1005	2055	1150	2370	+ 145	— 315	4569	1097	3472	4622
	04	975	1980	1090	2240	+ 115	— 260	4309	1143	3166	4256
	06	925	1900	1040	2130	+ 115	— 230	4079	1188	2891	3931
	08	890	1815	990	2030	+ 100	— 215	3864	1234	2630	3620
	10	850	1740	945	1935	+ 95	— 195	3669	1280	2389	3334
	12	825	1675	912	1857	+ 87	— 182	3487	1326	2161	3073
	14	800	1625	875	1787	+ 75	— 162	3325	1371	1954	2829
	16	775	1575	850	1725	+ 75	— 150	3175	1417	1758	2608
Total.	......	62,699	......	62,073	......	......	......	......	......	......	......

*Adjustment $= \frac{3175-1758}{31} = 45.71.$

†Arbitrarily assumed value.

the I and D curves in Fig. 7-14 were examined with a view to finding two identical pairs of simultaneous values of I and D, with both increasing or decreasing at the same rate. The only two nearly identical pairs occur at 0200 on June 3 and 1600 on June 5, and they accordingly became the beginning and ending points for the computations.

On completing the computation of column (9), we find ourselves out of balance by 1417 cfs. Before we proceed further, it is well to consider just what this amounts to and what the possible sources of the discrepancy may be. We note, first, that this represents roughly 6 per cent of the maximum amount that was in storage at any time during the flood and, next, that it is equivalent to a steady flow of approximately 23 cfs for the entire period, which is about 3 per cent of the minimum inflow and only about one-half of 1 per cent of the maximum inflow. As for its possible sources, we have to consider the following factors:

(1) *Retention by the Soil.* The river rises about 5 ft during the passage of the flood. If the banks have a slope of 2 to 1, this means that a strip 20 ft wide and 15 mi long is inundated for an appreciable length of time. If this strip were dry to begin with and the top 3 ft in depth took up a total of 1 ft of water which it did not return, this would account for about 30 per cent of the discrepancy.

(2) *Outflow Bypassing Gage.* Nothing is known (to us) in detail of the characteristics of this watershed. But it is quite possible that flows of the order of magnitude of several second-feet may leave the stream through cracks in the rock within this reach and thus not be included in the flow measured at the downstream gage.

(3) *Inaccuracy in Rating Curves.* It is only necessary to assume a 1½ per cent error in the plus direction on the Toboso rating curve above the 2500-cfs level, and a 1½ per cent error in the minus direction on the Dillon gage above the same level to account for the entire discrepancy.

With only the data as given at our disposal, it is not possible to pursue any of these possibilities further. All suggest, however, that the amount of the discrepancy is not unreasonable.

We now proceed to distribute the error uniformly with respect to time. This adjustment and the final result are shown in columns (10) and (11) of Table 7–5. Add columns (5) and (11), and we have in column (12) a set of values of $(2S/t) + D$, each value related to a specific pair of values of I and D. In Fig. 7–15 each value of D is plotted as an ordinate against the corresponding value of $(2S/t) + D$ as an abscissa, and each such plotted point is labeled with the corresponding value of $D - I$. A family of D versus $(2S/t) + D$ curves, with $D - I$ as parameter, is now constructed on these points. If data from two or three floods were available, the additional points would assist in interpolating these curves more accurately.

Use of Fig. 7–15 in solving flood-routing problems will be explained by example in Art. 7–6, beginning on page 181. First, however, it should be tested empirically. The test (omitted from the example) requires that data from several floods be available. It may be performed in either of two ways:

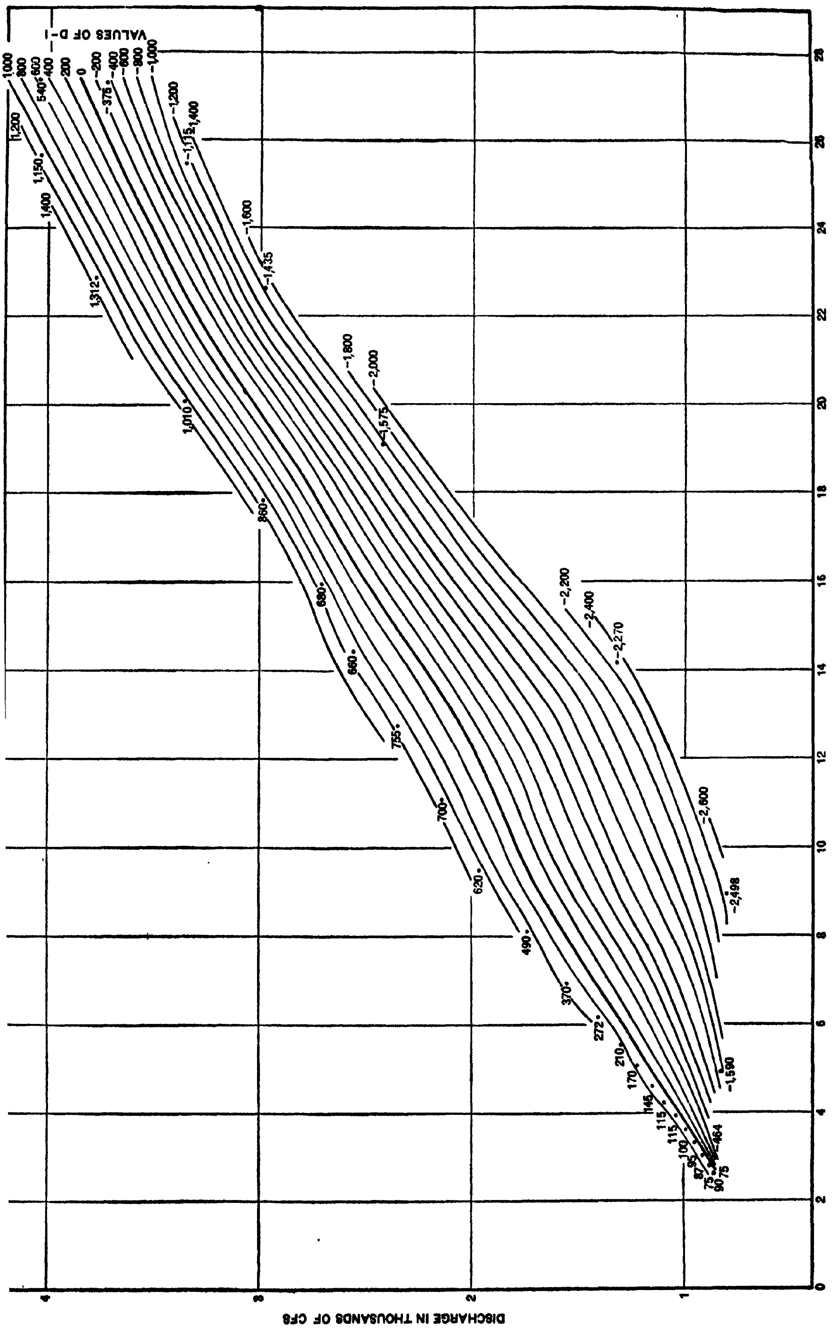

FIG. 7-15. $2S/t + D$ curves for Toboso-Dillon Reach, Licking River, Ohio.

(1) Base the curves on *part* of the available data, then route the remaining floods through the reach and compare the resulting discharge hydrographs with those actually observed. The more nearly they coincide, the stronger is the confirmation. This method has the advantage of giving a quantitative indication of the effects of any discrepancies that may exist.

(2) Use *all* the available data for computing values of $(2S/t) + D$ and plot as above. The values for closely adjacent points derived from the various floods should be mutually consistent. This method has the advantages of making use of all the data in plotting the curves, and of indicating the range of values of $D - I$ throughout which a consistent relationship is maintained.

(2) *Deriving the Storage Relationship from Surveys and Hydraulic Computations.* If hydrographs are not available for the downstream end of the reach (or for either end of the reach), the method used above cannot be applied, and the procedure for developing the storage relationship is as follows:

(1) Obtain actual rating curves or develop hypothetical rating curves for both ends of the reach and for as many intermediate sections as is possible and desirable.

(2) From detailed topographic and hydrographic maps, compute the cross-sectional area of the channel, for various elevations of water surface, at intervals along the reach.

(3) From (1) and (2) compute water surface profiles through the reach for a range of steady-flow conditions extending from some arbitrary minimum value of $I = D$ lower than the lowest value that will occur in the routing computation to a maximum value of $I = D$ at least equal to the maximum value that will occur in the routing computation.

(4) From steps (3) and (2) compute the volume in the reach under each of the water surface profiles. The storage under the lowest profile may be assumed and only the increase in volume due to an increase in flow considered. These volumes comprise a set of values for S, each value being related to one specific set of values of $I = D$. These values establish the D versus $(2S/t) + D$ curve for $D - I = 0$.

(5) Using as a basis the profiles computed in step (3), sketch additional profiles for various selected conditions of nonsteady flow and check, at least approximately, by hydraulic computations.

(6) Compute the volume in the reach under each of the profiles of step (5). These volumes comprise an additional set of values of S, each value being related to one specific pair of values of I and D. Given enough of these points, the D versus $(2S/t) + D$ curves for various values of $D - I$ can be established with reasonable accuracy, and this set of curves is then available for the routing of floods through the reach.

The amount of routine calculation required to develop a set of curves by this method for any but uniform channels is so great as to make a practical example unwarranted in the present text. It should be noted that no test of the reliability of the curves is possible in the absence of actual recorded hydrographs for each end of the reach for at least one flood.

7-6. Example of Flood Routing in a Stream

In Fig. 7-16 the given hydrograph (shown by a solid line) is that of a hypothetical flood in the Licking River at Toboso, Ohio. Given also is

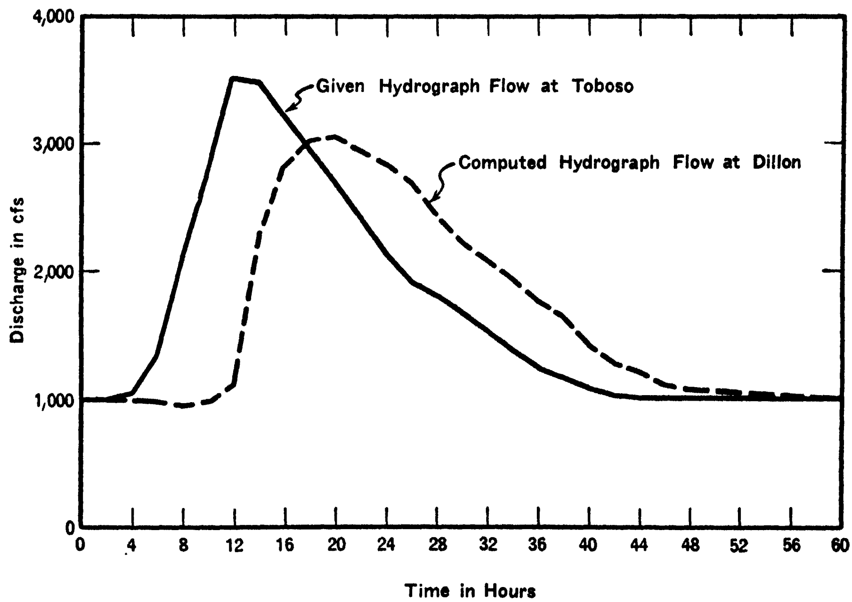

FIG. 7-16. Inflow and discharge hydrographs, hypothetical flood, Toboso-Dillon Reach, Licking River, Ohio.

the initial discharge at Dillon, 15 mi downstream. It is desired to compute the resulting hydrograph at Dillon. The curves of Fig. 7-15 apply to this reach and cover a sufficient range of flows to serve for routing this particular flood.

As before, we select a 2-hr integration interval. The routing computations, given in Table 7-6, are based on Eq. (7-4) and are similar to those used for the retarding-basin example. In the first line of Table 7-6, columns (2) and (5) are given. The value in column (4) may be determined from Fig. 7-15. Column (7) is column (4) *minus* twice column (5).

However, in subsequent lines the values of D and $D - I$ are both unknown, and values must be found by trial. To illustrate this trial procedure, in

TABLE 7-6

ROUTING COMPUTATIONS, TOBOSO-DILLON REACH, LICKING RIVER, OHIO

(1) Time (Hr)	(2) I (Cfs)	(3) I_1+I_2 (Cfs)	(4) $\frac{2S}{t}+D$ (Cfs)	(5) D (Cfs)	(6) $D-I$ (Cfs)	(7) $\frac{2S}{t}-D$ (Cfs)
	I_1			D_1		$\frac{2S_1}{t}-D_1$
	I_2	I_1+I_2	$\frac{2S_2}{t}+D_2$	D_2	D_1-I_2	$\frac{2S_2}{t}-D_2$
0	1000	....	3850	1000*	0	1850
2	1000	2000	3850	1000	0	1850
4	1040	2040	3890	1000	− 40	1890
6	1360	2400	4290	990	− 360	2310
8	2080	3440	5750	975	−1105	3600
10	2760	4840	8440	995	−1765	6450
12	3500	6260	12,710	1130	−2370	10,450
14	3450	6950	17,400	2330	−1120	12,740
16	3200	6650	19,390	2820	− 380	13,750
18	2960	6160	19,910	3040	+ 80	13,830
20	2660	5620	19,450	3060	+ 400	13,330
22	2400	5060	18,390	2960	+ 560	12,470
24	2120	4520	16,990	2830	+ 710	11,330
26	1920	4040	15,370	2670	+ 750	10,030
28	1790	3710	13,740	2440	+ 650	8860
30	1640	3430	12,290	2240	+ 600	7810
32	1490	3130	10,940	2110	+ 620	6720
34	1390	2880	9600	1940	+ 550	5720
36	1270	2660	8380	1780	+ 510	4820
38	1170	2440	7260	1650	+ 480	3960
40	1080	2250	6210	1440	+ 360	3330
42	1020	2100	5430	1310	+ 270	2810
44	1000	2020	4830	1200	+ 200	2430
46	1000	2000	4430	1120	+ 120	2190
48	1000	2000	4190	1070	+ 70	2050
50	1000	2000	4050	1040	+ 40	1970
52	1000	2000	3970	1030	+ 30	1910
54	1000	2000	3910	1020	+ 20	1870
56	1000	2000	3870	1010	+ 10	1850
58	1000	2000	3850	1000	0	....
Total	50,300	....	....	50,200	....	....

* Known at start of computation.

any except the first line of Table 7-6 column (2) is the given inflow hydrograph value, which is added to the preceding value in column (2) to give column (3). To this value of $(I_1 + I_2)$ is added $(2S_1/t) - D_1$ from the preceding line of column (7), giving the value of $(2S_2/t) + D_2$ in column (4). Entering Fig. 7-15 with this value, we must find values of D and

$D - I$ such that column (5) *minus* column (2) equals column (6). These values are recorded in columns (5) and (6), and column (7), is, then, column (4) *minus* twice column (5). The computed hydrograph at Dillon is shown as a dashed line in Fig. 7-16. The computed decrease in discharge at Dillon at the start of the period is a result of the assumption of similarity in hydrographs made in computing the $(2S/t) + D$ curves, which makes the discharge at Dillon decrease until the effect of the increased inflow at Toboso is felt.

Special Problems and Techniques

7-7. Taking Account of Intermediate Inflow

Thus far we have dealt only with floods originating entirely upstream from the reach in question. Local accretions, in the form of tributary inflow or contributions from ground water or precipitation within the reach, have been ignored. It was this simplification that permitted us to reduce the general hydrologic bookkeeping equation to the form of Eq. (7-1), and to define surface profile as simply as we did in the preceding section. Reaches that meet these simplified specifications are not too frequently encountered in actual practice.

In the general case there will be tributaries entering some, if not all, of the reaches into which the river must be divided for routing purposes; and the flow of these tributaries may be sufficient to mask completely the attenuating effects of storage in the main stream channel. Moreover, the flood hydrographs of the tributaries are not necessarily synchronous, or even likely to be synchronous, with the flood hydrograph at the upstream end of the reach, so that lag may also be masked—even to the extent that in extreme cases the peak discharge from a reach may precede the peak inflow to the reach.

To take proper account of storage changes, the *total* rate of inflow to the reach during each integration interval must be available. In routing through a natural river system, then, the rate of flow of tributaries entering each reach must be added to the rate of inflow of the main stream, interval by interval, throughout the flood period. This is a matter of some difficulty, particularly if the tributaries are not gaged. Unitgraph techniques are often employed in such cases to develop tributary hydrographs; and, as a last resort, one may even have to assume that ungaged areas are contributing flow at the same rate per square mile as adjacent gaged subbasins.

In developing the storage relationship (from hydrographs), another complication lies in the difficulty of "making the books balance." To accomplish this, the total volume of inflow during the flood period must

equal the total volume of discharge for the same period, within reasonable limits. But the errors in estimating tributary inflow are likely to be considerable, so that it is usually necessary to consider the first composite inflow hydrograph as an approximation, compute its volume, and then adjust the inflow ordinates sufficiently to make up the difference between that volume and the "correct" value given by the known discharge hydrograph.

7–8. Backwater Effects

Every point on a stream that is flowing at greater than critical depth is, of course, a point on a backwater curve. But, when hydrologists speak of a point as being "subject to backwater effects," they have more than this in mind. The phrase implies special effects, usually variable, produced by a reservoir or some other work of man or by tributary inflow or some other natural phenomenon. Such effects introduce severe complications in flood-routing procedure. Two examples are:

(*a*) In a series of reservoirs, like those of the Tennessee Valley Authority, the pool of one reservoir may at times be high enough to affect the tailwater elevation at the next dam upstream. This changes the discharge-elevation function of the outlet works at the upstream dam; consequently, storage in the upper reservoir can no longer be defined as a single-valued function of D.

(*b*) In a river of relatively gentle slope, like the Ohio, stage-discharge relations may be completely upset for a considerable distance upstream from the confluence of a major tributary during a high flood on the latter. It is even possible for the direction of flow of the main stream to be reversed in such circumstances.

7–9. Computing the Effects of Flood Control Projects

Routing studies are essential to the design of any important flood control project and must be sufficiently accurate and detailed as to provide a sound basis for comparing the economics of alternative designs. Studies made for such a purpose do not involve any essentially new principles, but they do involve special problems that may easily be overlooked.

Flood control may be effected by (1) reservoirs, (2) levees, (3) channel improvements, or (4) diversions. Reservoirs may be either of the uncontrolled type (retarding basins) or equipped with gates or valves to permit regulation of outflow. Channel improvements include any type of work designed to improve the carrying capacity of the stream—e.g., changes in alignment, dredging, cutoffs, overbank clearing, and removal of obstructions. The four basic types of project differ markedly in their principles of action and in their effects (both intentional and incidental) on the

height of flood crests and the shape of hydrographs. Table 7-7 has been prepared to clarify the major points of difference that have a bearing on

TABLE 7-7*

PRINCIPAL METHODS OF FLOOD CONTROL
(Information of Value in Planning a Flood-Routing Study)

Works such as →	RESERVOIRS	LEVEES	CHANNEL IMPROVEMENTS	DIVERSIONS
protect areas located →	downstream from the dam	landward from the levees	adjacent to and/or upstream from the improvement	adjacent to and/or downstream from the point of diversion
by →	holding back a part of the flood	confining the flow	reducing the stage required to carry a given flow	transferring a part of the flood to another watercourse
The operation of the works is →	either automatic (retarding-basin type) or subject to control	automatic	automatic	either automatic or subject to control
The intended effect on flood stages throughout the benefited areas is →	to reduce the *crest* stages of all floods, it being recognized that this is accomplished at the expense of prolonging the duration of intermediate stages	to eliminate flooding on the landward side of the levees. There is no *intended* effect on river stages, but a general *raising* of river stages may result incidentally	a general reduction throughout all floods, with maximum effects at the higher stages, up to bankful stage. (Lesser effects as the stage goes above bankful)	a general reduction of all stages above a predetermined stage at which the works are designed to go into action
Lands required for project construction include →	the reservoir areas and dam sites	strips on which levees are to be located; also borrow-pit areas, and possibly right-of-way for drainage ditches	none, except when the project involves realignments or cutoffs	the floodway areas
Some economic use may still be made of some of these areas, as indicated: →	Soil-tillage, pasturing, etc. may be continued in reservoir areas subject to varying degrees of interruption during every flood period	Levee tops may be used for highway or railroad right-of-way		Soil tillage, pasturing, etc. may be continued in floodway areas, subject to total interruption when project works go into action
Under certain conditions, the incidental effects of such works on crest stages or flood duration may increase the flood damage in areas located →	within or downstream from the benefited area	upstream, within, and/or downstream from the leveed reach	downstream from the improvement	downstream from the floodway outlet

(*Continued on next page.*)

TABLE 7-7*—*Continued*

PRINCIPAL METHODS OF FLOOD CONTROL—*Continued*

Works such as →	RESERVOIRS	LEVEES	CHANNEL IMPROVEMENTS	DIVERSIONS
Protection of project works against floods greater than that for which they are designed to provide flood control benefits is →	provided by excess spillway capacity and/or freeboard on dam	provided by freeboard on levees; and sometimes also by fuse-plug sections	not usually necessary	provided by freeboard at sides of inlet structure and freeboard on levees, if there are any, confining the diverted flow
In the above statement "Greater than design flood" means →	"greater in total volume." Height of inflowing crest is of no consequence unless it occurs when reservoir is full.	"greater in crest stage." Total volume of flood becomes important only in case freeboard is exceeded, in so far as damage to project structures is concerned	"greater in crest stage." Total volume of flood is of no consequence, in so far as damage to project structures is concerned	"greater in crest stage." Total volume of flood becomes important only in case freeboard is exceeded, in so far as damage to project structures is concerned
On occurrence of a flood greater than that for which project works are designed to provide flood control benefits, damage to *protected areas* will be →	more or less in proportion to amount that flood exceeds design value in total volume	nil	more or less in proportion to amount that flood exceeds design value in crest stage.	(*Exception:* If diversion is from a leveed reach, see remarks under "Levees")
	unless works are overtopped, in which case there may be major damage to works and possibly total loss of benefits to protected areas, plus additional damages			
Occurrence of two floods in rapid succession →	may result in reduced benefits from the works in the second flood period	has no effect on the performance of the project works		

*NOTES: (1) Works of two or more types are frequently combined in flood control projects. (2) In multipurpose projects, or in flood control projects that must be fitted into an over-all plan of water resources development, the requirements of irrigation, navigation, power development, recreation, and/or water supply may affect the design, the economics, and/or the operation plan of the flood control works.

flood-routing studies. Acquaintance with this table is assumed in what follows.

Reservoirs. In flood-routing studies for reservoir projects, attention should be given to the natural storage in the proposed reservoir area—that is, the storage existing in the reach before the reservoir is constructed.

In Fig. 7-17, I represents a flood hydrograph at a point on the stream in the vicinity of the backwater limit of the proposed reservoir, and D represents the hydrograph of the same flood at a point near the proposed dam site. D reflects a moderate amount of both lag and attenuation. A routing study to determine reservoir effects should be based on I and would yield the discharge curve D'. In evaluating the reservoir benefits, comparison should be made between curves D' and D rather than between D' and I, for the reservoir cannot properly be credited with attenuation that would have taken place without it.

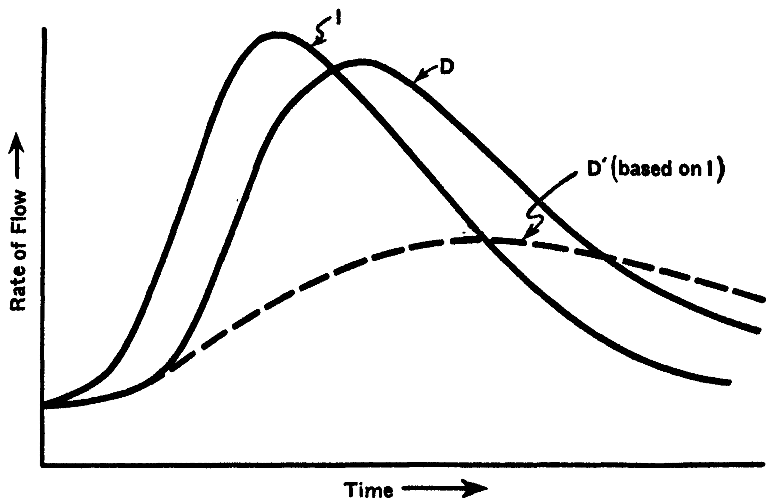

FIG. 7-17. Relationship between the hydrographs involved in a reservoir routing study.

It will be observed that in reservoir flood control, reduction of peaks is accomplished at the expense of prolonging the duration of intermediate stages. It is quite possible for damages to result from this effect if the protected area consists of artificially drained farm land; for, as long as river stages remain above the outlet elevation of the drains, the functioning of the latter is impaired. From the standpoint of getting crops in, a prolonged period of sogginess may be almost as damaging as a short period of inundation. Routing studies should accordingly give attention to intermediate stages as well as to peak flows.

Another point worthy of attention in reservoir-routing studies is the possibility that the attenuation may actually increase flood peaks at some point downstream. Consider, for example, a stream with two tributaries entering from opposite directions at about the same point; consider, further, that the normal direction of storm movement or differences in

drainage-basin characteristics may ordinarily result in a time difference of 48 hr between the cresting of these two tributaries. Under such conditions it may well be that a flood control reservoir on the quicker-cresting stream would so modify its hydrograph as to add to the peak flow in the main stream near its confluence with the two tributaries. Again, it is possible to conceive of a system of reservoirs each of which provides adequate protection for the tributary on which it is located, and yet which together increase peak stages in the main stream. A system may not disclose such tendencies when tested by the "official design flood" and yet may perform adversely even with a storm of lesser magnitude if the time distribution of precipitation is unfavorable. This lack of flexibility is a principal argument against the automatic, or retarding-basin, type of reservoir. Reservoirs of adequate capacity with controlled outlets can presumably be so operated as to avoid such consequences, provided that an adequately staffed and centrally directed flood prediction service is available to supervise the operation.

Levees. It is by no means easy to predict the effect of levees on the shape of the hydrograph or height of the flood crest. In the process of confining a flood, the levees deny to the river a considerable amount of storage formerly available to it in overbank areas. One effect of a levee system is to impede normal attenuation and thus *tend* to make flood peaks downstream from the system higher than they were before its construction. There are, however, other effects, somewhat less obvious, that must also be taken into account.

Fig. 7–18 is a conventionalized reach of river, with a proposed levee system indicated by dotted lines. Let the rating curves at A and B, prior to construction of levees, be identical, and assume that for a steady flow of 17,000 cfs the water is 5 ft deep over the overbank areas. Installation of the levees can have little effect on the rating curve at B, but it does increase the normal overbank depth for a flow of 17,000 cfs in the reach AB to approximately 7 ft. The backwater curve is on the order of 10 mi in length; hence if the leveed reach is 10 mi or more in length, the water will stand some 2 ft higher at A for a flow of 17,000 cfs than it did before. This suggests the possibility that a levee system may increase flood damages in the area upstream from the leveed reach, and it also indicates the importance of taking increased stages into account in levee design, in order to provide adequate freeboard to prevent damage from overtopping within the protected area itself.

Any alteration of the rating curve at A affects the storage relationship in the reach upstream from A. If in a particular case this effect is appreciable, it is bound to be in the direction of *increasing* the storage corresponding to a given rate of flow, thus adding something to the natural regulatory effect of the upstream reach. Conditions can be conceived under which this effect is sufficient to offset the loss of storage *within* the leveed

reach; in such cases the increase in flood peaks downstream from the leveed reach, previously mentioned as a possibility, might not take place.

Clearly, there is a wide variety of possible effects from a levee system and enough chance for major damages that a thorough routing study is

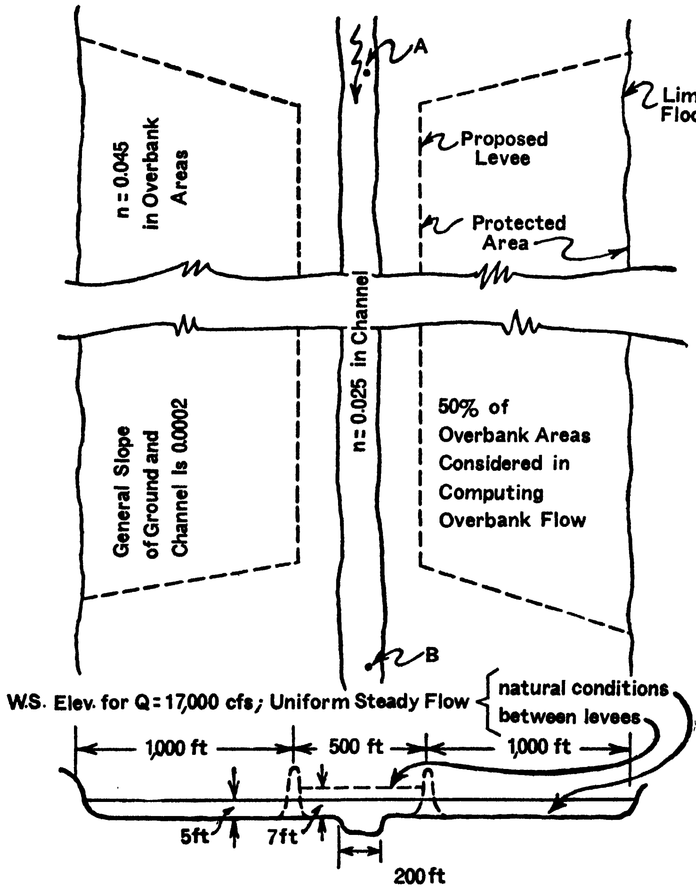

FIG. 7-18. Hypothetical reach and proposed levee system

warranted in the planning of any project of even moderate size. For a quick study the following steps should give results on the side of safety:

(*a*) Obtain or compute *natural* rating curves for *A* and *B* (as they would be before the levees were built).

(*b*) Make reasonable assumptions about the hydraulic properties of the leveed reach and, by the customary methods, compute backwater curves through the reach for various rates of steady flow.

(*c*) From the computations in step (*b*) prepare a *revised* rating curve for *A*, and compare it with the natural rating curve for that point. The differ-

ence in stages for a given flow is a fair indication of what additional flooding might be produced in the upstream reach, by the levees, for a flood of that size. It also provides flow-line data on which to base levee heights.

(4) To determine downstream effects, *ignore* any increase in storage in the upstream reach and assume that there is no attenuation in the leveed reach. In other words, transfer the natural hydrograph of the design flood at A from that point to B, with no modification except for time of travel, and compare it with the natural hydrograph of the design flood at B. The difference in peak stages is a fair indication (on the side of safety) of what additional flooding might be produced in the downstream reach, by the levees, for a flood of that size.

Channel Improvements. By lowering the stage corresponding to a given flow, channel improvements tend to modify the storage relationship in the direction of reducing the amount of storage in the reach adjacent to, and upstream from, the improvements. This reduces the natural attenuation and thus tends to increase flood peaks downstream. Whether the tendency is significant or not is a matter of degree and must be investigated in any particular case. We are on the side of safety in proceeding as follows:

(*a*) Starting at the control downstream from the improvement, compute the water surface profile for a *steady* discharge equal to the peak flow of the design flood (1) for the natural channel and (2) for the improved channel. Carry both computations back upstream until the profiles become essentially identical. This locates the upstream limit of effectiveness of the improvement.

(*b*) Obtain or compute a natural hydrograph of the design flood at the point thus determined, and route it through the *natural* channel to the downstream control point.

(*c*) Assume that this flood traverses the entire reach of *improved* channel without any attenuation. In other words, transfer the natural hydrograph of the design flood at the upstream point, through the reach, without any modification except for time of travel, and compare it with the natural hydrograph of the design flood at the downstream control. The difference in peak stages is a fair indication (on the side of safety) of what additional flooding might be produced in the downstream reach by construction of the improvement.

Diversions. No new difficulties of investigation are introduced by diversions. One must be sure, however, that routing studies are carried far enough downstream *on the channel to which the flow is diverted* to make certain of what, if any, adverse effects are likely to be produced there.

7-10. Mechanical Aids and Short Cuts

The complexities of flood routing have led a number of experienced engineers to the opinion that no important routing study should be con-

sidered complete until it has been verified by the use of hydraulic models. Certainly, the use of models for such purposes should always be given thorough consideration. The Corps of Engineers, U. S. Army, has been among the leaders in this type of work, and its model studies of the lower Mississippi River levee, floodway, and cutoff program are outstanding examples of the technique. Facilities for model work are not always directly available to the practicing engineer, but in all parts of the country there are laboratories able and willing to undertake such projects. Perhaps of even more importance than their facilities is their trained personnel; for models in the hands of unskilled technicians can yield results no more credible than those of any other method.

Mechanical integrators in various forms have also been devised, but they are special-purpose machines, expensive to build, and perform no function other than a purely mechanical one of making computations. Perhaps the distinction between their function and that of the hydraulic model can best be expressed by saying that, whereas the model *sets up* and solves a differential equation, the mechanical integrator can only solve the equation that is given to it.

The latest step in the development of aids to routing studies is the application of electronics. At Ohio State University the use of the cathode-ray oscilloscope has been shown to be feasible for this purpose. Like the mechanical integrators, this device can do no more than solve the equations given it; it can, however, solve equations considerably more complex than do the integrators. Its possibilities for relieving the computer of much tedious work deserve thorough investigation.

In the absence of mechanical aids, there is still available to the skilled technician a legion of short-cut methods that do much to reduce the work involved in a flood-routing study. They are thoroughly reported in the literature and, being mainly matters of technique, need scarcely be discussed here. Valuable as they are to the expert, *in the hands of the apprentice they may be risky tools.*

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CHAPTER 8

INFILTRATION THEORY AND THE ANALYSIS OF THE HYDROGRAPH

INTRODUCTION

The processes of infiltration have been discussed in Chapter 5. In this chapter we shall study infiltration in more detail and shall apply certain basic theories concerning retention, infiltration, and overland flow to the analysis of the hydrograph. For the sake of brevity we shall call this combination of ideas the "infiltration theory."

The infiltration theory is primarily based on the fact that total rainfall in any storm equals the sum of total surface runoff plus total infiltration, neglecting the negligible losses by evaporation. Other ideas that are basic to the infiltration theory are the following:

(*a*) A soil has a maximum rate of absorption of moisture, called its "infiltration capacity." Infiltration capacities vary seasonally and also during each storm.

(*b*) Unless the rainfall rate exceeds the infiltration capacity, there is no surface runoff.

(*e*) When rainfall rates exceed infiltration capacities, the excess rainfall begins to "pile up," initially filling small depressions in the ground ("depression storage").

(*d*) When depressions are filled to overflow, water begins to move toward streams by overland runoff. This water on the ground above the overflow level of depression storage is called "detention storage."

(e) The rate of surface runoff is a function of the average depth of detention storage, that is, the depth of detention storage is the head motivating runoff ("overland flow" theory).

The objectives of an analysis of a hydrograph are therefore (1) a curve of infiltration capacities and (2) a curve of the relation between surface runoff and detention storage. These data may be used to synthesize the hydrograph from rainfall data for other periods on the same stream. The relation curve of surface runoff–detention storage applies only to the gaging station for which it is derived, but infiltration capacities may apply, with varying accuracy, to larger areas of the same soil type or same soil characteristics. Infiltration capacities are therefore of use not only in "building up" flow records from rainfall data on streams in the same general area but also have many other applications, particularly to soil moisture studies.

Deriving Infiltration Curves from Data on Small Areas

8–1. Sprinkled-Plot Experiments

The rainfall-runoff relation, according to infiltration theory, depends on the infiltration capacities and on the relation between surface runoff and detention storage. An understanding of the above theory may best be obtained by the study of an infiltration experiment. We shall first take up the simplest case, that of sprinkled plots, with artificial rainfall applied at a constant rate.

For convenience the abbreviations and symbols used in this chapter are tabulated below:

Notation

q = Surface runoff in in./hr,
Q = Mass surface runoff in in.,
i = Rainfall intensity in in./hr,
P = Mass rainfall in in.,
V_d = Depression storage in in.,
D_a = Detention storage in in.,
F = Mass infiltration in in.,
f = Infiltration capacity in in./hr,
f_c = Ultimate infiltration capacity, the nearly constant capacity reached, usually after several hours of effective rainfall,
t_o = Time of beginning of rainfall,
t_v = Time of beginning of depression storage,
t_d = Time of beginning of detention storage,
t_s = Time of beginning of surface runoff,
t_c = Time when infiltration capacity becomes approximately constant,
t_e = Time of end of rainfall,

t_r = Time of end of runoff,
t_z = Time of end of infiltration,
Q_r = Mass residual runoff, i.e., total runoff after the end of rain,
F_r = Mass infiltration during residual runoff, from D_a,
q_r = Residual runoff in in./hr,
f_r = Residual infiltration in in./hr, from D_a,
I_s = Interception storage, in in.

The graphs of Fig. 8–1 are the measured rainfall and runoff from a 6.6 × 33-ft plot on Marshall silt loam, in Nebraska.*

For this experiment the average rate of applied rainfall was 3.46 in./hr. Runoff did not start until after 29 min of application. Initially, all the

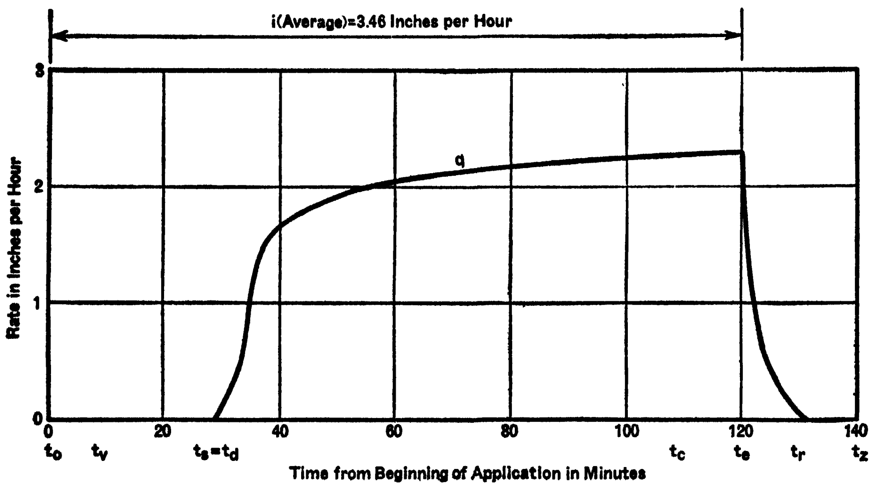

FIG. 8–1. Data from sprinkled plot experiment on Marshall silt loam. (From Duley and Kelly, *Circular 608*, Fig. 5, U.S. Department of Agriculture.)

applied rainfall was absorbed, so the infiltration capacity exceeded 3.46 in./hr. At some unknown time, before runoff started, the infiltration capacity decreased until it equaled the rainfall, and depression storage began. After 29 min, runoff started, depression storage being largely filled. In this case runoff started at the same time that detention storage began. After about 100 min the runoff became essentially constant (at time t_c). At 120 min the sprinklers were turned off (t_e), and runoff stopped at 131 min (t_r), when detention storage equaled zero. Depression storage continued to add to infiltration until some unknown time (t_z).

*F. L. Duley and L. L. Kelley, "Surface Condition of Soil and Time of Application as Related to Intake of Water," U.S. Department of Agriculture, *Circular No. 608* (August 1941).

The basic data from this experiment are plotted as mass curves in Fig. 8-2. By the analysis of these curves we shall determine the curves of mass infiltration and of infiltration capacity. By subtraction we obtain the curve labeled P − *Q*, or total accumulated rainfall *minus* total accumulated runoff. The total infiltration at any time is equal to total precipitation *minus* total runoff and *minus* all water temporarily stored

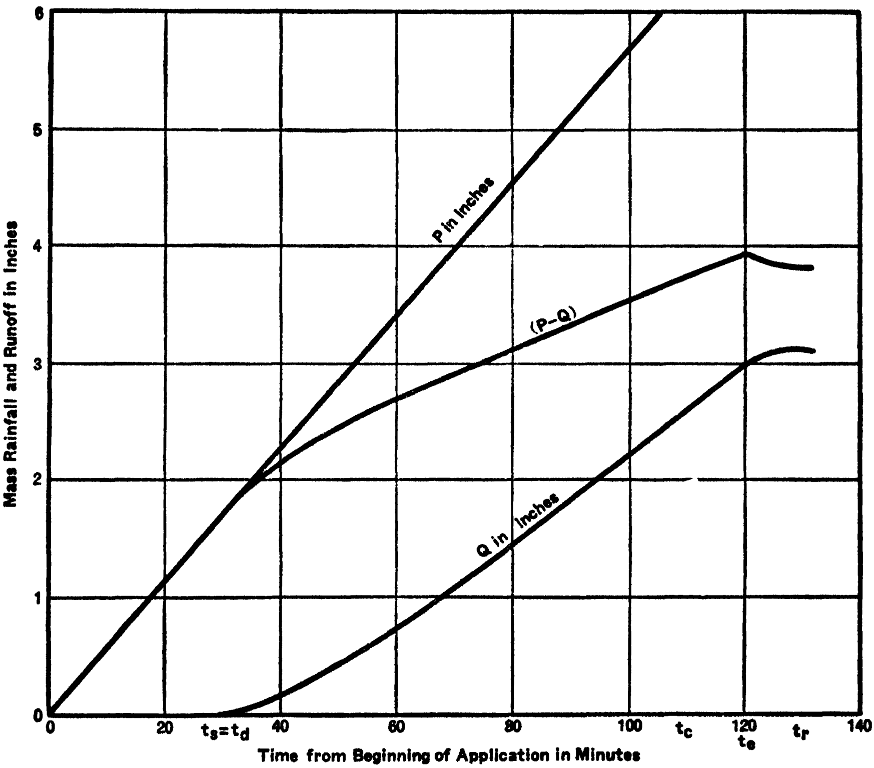

FIG. 8-2. Mass curves of artificial rainfall, runoff, and ($P - Q$).

above the ground surface. The storage factor consists of depression storage and detention storage. In other words, $P - Q = F + V_d + D_a$, F being mass infiltration, V_d being depression storage, and D_a being detention storage, all expressed in inches. In order to determine F, we must determine V_d and D_a, and the available data do not appear to give us much information on these two variables. We know that V_d began at an unknown time t_v, increased to a maximum at about t_c, was constant to t_e, and decreased gradually to zero at an unknown time t_z. D_a began at t_d, was a maximum and approximately constant from t_c to t_e, and de-

creased to zero at t_r. The $P - Q$ curve gives us no indication of the value of f, except during the short period from t_c to t_e, when the two storage factors are practically constant. For this period, since storage does not change, $i - q = f_c$, or $3.46 - 2.27 = 1.19$ in./hr infiltration capacity. But for the other periods the $P - Q$ curve or its slope $(i - q)$ does not indicate the infiltration rate because of changing storage. A more involved analysis is necessary to determine the storage changes. It should be noted carefully that the $i - q$ curve does not equal infiltration except during periods of constant storage.*

That part of the hydrograph after rainfall ceases may be used to determine a relationship between q and D_a, thereby reducing the number of unknowns by one. At the time that rainfall stops, V_d and D_a are practically constant and at their maximum values for this particular experiment. All runoff after rain ceases, called "residual runoff," is from D_a. Residual infiltration is from both D_a and V_d, but V_d has no effect on runoff, and we shall disregard it for the time being. D_a at any time during residual runoff, then, equals mass residual runoff plus mass residual infiltration from D_a as a source, or $D_a = (Q_r + F_r) = \Sigma(q_r + f_r)$. We know the relation between q and f at time t_e; it is

$$\frac{f_e}{q_e} = \frac{1.19}{2.27} = 0.525.$$

We shall assume that this ratio or division of D_a between q and f will apply throughout the residual period. We can then say

$$\begin{aligned} D_a &= \sum (q_r + f_r) = \sum q_r\left(1 + \frac{f_r}{q_r}\right) = \sum q_r\left(1 + \frac{f_c}{q_e}\right) \\ &= \sum q_r(1 + 0.525). \end{aligned}$$

By the above equation, summing q_r back from time t_r, values of D_a may be computed. Fig. 8–3 is a plot on logarithmic scales of computed D_a against q. The relation curve may then be used to compute a D_a curve for the entire hydrograph, from values of q. The D_a curve and the $F + V_d$ curve obtained by subtracting D_a from $P - Q$ are shown in Fig. 8–4.

This leaves V_d to be determined. Unfortunately, there is no method for computing it in this case, and it must be estimated.† However, there

*Some $i - q$ curves have been published as equal "for all practical purposes" to infiltration rates, a statement that is definitely incorrect.

†V_d can be determined approximately on sprinkled plots by starting a second artificial rain at time t_z, when depression storage disappears. The infiltration rate may be assumed to be approximately constant, so that all the variables are known except V_d, and the storage equation can be solved to determine V_d. But we are primarily interested in hydrographs of stream flow rather than sprinkled-plot experiments, and V_d must be estimated for stream hydrographs.

are several considerations to aid the judgment. Depression storage usually is less than ¼ in. (but this is, of course, not a rule). The F curve begins to flatten at time t_e and merges into the horizontal $P - Q$ curve at t_z. The lengths of time required for changes in D_a give clues to the lengths of time required for the accumulation of V_d and its decrease to zero after rain ceases. Based on these considerations, the F curve in

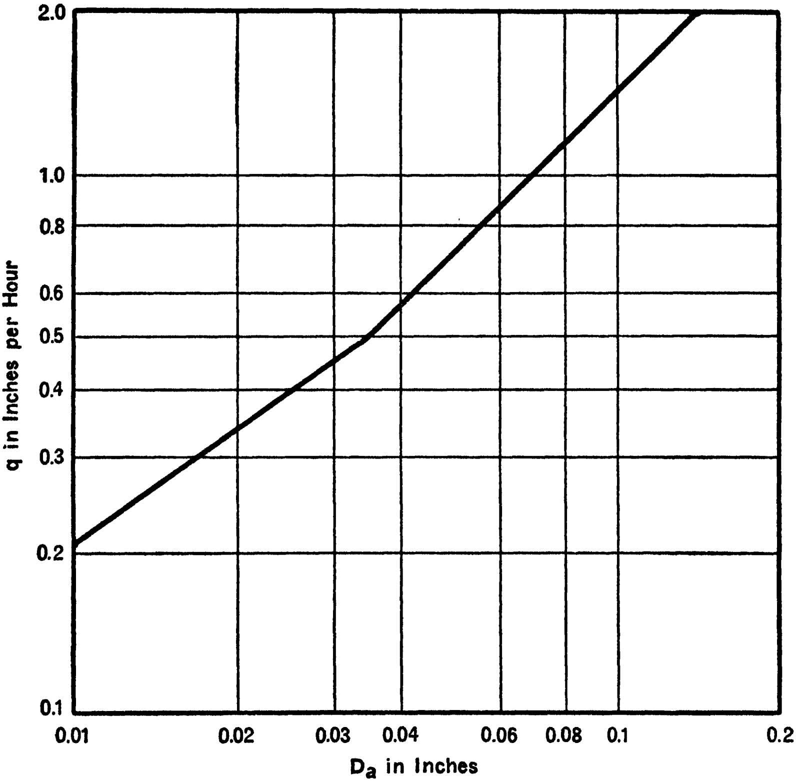

Fig. 8-3. Relation between q and D_a for experiment shown in Figs. 8-1 and 8-2.

Fig. 8-4 was sketched in, a distance equal to V_d below the $F + V_d$ curve. Time t_v is chosen as 26 min, and maximum V_d equal to 0.1 in. From the slope of the F curve the infiltration rates or capacities f now may be determined. Prior to t_v, or 26 min, the infiltration capacity is indeterminate, but the actual infiltration equals the rainfall rate. After t_e the infiltration rate is less than capacity. The f curve, based on the slope of the mass F curve, is the objective of the analysis, and is shown in Fig. 8-4. The ultimate infiltrative capacity, f_c, is the most important result of this study.

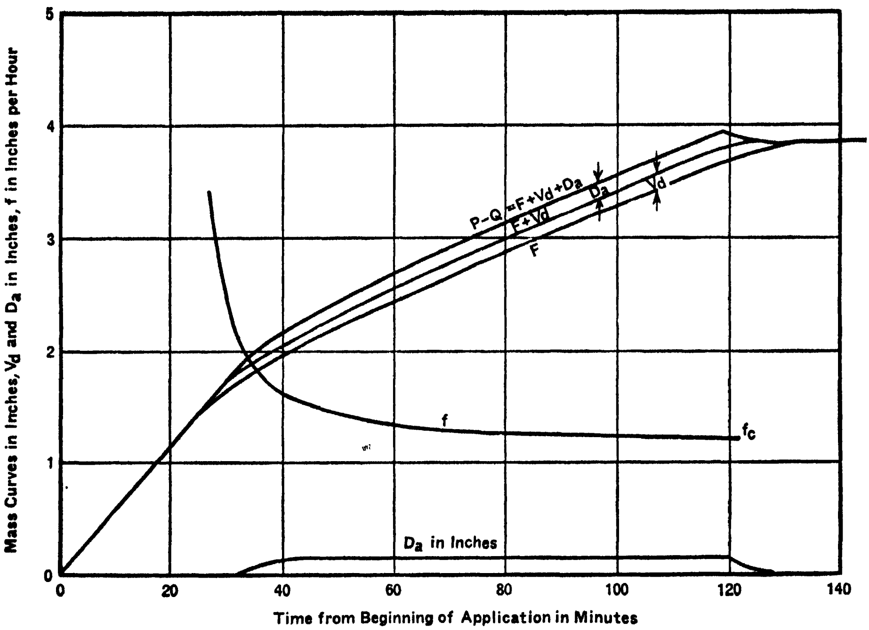

FIG. 8–4. Analysis of hydrograph.

8–2. Small Natural Areas

The same general principles apply to the analysis of rainfall and runoff for small natural drainage basins. For streams small enough not to be complicated by channel storage or ground-water seepage flow, the analysis will be similar to the example shown for a sprinkled-plot hydrograph, except that the precipitation rates are irregular and there may be a time interval or lag between points on the D_a curve and corresponding points on the q curve. Interception storage or loss must also be taken into account. With these exceptions there are no additional difficulties in the analysis for the more complicated natural runoff hydrographs.

Fig. 8–5 is a chart of the rainfall and runoff for a cultivated area of 1.69 acres near Coshocton, Ohio, for July 8, 1939.* The $P - Q$ curve in Fig. 8–6 is obtained by subtraction. A trial $F + V_d + I_s$ curve is sketched in as shown, with D_a beginning at 7:40 A.M., when runoff began. The negligible runoff after 8:20 A.M. may be disregarded, so the $F + V_d + I_s$ curve is horizontal beyond that point. By comparing times of peaks and troughs, it appears that there is about 1 min lag between D_a and q. This could be disregarded, but for purposes of illustration the trial D_a values

*"Hydrologic Data, North Appalachian Experimental Water Shed, 1939," *Hydrologic Bull. 1*, U.S. Department of Agriculture, 1941.

from Fig. 8-6 are plotted against the discharges 1 min later. The relationship is shown in Fig. 8-7. A straight line, roughly averaging the

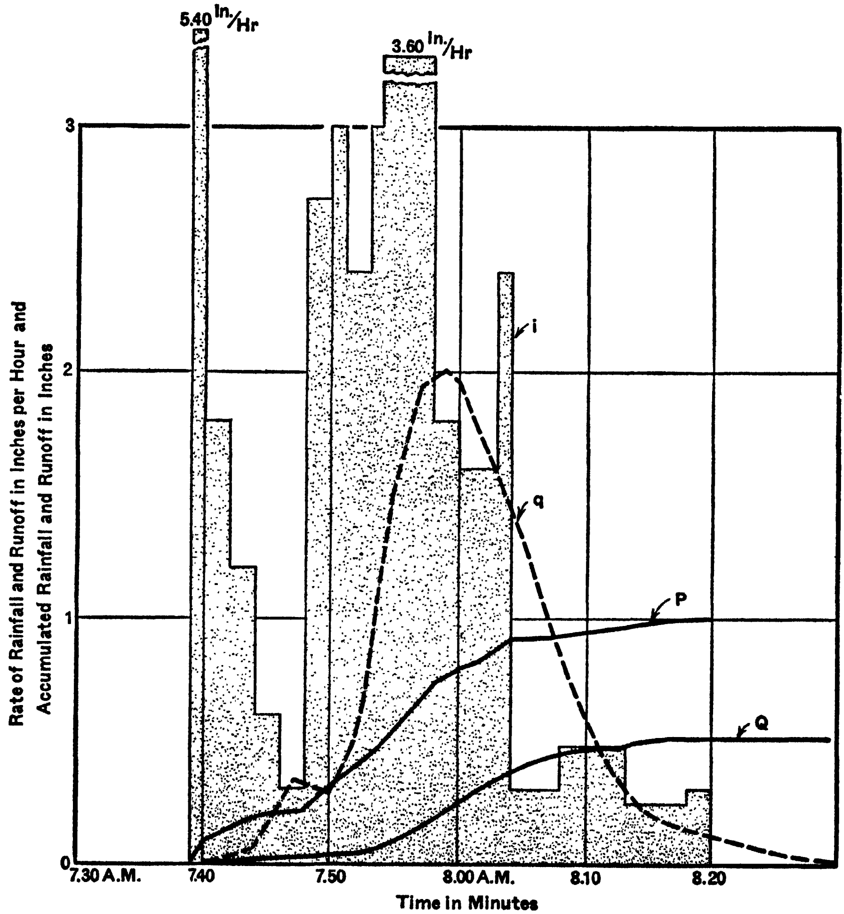

Fig. 8-5. Rainfall and runoff data for area 109, July 8, 1939, North Appalachian experimental area, Cochocton, Ohio. (From U.S. Department of Agriculture, *Hydrologic Bull. 1*, 1941.)

scattered points, has been drawn, and the D_a values computed from this line are plotted as a dotted curve in Fig. 8-6. Since the dotted line is at no point farther from the trial curve than 0.03 in., no new trial curve seems warranted.

The next step is to estimate the depression storage V_d and the interception I_s. Here again there is no simple rule-of-thumb to follow, and judgment and experience are required. V_d is at or near a maximum when D_a is at its maximum value, and it gradually decreases with diminishing rates of rainfall. I_s is estimated as 0.03 in. or less for cultivated crops. Based on the slope of the estimated mass infiltration F, the f curve of infiltration capacity is drawn. After 8:20 A.M. the infiltration is at a rate less than capacity because of diminishing supply. For this hydrograph

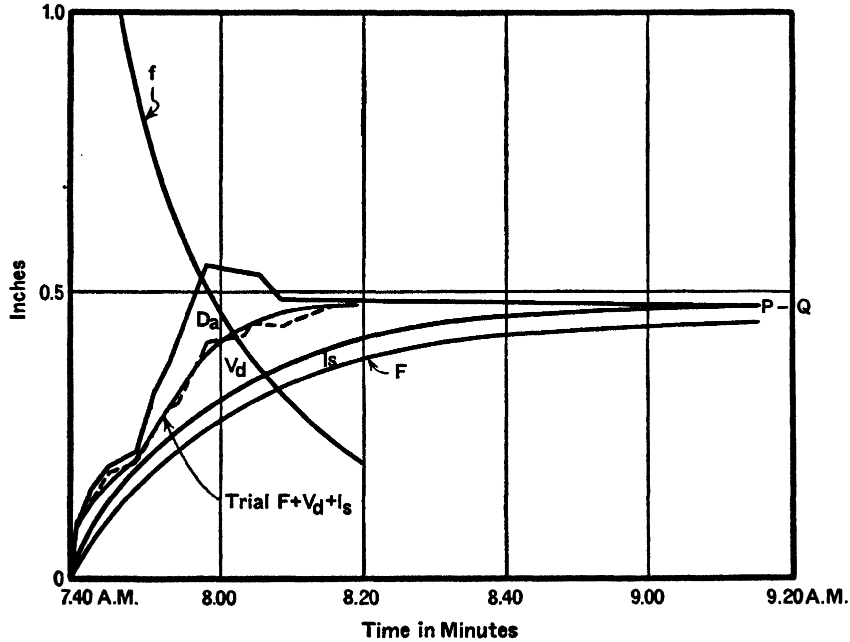

FIG. 8–6. Analysis of hydrograph for area 109, July 8, 1939.

there is no f_c or ultimate infiltration capacity, as the storm ceased before the f curve approached the horizontal.

A considerable amount of judgment is required in making computations of infiltration, and no two hydrologists would get exactly the same answer. This is a major disadvantage of the infiltration theory, but the same criticism applies to unitgraph computation and to nearly every other hydrologic computation. The student should work through each of the examples several times, using different estimated curves, so as to obtain several different results. Disregarding estimates that appear definitely illogical, it will be found that the range of results obtained is not very great.

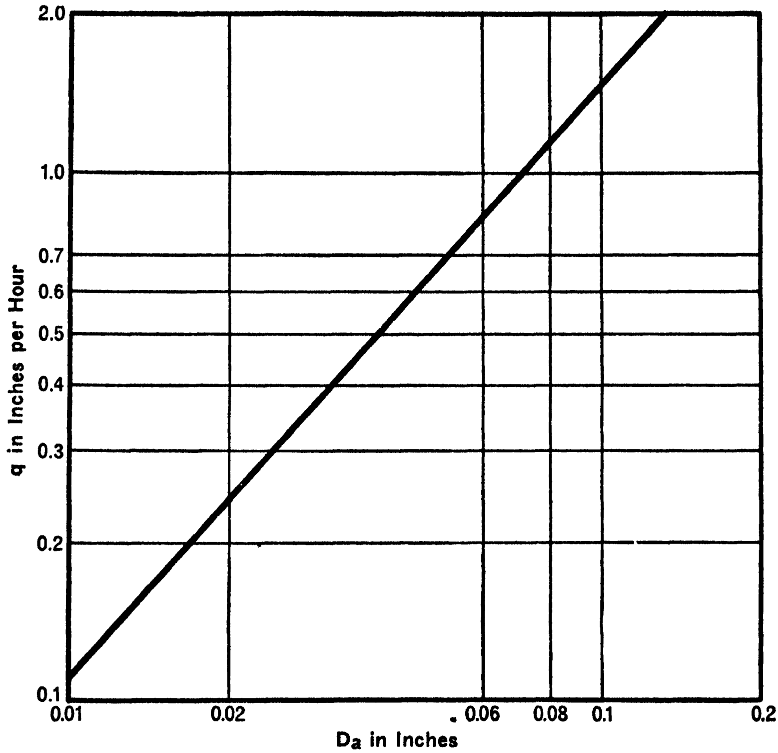

FIG. 8-7. Relation between q and D_a, for data shown in Figs. 8-5 and 8-6.

DERIVING INFILTRATION CURVES FROM DATA ON LARGER AREAS

8-3. Complications Introduced by Channel Storage and Ground-Water Flow

For streams with ground-water flow and channel storage the problem of analysis is much more complicated. The first step is the subtraction of estimated ground-water flow. Arcs of the normal depletion curve (see Chap. 5) are sketched in under the hydrograph, extending from the time of start of surface runoff forward and from the end of surface runoff backward. An **S**-shaped curve is then sketched in, joining the two arcs. This is identical to the procedure described in Chapter 6 for separating base flow from total flow. More care in estimating the base flow may be necessary in infiltration studies than in unitgraph computations because hydrographs of low peaks, with a large proportion of ground-water flow, may be pertinent to infiltration studies. In cases where ground-water flow is an important part of the total flow, records of ground-water levels

may be used to aid in sketching in the ground-water flow. Fig. 8–8 is a schematic diagram, showing a hydrograph of total flow and a hydrograph of ground-water flow based on arcs of the normal depletion curve.

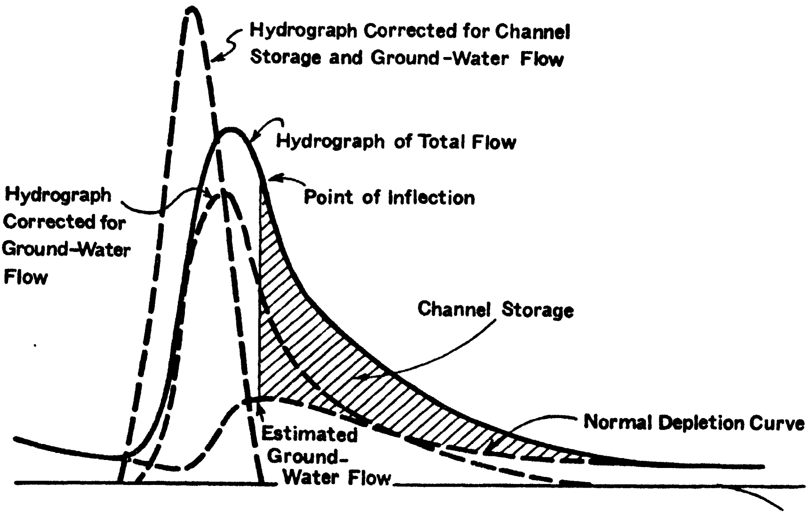

FIG. 8–8. Schematic diagram of hydrograph showing channel storage and ground-water flow.

8–4. Channel Storage Treated as of Reservoir Type

If we assume that all channel storage is of the reservoir type (see Chap. 7), then the computations of channel storage corrections are much simplified. Inflow and outflow will be equal at the peak of the surface-flow hydrograph, and all surface flow after the point of inflection on the falling side of the hydrograph may be assumed to be from channel storage. By summing the flow back from the time that surface flow ceases, the channel storage may be computed for any time after the inflection point, and a relation between surface-flow rates and channel-storage volumes plotted. By the use of such a relation curve, a hydrograph of inflow may be computed, reaching zero at the point of inflection and equal to the outflow at the outflow peak, with a total volume equal to the outflow volume and with a peak near the point of inflection on the rising side of the outflow hydrograph. An inflow hydrograph, corrected for channel storage and ground-water flow, is shown in the schematic diagram of Fig. 8–8. The inflow is the water flowing into stream channels by overland flow and is derived from detention storage. The inflow hydrograph, therefore, is the q to be plotted against D_a.

The weakness of the above method of analysis is that channel storage is seldom entirely of reservoir type. The storage on the falling side of the hydrograph is fairly well defined by a relation curve based on the

storage and flow after the point of inflection on the hydrograph, but this curve does not apply to rising stages. The errors involved may be quite

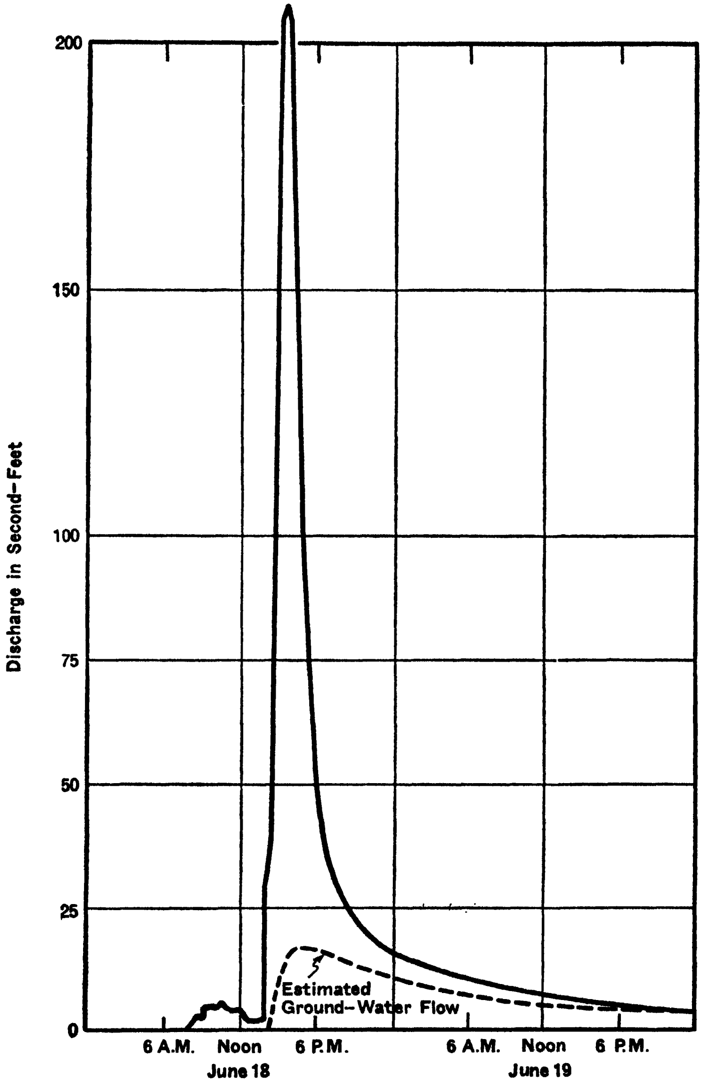

FIG. 8-9. Hydrograph, Home Creek, near New Philadelphia, Ohio, June 18-19, 1940.

large. However, we shall use this method in the next example to demonstrate the fundamentals of the problem.

Fig. 8-9 is a hydrograph of measured flow on Home Creek, near New Philadelphia, with a drainage area of 1.64 sq mi, for June 18-19, 1940.

The estimated ground-water flow is shown under the hydrograph of total flow and is based on the depletion curve *I* of Fig. 5–4*B*.

Fig. 8–10 shows the analysis of this hydrograph. The mass curve of precipitation is based on the average of four recording rain gages in the

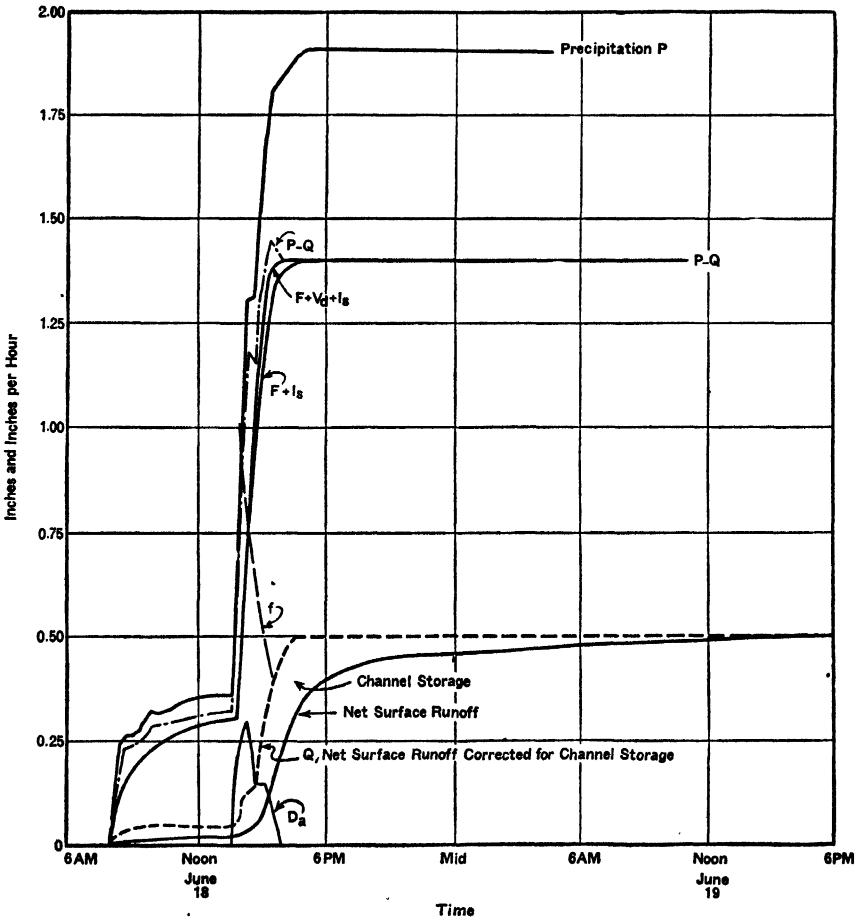

FIG. 8–10. Analysis of the hydrograph, June 18–19, 1940, Home Creek, near New Philadelphia, Ohio.

area. The mass curve of net surface discharge is the accumulated flow, in inches, computed from the hydrograph of Fig. 8–9, with ground-water flow subtracted. The point of inflection on the falling side of the hydrograph is at about 4:30 P.M. Accordingly, a horizontal line at 0.50 in., the total runoff, is extended back to 4:30 P.M. The vertical distances between

the net discharge curve and 0.50 in. are the total channel-storage values in inches for the period after 4:30 P.M. On Fig. 8-11 are plotted rates of discharge at various times after 4:30 P.M., against total channel storage at corresponding times, taken from Fig. 8-10. A straight line averages the plotted points very closely.

Assuming that the relation curve of Fig. 8-11 applies throughout the entire hydrograph, the channel-storage volumes corresponding to the flow rates prior to 4:30 P.M. have been taken from the curve and added to

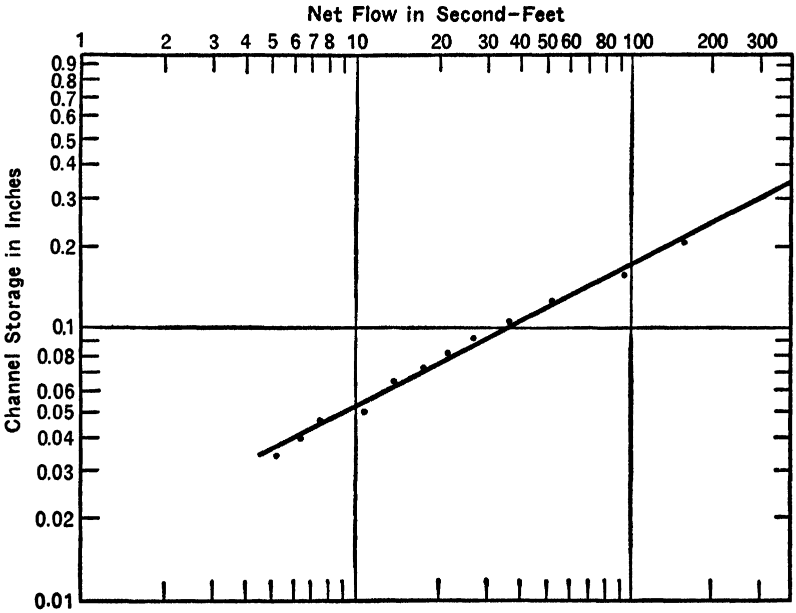

FIG. 8-11. Relation of net surface discharge and channel storage, Home Creek, near New Philadelphia, Ohio.

the mass curve of net surface runoff to obtain the Q mass curve of net surface runoff corrected for channel storage. Having computed Q, the $P - Q$ curve may be drawn.

An $F + V_d + I_s$ trial curve may now be sketched in, and the resulting trial values of D_a plotted against q. Here we run into complications because of the varying time interval or lag between D_a and q. Surface runoff begins at about 1:45 P.M., and D_a begins at about the same time. The peak of the outflow hydrograph is at 3:45 P.M., and correcting for channel storage brings the time of peak flow back to about 3:15 P.M. But the peak D_a value, by any reasonable trial curve of $F + V_d + I_s$, is 2:30 P.M. The lag interval between D_a storage and corresponding channel inflow q varies between zero and 45 min. An approximate relation curve

is shown in Fig. 8–12, but the variable lag obscures any close correlation. This is a major disadvantage of the infiltration–overland-flow theory and reduces its applicability, particularly to areas larger than about 1 sq mi.

Having decided on the D_a curve, we now complete the analysis in the same manner as for the smaller areas. In this example the interception

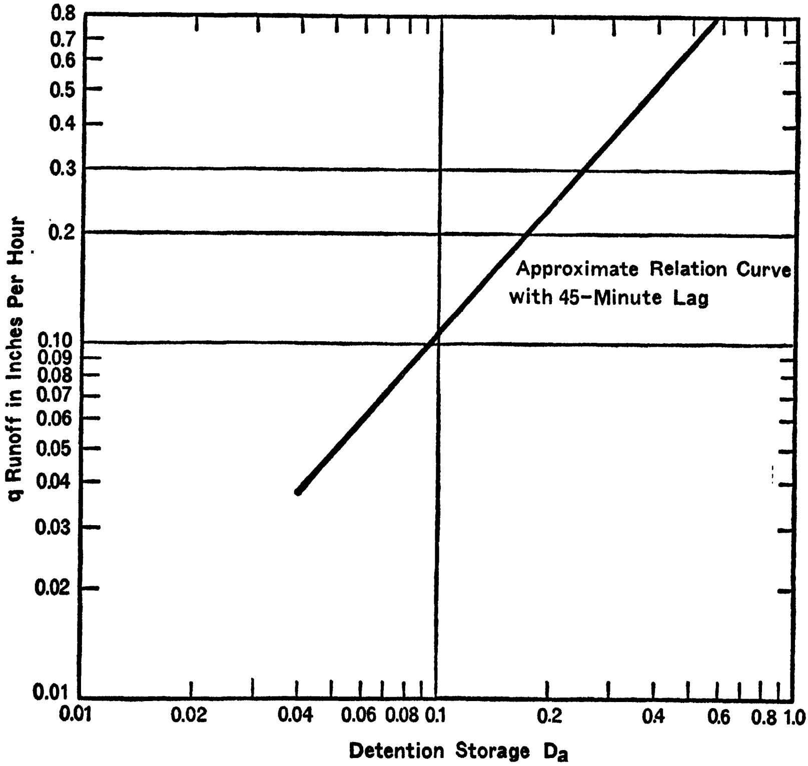

FIG. 8–12. Relation between q and D_a, Home Creek, near New Philadelphia, Ohio, June 18–19, 1940.

storage I_s has been considered to be small and relatively constant and has not been estimated. The rain is too short for an f_c to be reached. The derived f curve appears to be reasonable, but so much judgment has entered into the estimation of D_a, V_d, and I_s that no two hydrologists could be expected to obtain the same f curve.

8–5. A Refinement in Estimating Channel Storage

In the above example we assumed that channel storage was entirely of the reservoir type. From Chapter 7 we know that this cannot be

strictly true for a natural stream. If channel storage is plotted directly against discharge, a loop will be formed, since for a given amount of storage the discharge will be smaller on the rising side of the hydrograph than on the descending side. To avoid the formation of loops and to apply the information derived from the falling side of the hydrograph to both sides, the following method has been proposed.* The value of channel storage at any instant on the falling side of the hydrograph is plotted, not against the discharge at that instant, but against the geometric mean of the discharge at that time and the discharge obtained at a time interval T

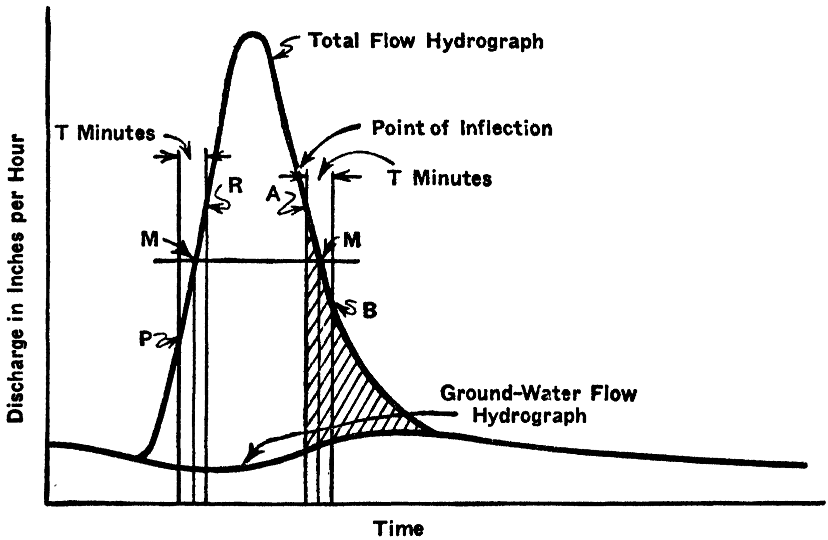

FIG. 8-13. Weighting discharge for estimation of channel storage during rising and falling stages. (From *Report on Analysis of Hydrologic Data for Index Areas, Muskingum River Basin, Ohio,* U.S. Engineer's Office, Huntington, West Virginia, August, 1943.)

minutes later, T being equal to thirty times the square root of the drainage area in square miles. Similarly, when the storage at a point on the rising side of the hydrograph is desired, the discharge at that instant is coupled with the discharge T minutes later, and the geometric mean is used in entering the channel storage curve. In Fig. 8-13 the channel storage at the time of the discharge P is equal to the channel storage at the time of A, and the geometric mean discharge M is plotted against this channel storage. The method is empirical but, if handled judiciously and flexibly, serves the purpose.

**Report on Analysis of Hydrologic Data for Index Areas, Muskingum River Basin, Ohio,* U.S. Engineer's Office, Huntington, W. Va., August 1943.

The discharge and channel storage data of Figs. 8–9 and 8–10 are plotted by this method in Fig. 8–14. The corrected mass curve of surface

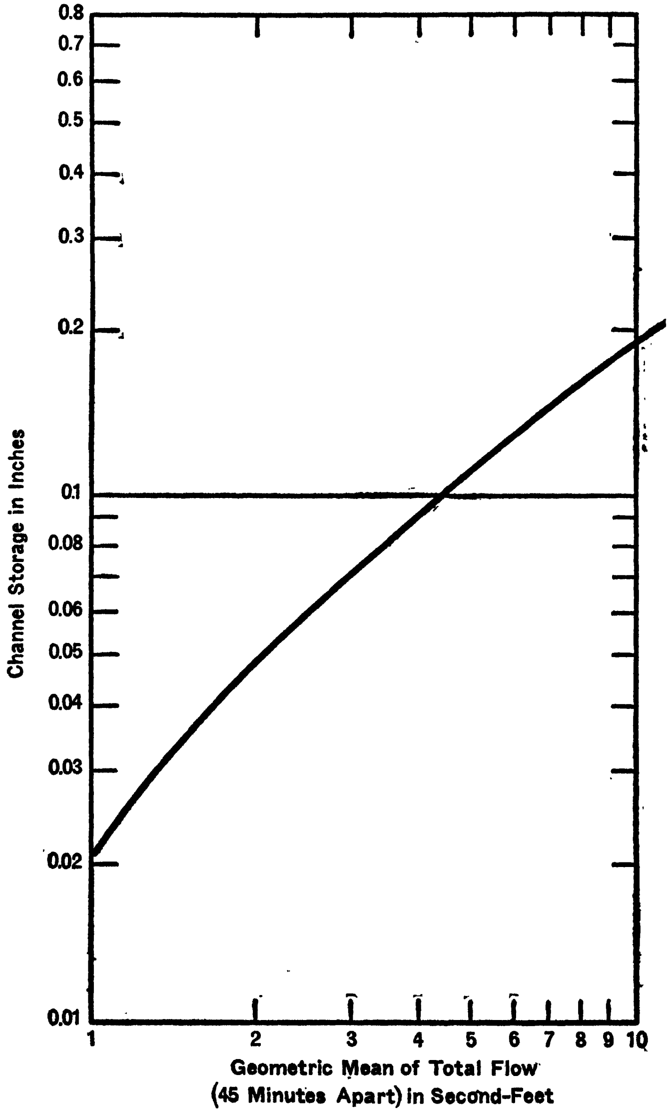

FIG. 8–14. Channel storage curve, Home Creek, near New Philadelphia, Ohio.

runoff so derived is compared in Fig. 8–15 with the mass curve used in the example of Fig. 8–10. If we assume that the $F + V_d + I_s$ curve of

Fig. 8–10 is correct, the corrected channel storage has the effect of reducing D_a to the values plotted in Fig. 8–15 and reducing the apparent lag by about 15 min. The new D_a values do not appear reasonable, but slight adjustments in V_d would take care of this without changing the

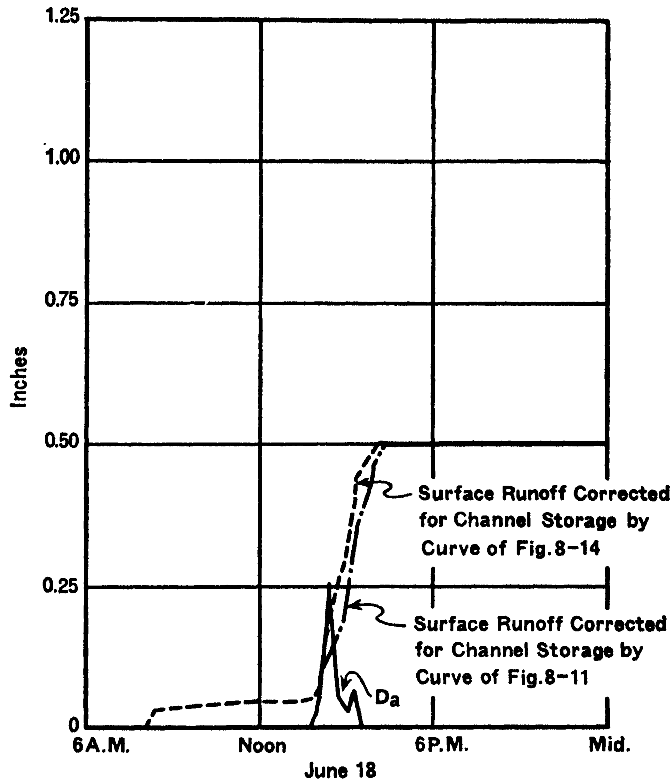

FIG. 8–15. Comparison of channel storage corrections, Home Creek, near New Philadelphia, Ohio.

derived f curve. For this problem the improved channel storage estimate has little effect on the end-result.

Applications and Limitations of Infiltration Theory

Perhaps the most important application of the infiltration theory is to land-use management and allied fields. Infiltration generally is a gain to the agriculturist, for it increases the water supply available to plants in both the aerated zone and the saturated zone of the soil. Any land management which increases infiltration will not only increase the water supply for crops but also reduce erosion. Increased infiltration into poorly drained soils may, at times, be harmful to crops. Infiltration is therefore

an important phase of the hydrologic cycle, of interest to both soil moisture studies and to ground-water investigations.

Again, the infiltration theory, in spite of its obvious limitations, may be of value as a means of estimating the percentage runoff factor to be applied to total rainfall before the application of unitgraph procedures. (It should be recalled that the unitgraph procedures described in Chap. 6 are "means of determining the time of distribution of surface runoff, but they do not solve the problem of determining how much runoff will occur under any given set of conditions.")* Knowledge of infiltration capacities is extremely valuable to flood forecasting and to the estimation of flood frequencies from rainfall records.

Some of the limitations of the theory are obvious. The wide range of infiltration capacities for a small area, not only seasonally but also within a single storm, limits the applicability of computed capacities. The wide latitude for judgment in analyzing hydrographs is another fault, though the same criticism might be made of many other hydrologic theories. The limited size of drainage area for which hydrographs can be satisfactorily analyzed, together with the limited applicability to large areas of infiltration capacities derived from small areas, are other major limitations.

The final test of a method of analysis of the hydrograph is the synthesis of the hydrograph, and in this the infiltration–overland-flow theory falls down. Channel storage curves derived from one hydrograph do not apply closely to another. The direction of storms, up the valley or down the valley, affects the channel storage relationship. Curves of discharge-detention storage are not constant. Partial-area storms are as difficult to synthesize as they are by the unitgraph procedures. It may be noted that the difficulties are with the application of the overland-flow theory rather than from failures of the basic infiltration theory. The combined infiltration–overland-flow theory as here presented has been applied successfully to rainfall-runoff problems of small areas, such as airport drainage,† highway drainage, and soil moisture studies. For larger areas the use of infiltration theory in computing net effective rainfall in the unitgraph method has improved the accuracy of results. Depending on the type of problem, certain phases of the combined infiltration–retention–overland-flow theory have more applicability than others.

Despite the limitations, the infiltration theory is a valuable tool, particularly in the study of rainfall-runoff relations on small areas, and knowledge of its principles and limitations is essential to an understanding of the involvements of the hydrologic cycle.

*F. F. Snyder, "Synthetic Unit Graphs," *AGU Transactions*, 1938.

†See Stifel W. Jens, "Drainage of Airport Surfaces—Some Basic Design Considerations," *ASCE Transactions*, 1948, pp. 785–836.

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CHAPTER 9

THE HYDROGRAPH AS A FUNCTION OF DRAINAGE BASIN CHARACTERISTICS

INTRODUCTION

9-1. The General Problem

To be able to examine a topographic map, perhaps walk over the ground, and then predict the flood regimen of an ungaged drainage area—that is at once the grand dream and the despair of hydrologists. The problem is almost infinitely complex, and the present approaches to it are mainly frontier trails. Nevertheless, enough has already been accomplished to provide practical solutions in a number of cases, and the published studies of several of the pioneers can be reduced to fairly simple terms.

A common empirical approach to a solution involves (1) selecting a number of drainage basin characteristics (e.g., shape, slope) that seem

likely to have an effect on the shape of the hydrograph; (2) selecting a number of gaged drainage basins possessing these characteristics in varying degree; (3) looking for correlations between these characteristics, on the one hand, and the observed flood regimens of the various basins, on the other; and (4) expressing the most significant correlations either graphically or mathematically in such form that they can be used to predict the flood regimen of ungaged basins.

The shape of any actual flood hydrograph is determined not only by drainage basin characteristics but also by the pattern of the storm that produces it. Thus the effect of the latter variable must be eliminated before the study of correlations in step (3) can be undertaken. For many years this was one of the major stumbling blocks in the way of an effective analysis. Discovery of the unitgraph principle did much to clear the way, by making it possible to reduce the flood hydrographs of a wide variety of streams to functions of drainage basin characteristics alone. This principle is, in fact, the basis of most of the useful approaches developed to date. In the present chapter, three of these approaches (Bernard's, McCarthy's, and Snyder's) will be presented in outline; in each of these it is assumed that unitgraphs (or distribution graphs) are available for all basins used in the study, and the problem is considered solved when an empirical relation is found between one or more parameters of the unitgraph, on the one hand, and one or more drainage basin characteristics, on the other, such that an acceptable unitgraph (or distribution graph) can be synthesized for basins not included in the study.

A fourth approach, that of Clark, is also outlined here. In its present form it does not constitute a "complete" solution to the problem as we have stated it, because a certain amount of hydrologic data for a stream is required before its flood regimen can be predicted. In other words, it is more properly a method for *deriving* the unitgraph than for *synthesizing* it. Nevertheless, it ties in logically with the present subject matter, and it is quite possible that future developments may relieve it of its present limitations.*

9-2. Physical Characteristics To Be Considered

Before discussing the various approaches, it will be helpful to list a number of the physical characteristics that may be expected to have some effect on the dimensions or shape of the unitgraph:

Area. The total volume in a unitgraph is rigidly proportional to the area of the drainage basin. If two drainage basins of different area are "hydraulically similar" in all respects, then (mathematically) the horizontal (time) dimensions of the corresponding unitgraphs are in the same ratio as the one-fourth power of the areas, and the vertical dimensions (discharges) are in the same ratio as the three-fourths power of the areas.

*In this connection, see Addendum at end of this chapter.

Overland Slope. This characteristic influences the velocity of overland flow and thus may be expected to be of importance in small basins where the time spent in overland flow is an appreciable part of the total time required for the water to reach the gaging station. As size of basin increases, this factor decreases in importance, and in any case it is easy to overestimate its significance. There may not be much difference in velocity of overland flow over a considerable range of slopes, and, where slopes are slight, the direction in which a farmer plows his field may be more significant than the slope as measured from a topographic map.

Channel Slope. Other things being equal, the steeper the channel slope, the greater the velocity of flow and the more peaked the unitgraph.

Size of Channel. As between two channels of equal slope, the one with the larger cross section has more storage capacity per mile and may therefore be expected to exert a greater regulatory or attenuating effect on the passage of a flood wave.

Condition of Channel. Low values of n result in higher velocities of flow and therefore, possibly, higher peaks.

Stream Pattern. A fan-shaped area with streams radiating more or less from a common point suggests the possibility of synchronized peaks from the constituent subareas, while an elongated area traversed by one major stream with more or less uniformly spaced tributaries suggests the possibility of a slower and less pronounced rise and recession.

Stream Density. Closely spaced tributaries suggest the possibility of more rapid runoff. As in the case of overland slope, however, it is easily possible to overestimate the importance of this characteristic.*

It is no small problem to develop an adequate, objective, quantitative measurement of some of these characteristics, yet this is essential to a solution. We cannot be content with subjective classifications like "flat" or "rolling" or "hilly," which have one set of connotations for the man who has spent his life in Illinois and an entirely different set for a resident of Utah. Even the description of a drainage system as "fan-shaped" or "elongated" involves a subjective interpretation and, at best, provides only a qualitative, rather than a quantitative, measure. And how is channel slope to be evaluated in such a way as to distinguish between two streams having the same *average* slope, if one of them drops at a fairly uniform rate and the other is made up of a series of relatively flat reaches separated by waterfalls or steep chutes?

Various investigators have developed various schemes of evaluating all these characteristics, and it is not necessary to catalog them here. Those required for an understanding of what follows will be explained as they are introduced; the most important thing for the beginner to remember is that the same term may be defined differently by different writers, and it is never safe to apply any method of analysis until these definitions are thoroughly understood. It is also worth noting that not all the items

*And may it not on occasion operate in the opposite direction—i.e., the more closely spaced the streams, the greater the storage and thus the smaller the peak?

in the preceding list are totally independent of one another, and thus it is sometimes possible to develop a single measurement that to a greater or less extent takes care of more than one characteristic. For example, over an extensive area, there may be a reasonably constant relation between overland slope and channel slope. Again, if the topography of a number of drainage areas is not too widely different, there is likely to be at least a moderate degree of correlation between area, on the one hand, and channel slope and channel size, on the other. Finally, there are ways of expressing basin shape numerically that take account, at least in part, of both stream pattern and stream density.

Bernard's Approach

Among the first to take advantage of the unitgraph principle in an attempt to analyze and synthesize the hydrograph from drainage basin characteristics was Bernard, whose first paper on the subject appeared in 1934.* His basic data consisted of 24-hr total rainfall records and daily average discharge records for six drainage areas in Ohio, varying in size from 500 to 6000 sq mi. Because he believed that any method developed should utilize "available data" only—namely, the published records of the U.S. Weather Bureau and the U.S. Geological Survey—he did not extend his study to the smaller drainage areas, for which time units of less than a day would have been required. For the same reason, he chose to introduce the concept of the "distribution graph" (see Chap. 6, p. 143) and to deal simply with the percentage of total runoff that occurred in each day of the surface-runoff period rather than to attempt any prediction of instantaneous peak flows. This self-imposed limitation is less necessary now than it was in 1934, because of the greatly increased number of recording rain gage and stream gage records available. Also it places a severe restriction on the usefulness of the results. Notwithstanding, Bernard's paper remains today one of the outstandingly important and original contributions to the subject.

Briefly, Bernard's approach is this:

(1) From a large mass of data (including many streams other than the six used in the paper here discussed) he segregated from the general expression of runoff the factors representing watershed characteristics into an expression of the form

$$U = f\left(\frac{P}{L}, F, S\right).$$

Here P is a constant that depends on the shape of the drainage basin and its "manner of concentration"—i.e., the stream pattern; L is the distance from the most remote portion of the drainage area to the outlet; F is a constant that ex-

*Merrill M. Bernard, "An Approach to Determinate Stream Flow," *ASCE Proceedings*, January 1934; reprinted with discussion in *ASCE Transactions*, 1935, pp. 347–395.

presses the shape and condition of the main channel; and S is the slope of the main channel. U may be called the "drainage basin constant."

(2) After examining the rainfall and discharge records, he selected two storm periods on one of the six streams used in the study and three storm periods on each of the other five and derived the corresponding 24-hr distribution graphs. For each stream the distribution graph used in the study was the average of the individual graphs for that stream.

(3) From topographic maps and general knowledge of the region, he computed or estimated the values of P, L, F, and S for each basin and substituted these values in the expression for U, thereby determining the drainage basin constants.

(4) On log-log paper, the 24-hr percentages from the distribution graphs were plotted against the U-values for the corresponding drainage basins (see Fig. 9–1). each day being designated by a special symbol and by number. In this form of plotting, each distribution graph becomes a series of points ranged along a vertical line.

(5) Straight lines were drawn diagonally across the diagram, each one being fitted as well as possible (presumably mainly by eye) to the various plotted points representing the percentages for a particular day. Thus the line labeled "fourth day" passes slightly above two of the fourth-day points, slightly below two others, and somewhat farther above the remaining two. Observe that the fitting aims to give over-all consistency to the diagram rather than a "best fit" for any one line.

The results (Fig. 9–1) were proposed to be used as follows: Suppose that it is desired to synthesize a distribution graph for an ungaged drainage basin. From topographic maps and other information the data necessary for the computation of U would be assembled, and U would be computed. The chart could then be entered at the bottom with that value of U, and the distribution graph percentage for each day read off at the intersection with the correspondingly designated sloping line. Bernard calls attention to the fact that the study approaches the lower limit of area to which the 24-hr interval is applicable.

We do not give here the specific form of the U function or the method for arriving at the proper values of P and F. The function is rather complicated, and F depends on data not obtainable from a U.S. Geological Survey map. Interested readers should consult the basic paper and the references given in it for details. It should be noted, however, that Jarvis,* in discussing the paper, suggested a simplified substitute expression that, under certain conditions, might be expected to give a fairly good approximation of the value of U, namely,

$$U \text{ (approx.)} = \frac{S^{0.5}}{L}.$$

This retains length and slope, which Jarvis considered to be the most important factors, and eliminates the others, which are perhaps less

*C. S. Jarvis, discussion, *ASCE Transactions*, 1935, pp. 363–365.

important and are certainly more difficult of determination. S is the fall, in ft per 1000, of the main channel of flow, and L is the length, in ft, which water must traverse in running from the most remote portion of the drainage area to the outlet.

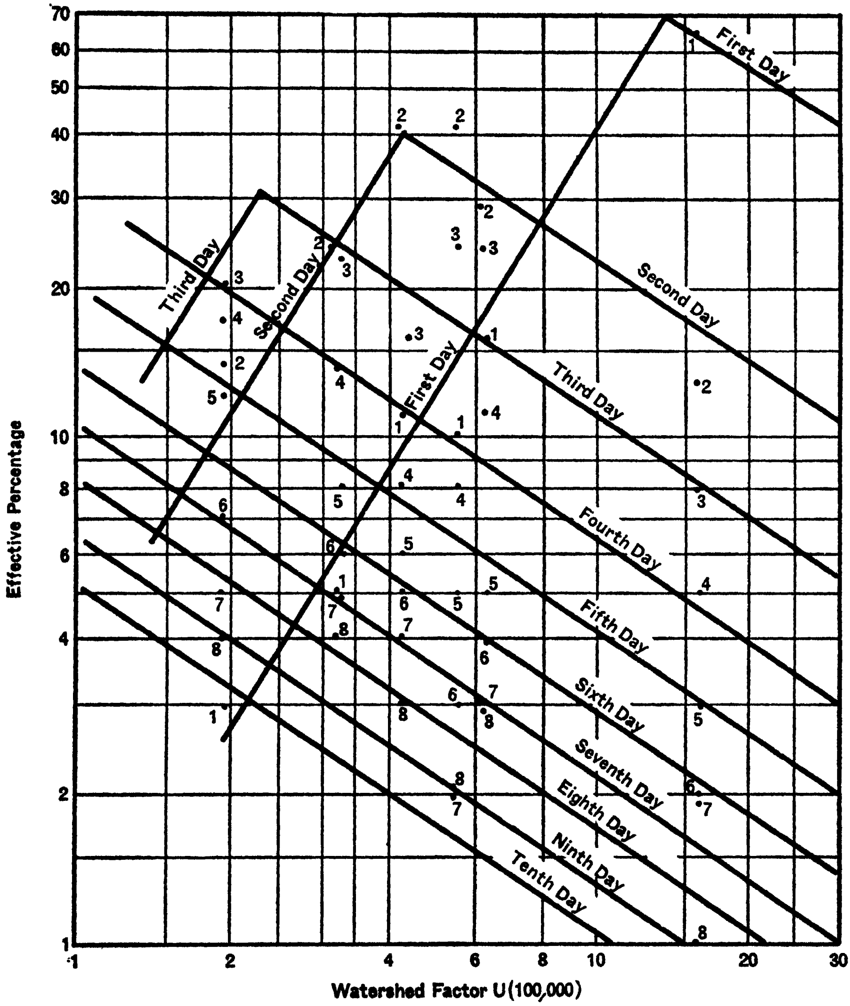

FIG. 9-1. Relation of watershed factor U to daily increments of various distribution graphs. (From Merrill M. Bernard, *Trans. A.S.C.E.*, 1935.)

McCARTHY'S APPROACH

9-3. General Discussion

McCarthy's approach to the synthesis of the unitgraph is widely referred to in hydrologic literature but has itself never been published

except as a mimeographed pamphlet not available to the public.* His basic data consisted of 6-hr unitgraphs of 22 streams in the Connecticut River basin, ranging from 74 to 716 sq mi in area. For study, three parameters of the unitgraph were selected, namely: peak discharge, lag-to-peak from beginning of rain, and total base length. These three parameters were plotted against various topographic characteristics and combinations of characteristics, more than a hundred such plottings being

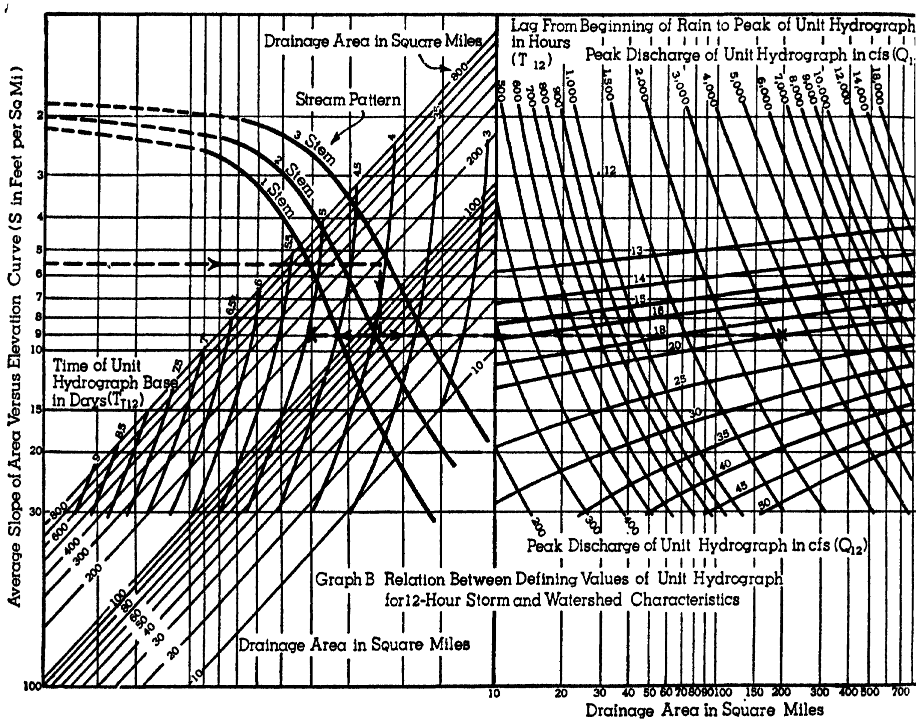

FIG. 9–2. Unit hydrograph relations for 12-hr storm and watershed characteristics. (From Gerald T. McCarthy, *The Unit Hydrograph and Flood Routing*, U.S. Engineer's Office, Providence, R.I. (1938; rev. 1939.)

made in all. As a result, the following three topographic characteristics were found to be predominant in their effect on the unitgraph: size of area, slope of area-elevation graph, and stream pattern expressed as the number of major stems in a drainage area. The results are summarized in Fig. 9–2† in such a way as to permit an estimate of the three unitgraph

*Gerald T. McCarthy, Senior Engineer, U.S. Engineer's Office, Providence, R.I., *The Unit Hydrograph and Flood Routing* (1938; rev. 1939).

†Being part of Fig. 2 of Appen. 2 of McCarthy's paper. Reproduced through courtesy of the Division Engineer, Corps of Engineers, U.S. Army, Boston, Mass.

parameters for an ungaged drainage area when the three determining topographic characteristics are known.

Definitions of watershed slope factor and stream pattern are required. Watershed slope factor is not a dimensionless slope but a somewhat arbitrary convention expressed in ft/sq mi. It is computed as follows: (1) Plot a graph of drainage area versus elevation equaled or exceeded; (2) planimeter the area between this graph and a horizontal line drawn through the minimum elevation of the watershed; (3) divide the planimetered area by half the square of the total drainage area in sq mi. The result of the last step is the *average* slope of the graph in ft/sq mi and is the watershed slope factor. Stream pattern is expressed in terms of the number of major stems in a drainage area and is determined by inspection of the topographic map. A one-branch stream is one having no single tributary that drains more than 25 per cent of the total drainage area. A two-branch stream has two major branches of approximately the same size, which together drain at least 50 per cent of the total area. A three-branch stream has three major branches draining at least 75 per cent of the total area. If it appears that peaks from two branches are likely to be considerably out of synchronization, the classification is reduced from 3 to 2 or 2 to 1, as the case may be. Some borderline cases may call for a classification of 1½ or 2½; personal judgment enters here to a considerable extent.

9-4. Procedure

Development of the relationships indicated by Fig. 9-2 was carried out as follows:

(1) The six variables (three unitgraph parameters, three drainage basin characteristics) were reduced to five by applying the laws of hydraulic similitude and reducing all drainage areas to a single size (in this case 10 sq mi) (for details see next paragraph).

(2) The peak discharges of the resulting "model" unit graphs were plotted against their respective base lengths and their respective lags, and a good correlation was found in both cases. It was thus necessary to relate only one of the unitgraph parameters to the remaining two topographic characteristics.

(3) Unitgraph peaks were accordingly plotted against model watershed slope factors,* with stream pattern as parameter. Construction of Fig. 9-2 followed.

Reduction of the unitgraphs to model dimensions needs explanation. By the laws of hydraulic similitude, rates of flow in the model are to

*Because the slope factor has the dimension L^{-1}, the *model* slope factor is n times the prototype slope factor, where n is the linear scale ratio, i.e., (prototype area ÷ model area)$^{0.5}$.

rates of flow in the prototype as $n^{-2.5}$, and time in the model is to time in the prototype as $n^{-0.5}$, n being defined as the linear scale ratio—i.e., the square root of the ratio of the area of the prototype to the area of the model. However, the model hydrograph computed by these relationships would be only the $1/n$th part of a unitgraph; hence to convert prototype unitgraph to model unitgraph, $n^{-1.5}$ is used instead of $n^{-2.5}$. The time scale remains $n^{-0.5}$. The example given by McCarthy is reproduced here: Given an actual 200-sq mi watershed with slope factor of 5.5 ft/sq mi and a stream pattern of two stems, producing a 12-hr unitgraph with a peak of 3200 cfs, a base length of 4.9 days, and a lag of 20 hr; to compute the model unitgraph for a 10-sq mi model.

Solution:

$$n = \left(\frac{A}{a}\right)^{0.5} = \left(\frac{200}{10}\right)^{0.5} = 4.46;$$

$$s = nS = (4.46)\,(5.5) = 24.5 \text{ ft/sq mi};$$

Stream pattern = 2 stems (dimensionless; no change);

$$q = Qn^{-1.5} = (3200)\,(4.46)^{-1.5} = 340 \text{ cfs},$$

$$t_t = \text{Base length} = (4.9)\,(4.46)^{-0.5} = 2.3 \text{ days},$$

$$t_r = \text{Lag to peak} = (20)\,(4.46)^{-0.5} = 9.4 \text{ hr},$$

$$d = \text{Unit period} = (12)\,(4.46)^{-0.5} = 5.7 \text{ hr}.$$

It will be noted that the model unitgraph is not a 12-hr graph but a 5.7-hr graph. Since the model unitgraphs are to be compared with one another, they must all be adjusted to conform to the same storm duration. The procedure is as follows:

(1) Using the prototype unitgraph, which is a 12-hr graph, compute the peak of a 24-hr graph by summing the peak flow and the flow 12 hr after the peak, and dividing by 2. By similar methods, compute the peak of a 36-hr graph.

(2) Reduce these prototype peaks and rainfall durations to model peaks and model rainfall durations, and plot the q's against the d's.

(3) From the resulting q versus d curve, read off the peak discharge of a 12-hr model unitgraph. (The work is not shown here; the result was a model peak of 297 cfs as against the model peak of 340 cfs for the 5.7-hr graph.)

(4) The base length of the 12-hr model graph is $(12 - d)/24$ days longer than the base length of the d-hr (5.7-hr) unitgraph, and the lag is $(12 - d)$ hours longer than the lag of the 5.7-hr unitgraph.

It should be emphasized that application of these model relationships is not essential to the *use* of Fig. 9–2, in which all dimensions are prototype dimensions. The model relationships *are* essential, however, to the *con-*

struction of Fig. 9-2, as they provide the means for eliminating one variable (area).

9-5. Limitations

McCarthy's own comments on the limitations of his approach are worth noting:

> The relations developed between unit hydrograph properties and drainage-area characteristics are fairly well defined by the data used although refinements and improvements are desired that were not made because of limitations in time and data. It is desired that the curves be better defined and substantiated by computed points developed from watersheds in other parts of the country. It is believed that a more exact expression may be found for evaluating the stream pattern of a basin and that a refinement may be introduced to distinguish between watersheds for which the area-elevation curves have the same average slope but markedly different shapes.

To these comments may be added the following: (1) The fixed relation between peak, lag, and base length of unitgraphs is somewhat restrictive, for actually two unitgraphs might have the same peak, lag, and base length and still be quite different in shape; (2) the work involved in computing the watershed slope factor is considerable; and (3) the complications of model analysis are likely to discourage many hydrologists from following similar procedures in studying unitgraph relationships in other parts of the country. None of these criticisms is basic. As for (1), it is quite possible that additional unitgraph parameters could be introduced and analyzed by the same method if sufficient data were available. As for (2) and (3), it must be remembered that short cuts can be justified only after a sound foundation has been laid and that McCarthy, like Bernard, was helping to lay the foundation.

Snyder's Approach

9-6. General Discussion

Snyder's work* was more or less contemporaneous with that of McCarthy. His basic data were mainly records of streams in the Appalachian Highlands. Like McCarthy, Snyder selected three parameters to define the unitgraph—lag, peak flow, and total base length. Lag, however, was measured not from the beginning of rainfall but from the time center of mass of effective rain. As for topographic characteristics, Snyder selected area and shape as controlling and relegated slope to a secondary position, introducing a coefficient to take care of it "when changing from one area to another with great differences in channel slopes."

*Franklin F. Snyder, "Synthetic Unit-Graphs," *AGU Transactions*, 1938, pp. 447-454.

Another coefficient is available to express differences in flood wave and storage effects between basins. The results were summarized in three empirical equations that permit an estimate of the three unitgraph parameters for an ungaged drainage area when the two principal determining topographic characteristics are known, provided that there are at hand sufficient data on similar streams to indicate reasonable values for the slope and storage-effect coefficients.

9–7. The Lag Equation

The basic theory of the approach stems from Bernard's concept of time contours developing about the point of concentration and enveloping progressively larger portions of the drainage area as each portion in turn makes its contribution to the creation and acceleration of the flood wave. Snyder suggests that these contours of travel time could be located by hydraulic computations for steady flow, using the Manning formula, and that the results, when plotted with time as abscissa and increments of area between time contours as ordinates, would express the shape of a drainage basin by giving the increment of the area that is at any particular travel time distant from the gaging station. Such a diagram is presented in Fig. 9–3, together with a plot of the corresponding hydrograph of surface runoff. The relation between the two is almost obvious; the area-time curve can be conceived as the hydrograph that would be produced by the basin if its channel system had zero storage and no flood wave effect, while the actual hydrograph reflects the modification of this hypothetical hydrograph by the actually existing storage and by flood wave action. May there not be some helpful relation, then, between the lag-to-peak of the area-time curve, on the one hand, and the lag-to-peak of the actual hydrograph, on the other? After investigating this, Snyder concluded that a somewhat more stable relationship exists if the time abscissa of the *center of area* of the area-time curve is used in place of the lag-to-*peak* of that curve.

Ideally, the procedure at this point would have been to develop area-time curves for all the basins in the study and locate their time centers. However, the time that would have been required for deriving these curves was prohibitive, so a simple substitute was adopted. This substitute was the length in miles, measured along the principal channel, from the gaging station to the geographical center of the drainage basin. The geographical center was located by suspending a cardboard cutout of the basin from a pin through a point near its edge, drawing a vertical line, and repeating the process at another point to establish an intersection.

By substituting miles for hours, this simplification introduced the assumption that channel velocities are essentially the same throughout the basin—or, at least, that the effect of differences in velocities in various parts of a basin is more or less the same in all basins. Further, by substitut-

ing the mass center of the geographical area for the mass center of the area-time curve, the simplification tended to obscure the possible effect of variations in stream pattern. To avoid this, irregular basins with fair-sized tributaries were broken up into subareas, the geographical

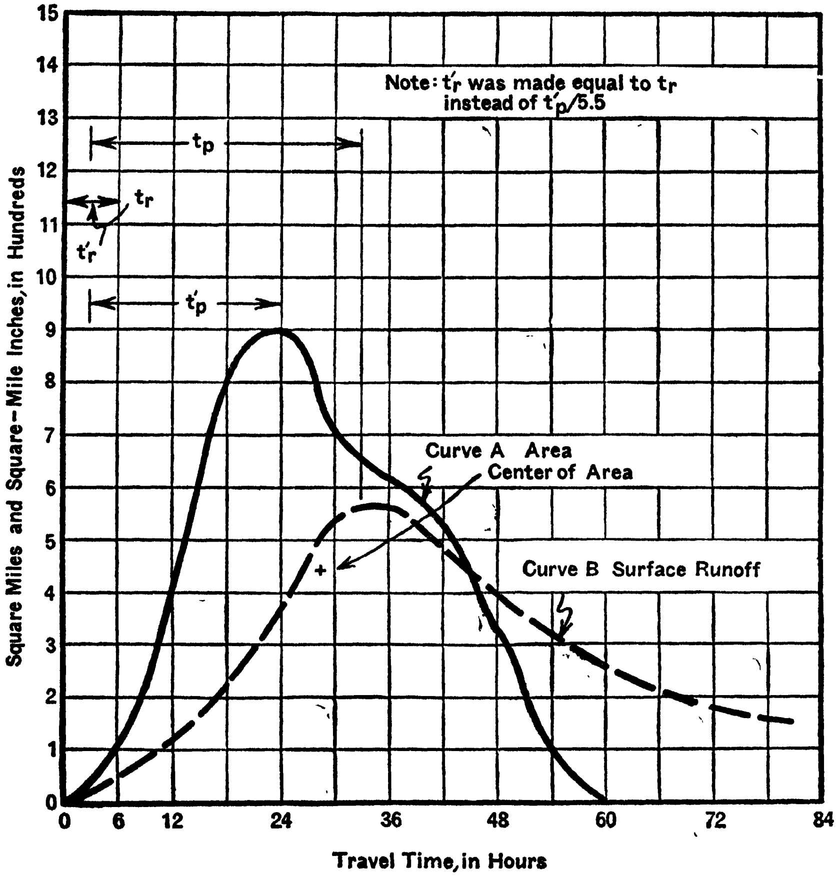

FIG. 9-3. Shape diagrams, French Broad River Basin, above Dandridge, Tennessee. (From F. F. Snyder, "Synthetic Unitgraphs," *Trans. A.G.U.*, Part I, 1938, pp. 447–454.)

centers of which were located independently and then recombined, with proper weighting for area and for distance from the gaging point.

Snyder plotted t_p (lag-to-peak of unitgraph in hr) against L_{ca} (distance from gaging station to center of area, in mi) for a large number of basins and found that t_p varied with $L_{ca}^{0.6}$. Ultimately, however, he chose to replace $L_{ca}^{0.6}$ with $(L_{ca}L)^{0.3}$, where L is the total length of the basin in mi. The idea behind this is that L_{ca} is, in any case, only an approximation

of the factor actually desired and that L can be more definitely determined than can L_{ca} and therefore tends to make the relation more stable. There resulted the empirical equation

$$t_p = C_t(L_{ca}L)^{0.3}, \tag{9-1}$$

in which C_t must be varied to take care of differences in slope and storage.* C_t had an average value of 2 for the areas studied and did not vary greatly for individual areas, all of which were of somewhat similar characteristics. This equation permits the computation of lag, the basic parameter of the unitgraph in the Snyder approach, when length of drainage basin and distance to geographical center of basin are known.

9-8. The Peak-Flow Equation

It was next necessary to relate peak discharge of the unitgraph to lag. Since 1 in. of runoff per hr from 1 sq mi is the equivalent of 640 cfs, Snyder reasoned from a consideration of the area-shape diagram that the peak per square mile should be $q_p = (640/t_r)(a_p/A)$, in which t_r is the unit period, a_p is the portion of the area contributing to peak flow, and A is the total area of the basin. In order to introduce lag, the preceding equation may be re-written as

$$q_p = \frac{640}{t_p} \cdot \frac{t_p a_p}{t_r A} = \frac{640}{t_p} \cdot C_p. \tag{9-2}$$

To make C_p independent of t_p and t_r, for each unitgraph t_r must be chosen so that the ratio t_p/t_r for all unitgraphs is the same. Snyder found it convenient to divide the rising limb into six equal parts; hence in his study $t_r = t_p/5.5$ for all unitgraphs. Thus

$$q_p = C_p \frac{640}{t_p} \tag{9-3}$$

gives the peak discharge in cfs/sq mi for the $(t_p/5.5)$-hr unitgraph of a basin whose lag is t_p. The coefficient C_p takes account of the flood wave and storage effect of the basin; for the streams included in his original study Snyder found it to vary from 0.56 to 0.69.†

*Applying the laws of hydraulic similitude, the exponent of $(L_{ca}L)$ should be 0.25 instead of 0.3. The question may be raised, therefore, as to whether C_t is completely free of scale effects.

†In applying a similar method of analysis to another region, an investigator should compute C_p by means of the above equation for each drainage basin for which he had derived a unitgraph. This would indicate a range of values of C_p applicable to the region. Then, for synthesizing a unitgraph for an ungaged area, he would either use an average value of C_p or, preferably, would select the value corresponding to the drainage basin that most nearly resembled the ungaged basin in such respects as channel size and channel slope.

Application of Eq. (9-3) for purposes of synthesis gives the peak of the $(t_p/5.5)$-hr unitgraph. If it is desired to synthesize a unitgraph of some pre-specified unit period, as, for example, 6 hr, a modification of Eq. (9-3) is necessary. An empirical modification, recommended by Snyder for this purpose but not thoroughly explained in his published work, is

$$q_{pR} = C_p \frac{640}{t_p + \frac{t_R - t_r}{4}}, \tag{9-4}$$

in which t_R is the desired unit period and q_{pR} the corresponding peak per sq mi; other symbols as before.

9-9. The Base-Length Equation

Base length, or duration of surface flow—the third parameter of the unitgraph—is related to lag in the Snyder approach through the relationship

$$T = 3 + 3\frac{t_p}{24}, \tag{9-5}$$

in which T is the base length in days. This expression is based on observation of a large number of unitgraphs; but, as Snyder pointed out, the length of base depends largely on the method adopted for handling base flow. The long base lengths for small areas given by Eq. (9-5) are characteristic of unitgraphs that include subsurface storm flow.

9-10. Example

To illustrate the synthesis of a unitgraph in his paper, Snyder chose the 4450-sq mi drainage area of the French Broad River above Dandridge, Tennessee. The length of basin, L, was measured as 130 mi, by using a map measurer on a 1: 500,000-scale map. L_{ca} was measured as 67 mi, and no adjustments were made (i.e., the area was not broken down into subareas) because the stream pattern is uniform. Then, using the "average" value of 2 for C_t and 0.625 for C_p, he obtained

$$t_p = 2(67 \times 130)^{0.3} = \mathit{30.4\ hr},$$

$$q_p = \frac{0.625 \times 640}{30.4} = 13.2 \text{ cfs/sq mi},$$

$$Q_p = 4450 \times 13.2 = \mathit{58{,}700\ cfs},$$

$$t_r = \frac{30.4}{5.5} = 5.5 \text{ hr},$$

$$T = 3 + 3\left(\frac{30.4}{24}\right) = \mathit{6.8\ days:}$$

The value of t_r is close enough to 6 that this may be considered as a 6-hr unitgraph without correction.

9–11. Concluding Notes

As an aid to shaping the unitgraph after the three parameters are computed, Snyder provided in his paper a graph in which 24-hr distribution-graph percentages are plotted against lag. This is similar to the Bernard graph previously discussed (Fig. 9–1) except that abscissae are lag instead of U.

It should be noted that Snyder is still continuing his studies of unitgraphs for the Corps of Engineers, U.S. Army, and that, since the publication of his original paper, large quantities of additional data have been made available to him from the various district offices. The outline of the "Snyder approach" given in the preceding paragraphs is limited to a digest of his basic paper; it serves our present purposes of illustrating a method of approach to the problem but should not be construed as necessarily representing the full scope of his present theories or techniques.

Clark's Approach

9–12. General Discussion

We have already stated that Clark's approach is not so much a procedure for synthesizing a unitgraph for an ungaged area as it is one for deriving a unitgraph for an area for which a certain amount of hydrologic data is available.* Its value is twofold: (1) It makes possible the derivation of a unitgraph of any desired unit period, from a single record of flood discharge, without the necessity of knowing anything about the distribution of runoff-producing rainfall except the time of its ending; and (2) it clarifies the inherent relationship between the concept of the unitgraph and the concept of flood routing and relates the shape of the unitgraph in a logical manner to the shape of the drainage basin that produces it. Clark's own analysis† would be somewhat difficult for a beginning student to follow; for that reason we present here our own interpretation of the underlying principles rather than a review of his paper.

In the discussion of flood routing (Chap. 7) it was pointed out that most natural channels feature a combination of two routing effects—lag, or transposition of a flood wave in time without modification in shape, and attenuation, or flattening and elongation. In a sense, lag represents a time of travel, but it is not so much the time of travel of a particular

*See Addendum at end of this chapter.

†C. O. Clark, "Storage and the Unit Hydrograph," *ASCE Transactions*, 1945, pp. 1419–1488.

particle of water as it is the time of travel of a wave or impulse. The essence of Clark's approach is the separation of these two types of storage effect and their application, separately, to individual elements of inflow. To separate and evaluate the two storage effects, recourse is had to an observed hydrograph of flood flow; to set up the individual elements of inflow, a topographic map is used and an area-time diagram constructed.

9-13. Measuring the Storage Effects of the Basin

The recession limb of a flood hydrograph usually contains a point of contraflexure. It appears that this point marks approximately the time of arrival at the gaging station of the flood wave effect from the most remote portion of the drainage area. Since the time of cessation of rainfall marks the end of the generating period of this wave, it follows that the time interval between cessation of rainfall and point of contraflexure measures the maximum lag effect of the drainage basin on any element of inflow. Elements of inflow originating at less remote points are subjected to lesser lag effects, down to a minimum of zero for the element that originates in the immediate vicinity of the gage. On the assumption that lag effect is directly proportional to distance from the gage, measured along the watercourses, it is then possible to construct on a map of the basin a set of time contours indicating the lag at any point. The area between any two time contours, multiplied by the depth of net effective rainfall on that area, is, clearly, the element of runoff (i.e., inflow to the channel system) that is subject to an average lag effect equal to the average of the time values of the bounding contours. Thus an area-time curve with a base length equal to the maximum lag is equivalent to an instantaneous unitgraph of the basin, *uncorrected for reservoir-type storage effect.*

In the Clark method all elements of the area-time curve are routed through the same amount of reservoir storage. This is admittedly an approximation, but one which in many cases does not appear to have any serious effect. The amount of reservoir-type storage to use, expressed in hours, is equal to the rate of discharge at the point of contraflexure of the observed flood hydrograph divided by the rate of change of discharge per hour at that point. The reasoning behind this is as follows:

(*a*) For any point on the hydrograph after inflow to the main channel system has ceased (i.e., to the right of the point of contraflexure), we may write

$$\frac{dS}{dt} = -Q,$$

where S is storage (say, in cu ft) and Q is discharge (say, in cfs).

(*b*) For a stream that behaves in accordance with unitgraph theory, storage is approximately proportional to discharge, i.e.,

$$S = KQ,$$

in which K has the dimension of time. Clearly, then, $dS = KdQ$.

(*c*) Substituting KdQ for dS in (*a*) and solving, we obtain

$$K = -\frac{Q}{\frac{dQ}{dt}};$$

and, if dQ/dt is measured in cfs/hr, K will be given in hr and will be positive, since dQ/dt is negative.

(*d*) Theoretically, it should be possible to compute K from the ordinate and slope of the observed hydrograph at the point of contraflexure or at any point to the right thereof. Actually, K begins to increase in value to the right of the point of contraflexure because the inflow from ground water does not decrease as rapidly as the storage-controlled surface runoff does, and it thus becomes an increasingly larger percentage of the total discharge. It follows that the best value of K will be given by performing the computation at the point of contraflexure.

9–14. The Routing Procedure

A convenient method of routing is the so-called "Muskingum method," described by McCarthy in the second part of his paper on "The Unit Hydrograph and Flood Routing." The operating equation is in the form

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1,$$

where Q_2 is the unknown discharge at the end of the interval, Q_1 is the known discharge at the beginning of the interval, I_2 and I_1 are the known inflows at the end and beginning of the interval, respectively, and the C's are constants which, for reservoir-type storage, can be reduced to the form

$$C_0 = C_1 = \frac{0.5T}{K + 0.5T},$$

$$C_2 = \frac{K - 0.5T}{K + 0.5T},$$

T being the length of the interval.

The I's in the operating equation are the ordinates of the area-time curve, and, if the latter is presented in histogram form, the computation can be simplified, by taking $I_1 = I_2$ in each interval, to

$$Q_2 = I \frac{T}{K + 0.5T} + Q_1 \frac{K - 0.5T}{K + 0.5T}.$$

This is the form in which the equation is applied in the example that follows.

9-15. Example

The method will be illustrated by applying it to derive the unitgraph for Big Walnut Creek, at Central College, a drainage basin of 191 sq mi

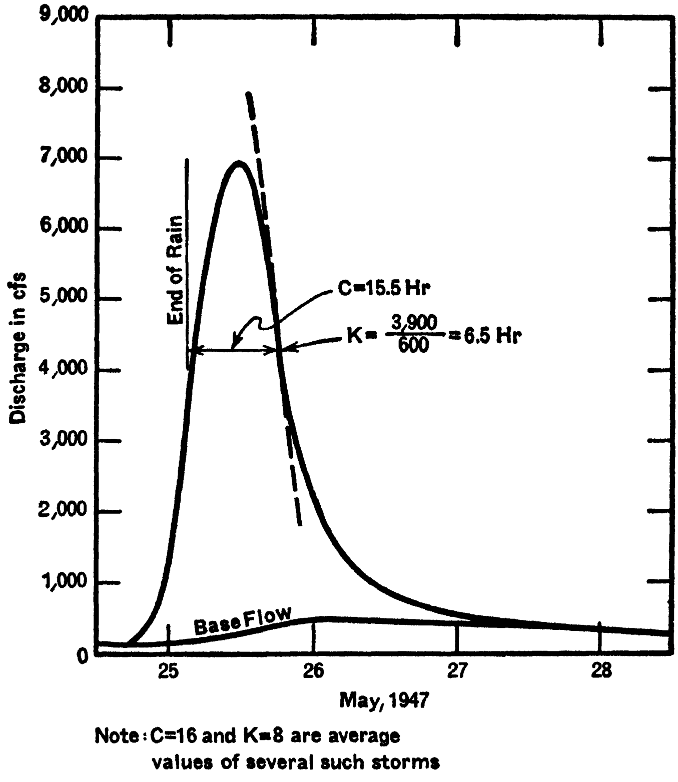

FIG. 9-4A. Flood hydrograph, Big Walnut Creek, at Central College, Ohio.

located northeast of Columbus, Ohio. Fig. 9-4A is a flood hydrograph produced by the basin; indicated on it are the approximate time of cessation of runoff-producing rainfall, the point of contraflexure on the recession limb, and the slope of the hydrograph at that point. These data give 15.5 hr for the base length of the area-time curve and 6.5 hr for the value of K.*

*Note that in Fig. 9-4A, K is computed from *surface* runoff rather than from *total* runoff. This is a departure from the Clark procedure which we believe to be a worthwhile refinement. The obvious logic of deducting base flow is borne out by the observed fact that K's computed from total flow tend to vary systematically with the size of the rise, while K's computed from surface runoff do not.

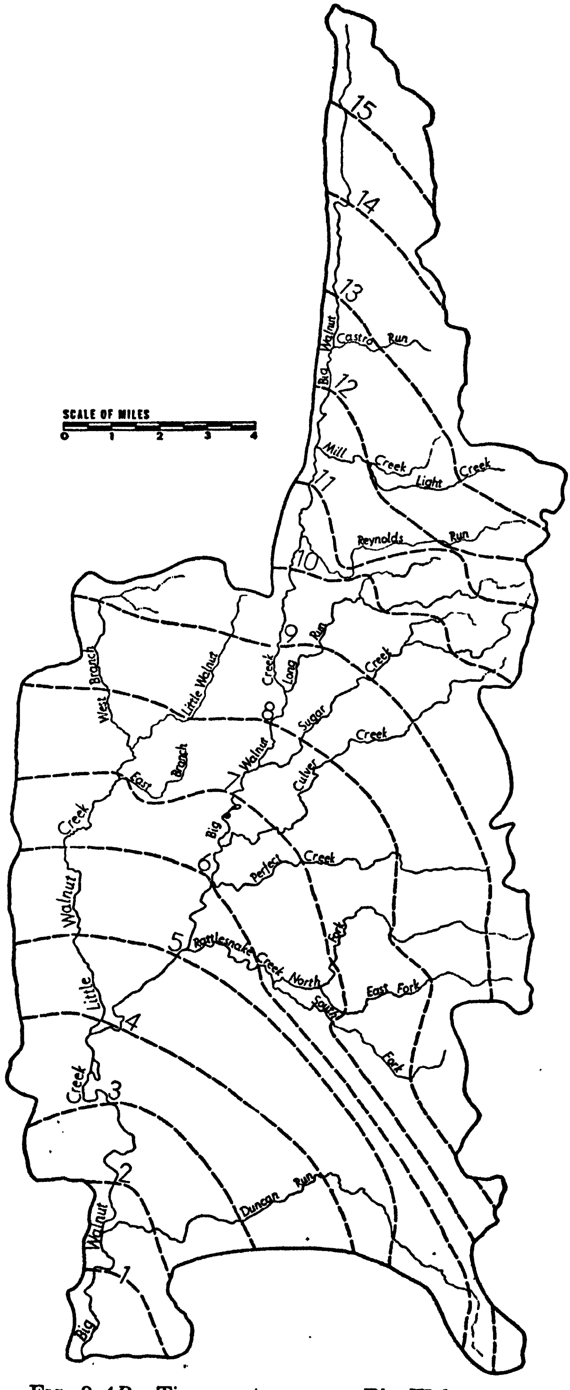

FIG. 9–4B. Time contour map, Big Walnut Creek, above Central College, Ohio.

Examination of a number of such hydrographs, however, resulted in *average* values of 16 hr for base length and 8 hr for K. These average values are used in what follows. Fig. 9-4B shows the drainage basin, with time contours marked on it. These contours were located by dividing the developed length of the main channel (extended to the basin boundary) into sixteen equal parts and establishing an additional point on each contour at its intersection with each tributary by stepping off the same lengths along the tributary channels with dividers. The area-time curve may be obtained by planimetering the areas between successive contours.* This curve is not shown graphically here, but its ordinates are tabulated in Column 2 of Table 9-1. The remaining columns of that table show the routing of the area-time curve through 8 hr of reservoir-type storage, using a T-interval of 1 hr; the next-to-last column is the result of the routing,

TABLE 9-1

COMPUTATION OF UNITGRAPH FOR BIG WALNUT CREEK BY CLARK METHOD

Base length of area-time curve = 16 hr
K = 8 hr
T = 1 hr

$$\frac{T}{K + 0.5T} = \frac{1}{8.5} = 0.118$$

$$\frac{K - 0.5T}{K + 0.5T} = \frac{7.5}{8.5} = \frac{0.882}{1.000} \text{ (check)}$$

1 in. per sq mi per hr = 645 cfs.

(1) Hour	(2) Sq Mi	(3) 0.118×(2)	(4) 0.882×(5)	(5) (3)+(4)	(6) Inst. U.G. (5)×645 (cfs)	(7) 6-Hr U.G. (cfs)
0....	0	0	0	0	0	0
1....	3.05	0.36	0	0.36	232	20
2....	5.92	0.70	0.32	1.02	657	93
3....	8.40	0.99	0.90	1.89	1220	250
4....	19.10	2.25	1.67	3.92	2530	563
5....	15.10	1.78	3.45	5.23	3370	1054
6....	17.00	2.01	4.61	6.62	4270	1690
7....	13.56	1.60	5.83	7.43	4800	
8....	21.93	2.59	6.55	9.14	5900	
9....	21.93	2.59	8.05	10.64	6860	
10....	17.58	2.07	9.50	11.57	7460	
11....	10.31	1.22	10.20	11.44	7380	
12....	12.60	1.49	10.10	11.59	7470	6045

(*Table 9-1 continued on next page.*)

*Actually, the authors use a different method for obtaining this curve, based on subdividing the drainage basin into thirty or more subbasins, instead of drawing time contours. The procedure is somewhat complex and need not be explained here; but it has certain advantages of flexibility when the problem of eliminating the effect of one portion of the drainage basin arises, and it also permits making allowance for differences in channel slope, both along the main stem and along the tributaries.

TABLE 9-1—*Continued*

(1) Hour	(2) Sq Mi	(3) 0.118×(2)	(4) 0.882×(5)	(5) (3)+(4)	(6) Inst. U.G. (5)×645	(7) 6-Hr U.G.
13....	9.73	1.15	10.21	11.36	7320	
14....	7.06	0.83	10.01	10.84	7000	
15....	4.01	0.47	9.57	10.04	6480	7217← Peak
16....	3.62	0.43	8.86	9.29	6000	
17*...					5280	
18....					4660	6358
19....					4110	
20....					3620	
21....					3200	
22....					2820	
23....					2480	
24....					2190	3276
25....					1930	
26....					1705	
27....					1496	
28....					1322	
29....					1162	
30....					1025	1537
31....					903	
32....					794	
33....					703	
34....					620	
35....					546	
36....					481	736
37....						
38....						
39....						
40....						
41....						
42....					226	353
43....						
44....						
45....						
46....						
47....						
48....					106	166
49....						
50....						
51....						
52....						
53....						
54....					50	78
55....						
56....						
57....						
58....						
59....						
60....					24	37

*Beginning with Hour 17, entries in Column 2 are zero. Thus Column 3 entries are also zero, and Column 5 entries would be merely a duplication of Column 4. Time is therefore saved by recording entries in Column 6 only, each entry being 0.882 times the preceding one. Beginning with Hour 36, entries are recorded every six hours, the factor being $(0.882)^6$.

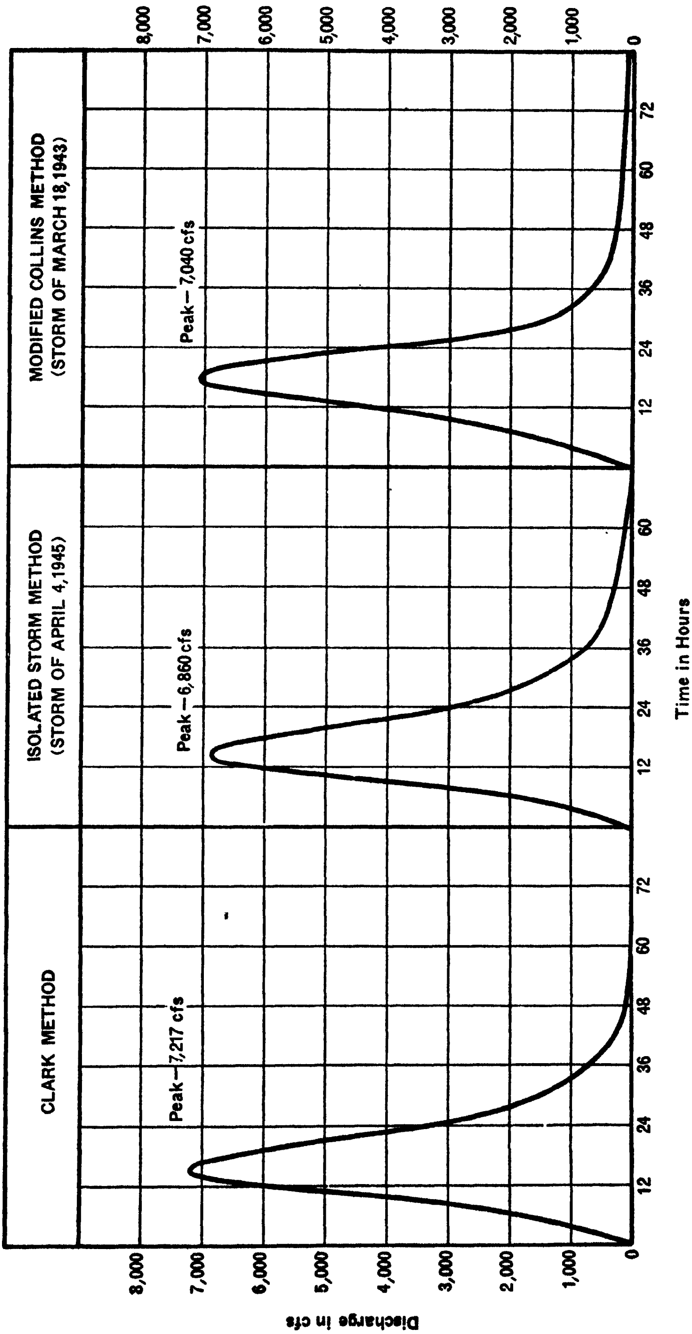

FIG. 9-4C. Unitgraphs, Big Walnut Creek, at Central College, Ohio.

expressed in cfs, and these values are the ordinates, at 1-hr intervals, of the instantaneous unitgraph.

From the instantaneous unitgraph, a unitgraph for any desired unit period can be developed by "subdividing the instantaneous graph into periods of the desired unit length and averaging the ordinates over the preceding periods of time. Thus, in a 6-hr unitgraph, the rates of discharge at the end of 6, 12, and 18 hours, respectively, are the average ordinates to the instantaneous hydrograph in the 6-hr periods preceding the sixth, twelfth, and eighteenth hours."* Following this procedure, the ordinates for a 6-hr unitgraph have been computed and entered in the last column. A plot of the same data is shown in Fig. 9–4*C* (p. 233), with unitgraphs of the same basin derived from a multiperiod storm (by the Collins method) and from an isolated storm. The similarity is a fine confirmation of the Clark method, and the writers have obtained equally good results on many other streams, including multiple-branched streams whose unitgraphs are characterized by double peaks and other irregularities.

The student should observe that in the Clark method no arbitrary relationship between peak flow, lag-to-peak, and base length is imposed. All these parameters develop independently in routing the area-time curve.

Concluding Note

In singling out four papers on the general subject matter of this chapter, we have not had any intention of belittling the work of any other investigators. Noteworthy among these are Langbein, Horner, Turner, Barrows, Kinnison, and Colby—to name but a few, again at the risk of unintended omissions. Our selections were made with a view to covering as many aspects of the problem as possible, in as elementary a way as possible, and always with reference to the unitgraph, which is the best point of departure for the beginner. The student should understand, however, that other approaches not based on unitgraph theory are available; consider, for example, the "modified rational method" of Gregory and Arnold, to which Bernard also has contributed.

Finally, we have not intended to imply, by labeling the various "approaches" with names of individuals, that they alone are to be credited with all the ideas set forth in the respective sections. Totally original contributions to any science are rare, and the authors cited here all have, in their original papers, graciously acknowledged their dependence on the work of their predecessors and contemporaries. In each case, however, it appeared to us that there was some distinctive factor in each man's approach that warranted its designation by his name.

ADDENDUM

Since preparing Chapter 9, the writers have had the privilege of working with the Scioto-Sandusky Conservancy District, Ohio, in the development of a method for syn-

*Clark, *op. cit.*, p. 1445.

thesizing unitgraphs for ungaged drainage areas within the Scioto-Sandusky basin. The basis of the method finally adopted is the Clark approach, and the novelty of the Scioto-Sandusky work lies in evaluating the two parameters (C, or base length of the area-time curve, and K) in terms of drainage basin characteristics.

It was found that C could be expressed by an equation of the form

$$C = ar^n \left(\frac{L}{\sqrt{S}}\right)^m,$$

in which a, n, and m are experimentally derived constants, L is the length of the main channel in miles, S is the equivalent uniform slope of the main channel in feet per mile, and r is a "branching factor" purporting to express numerically the stream pattern of the basin.

Similarly, K was represented by

$$K = b + c\,\frac{W}{R},$$

in which b and c are experimentally determined constants, W is the effective width of the drainage basin in miles, and R is the general average overland slope in feet per mile.

The method was tested by deriving synthetic unitgraphs for several drainage basins that had not been used in deriving the general expressions for C and K and by comparing the synthetics with unitgraphs derived for the same basins by standard methods. Comparisons were good as regards instantaneous peak flows, lag-to-peak, duration of flow in excess of 10% of peak flow, and general shape, including such irregularities as bulges and secondary peaks.

We refrain from giving the numerical values of the experimental constants because successful use of the empirical equations depends not only upon those values but upon the observance of especially prescribed techniques for measuring r, L, S, W, and R, which space limitations preclude describing here. The suggestions given above should be sufficient guide for the interested student in developing a similar study in another area. If the number of streams available for study is small and if the basins are not too widely different in stream pattern, such a study may possibly be simplified by omitting r and considering C as a function of $L/\sqrt{S}$ only. (There is good theoretical justification for treating $L/\sqrt{S}$ as a single variable in any case.) Similarly, it may be acceptable to omit R from the expression for K. For good results, C and K values used in deriving the empirical equations should be an average of several measurements—preferably from flood peaks of various sizes—and the K values *must* be computed from surface runoff rather than from total runoff.

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CHAPTER 10

APPLICATIONS OF STATISTICAL ANALYSIS TO HYDROLOGY

INTRODUCTION

10–1. General Remarks

The hydrologist is often faced with the problem of analyzing and interpreting a large mass of data. To do this effectively and efficiently, he must employ methods of statistical analysis. Broadly speaking, statistical methods consist of the collection, presentation, analysis, and interpretation of data. By this definition every hydrologic problem involving the collection of data is a problem in applied statistics. The methods of statistical analysis have applications to all phases of research problems, including: (1) the designing of experiments, i.e., setting up the investigation so that the answers to research questions may be determined definitely and efficiently; (2) the collection and assembling of the data; (3) the analysis of the data so that all the relevant information (and nothing irrelevant) is obtained from the investigation; and (4) the

presentation of the condensed data and the results in a form easily understood.

Statistics, like hydrology and most other sciences, is in a state of rapid evolution. New ideas and techniques are being proposed constantly, superseding or modifying methods previously in use. Statistics is a large branch of mathematics, with applications to all sciences and, in fact, to nearly every walk of life. In this chapter only a few of the more fundamental applications of statistical methods to hydrology are discussed, together with elementary examples. The emphasis, as elsewhere in this book, is on ideas rather than on techniques, and the objectives are to encourage the student to pursue the study of statistics further and to stimulate the more experienced to apply statistical methods to hydrologic problems.

10-2. Principal Uses of Statistics

Some of the more important uses of statistics, not only in hydrology but with broad general application, are summarized below:

- (*a*) To design the investigation so that the data required for satisfactory analysis will be obtained
- (*b*) To estimate the reliability of a measurement
- (*c*) To determine whether the difference between two values is or is not significant
- (*d*) To derive empirical relationships between two or more variables
- (*e*) To estimate the frequency of occurrence of an equal or greater (or less) event
- (*f*) To determine whether departures from a hypothesis are or are not significant

It should be noted that the design of the analysis of some hydrologic problems may be limited by the lack of data. Rainfall cannot be turned on and off as can a sprinkler, and a 25-yr record of stream flow cannot be doubled in length in less than another 25 yr. The opportunities for replication of experiments are often limited. In hydrology we can rarely have successive measurements of the same event.

The use of statistics listed under (*d*) above is a broad one, for it embraces the entire field of correlation analysis. It is usually a simple matter to obtain empirical relationships between variables, but to determine the significance of the relationships is more difficult and involved.

Frequency studies were among the first applications made of statistics to hydrologic problems. The technical literature on this important subject is so voluminous and involved that it well-nigh defies classification. Despite the attention given this type of problem, there is still much to be learned. The short length of most hydrologic records is responsible for the need for the extrapolation of frequency curves and,

at the same time, is the cause of doubt as to the reliability of extrapolated estimates. At least until much longer records are available, frequency theories will be subjects of discussion and study.

10–3. Popular Misconceptions of Statistics

There are several confusing and erroneous conceptions of statistics which unfortunately are widely held. A little clarification is in order, and a few of the more popular misconceptions together with brief corrections are as follows:

"There is too much mathematics involved in statistical analysis for it to be practical for ordinary use." The basic theories of statistical analysis involve some of the most complex mathematics, it is true, but the practical applications involve no more than a moderate facility with elementary algebra.

"Hydrologic data are too rough to warrant analysis by statistical methods." The very roughness of the data is one of the best arguments for applying to it the best that statistical theory has to offer.

"You can prove anything you want to by statistics." This oft repeated statement is as stupid as it is incorrect. Statistical analysis, properly applied, is not a hocus-pocus procedure by means of which one can twist the data to give a desired answer, but is an orderly, scientific means of drawing from any mass of data the maximum information that it is capable of providing.

"Statistics prove that such and such is true." This variant of the preceding statement is also incorrect. Statistical analysis, surprising as it may seem on first thought, is ordinarily incapable of proving anything. Statistics deals with probabilities, not proofs. Generally, statistical analysis tells us whether we have *good* reason to doubt a given hypothesis, or whether we have *no* reason to doubt it. We cannot *prove* the falsity or the truth of any hypothesis by statistical means.

"By the law of averages, such and such is bound to happen." This is a common expression that means nothing. Presumably, the "law of averages" is intended to mean "laws of probability"; but the very concept of probability negates the concept of compulsion. If an event has "even chances" of occurring or not occurring in a single trial, then the chances of its happening at least once in a series of many trials is obviously great—but it is not *bound* to happen in any finite number of trials.

"Things vary according to the normal curve." A wide variety of data do follow closely the normal distribution, and most distributions tend toward the normal *in the limit*; but there are other important distributions, particularly the Poisson and the binomial, that should not be forgotten, especially if the samples are small.

General Principles

10–4. Sampling Theory

Statistical inferences or deductions are made from a given set or *sample* of observations about a larger set or *population* of potential observations. Large masses of data are unwieldy and uneconomical to analyze, and often the population is infinite or approaches infinity in size, and we therefore resort to sampling. An example familiar to all is the popular

public opinion poll. In this case the population is the opinion on a certain question of everyone in the country. The samples are the opinions of selected individuals. If the individuals are properly selected, their combined opinion will be the same, within certain limits, as the combined opinion of the entire population. Every effort is made to make the sample unbiased, that is, to select the sample without prejudice. A sample for which the selection is entirely by chance is called a *random* sample. Statistical theory is based on simple random sampling; that is, the sample is considered to be of the type obtained by placing the "population" in a hat, drawing one sample, and replacing it before drawing again. Selecting a sample from a telephone directory by lot, however, would not give a random sample, as not everyone has a telephone, and the sample would therefore be biased. To prevent the sample's being biased, it should be as representative as possible of the total population. In opinion polls it is often the practice to select the sample in such a way that every income group is represented in proportion to the size of the group in the total population. Such a representative sample is called a *stratified* sample. A hydrologic example would be the design of the collection of weather data for a drainage basin: the stations would be located so that no large part of the basin would be without representation, and the stations would also be scattered so that there would be valley, hillside, and hilltop locations. A *spot* sample, on the other hand, is one taken without regard to representativeness, from one small area or class of the population. Examples are an opinion poll based on questioning in one city block, and the estimation of mean rainfall from a group of gages close together in one corner of a drainage area.

Statistical theory presupposes that samples are not only random but also independent and from a homogeneous population. These two additional requirements may best be explained by examples. If two rain gages are operated side by side, then the two records are practically identical and obviously should be considered one gage in computing mean rainfall. Two such gage records would lack *independence*. Records of temperatures taken in the sun should not be averaged with temperatures in the shade, not that there is anything wrong with temperatures taken in the sun—but they are measurements of different things, not part of the same parent-population, i.e., when combined with shade temperatures the combined records would lack *homogeneity*.

Hydrologic observations seldom completely satisfy the requirements of theory. For example, rain gages usually are not uniformly distributed over an area. Furthermore, hydrologic measurements often are not completely independent of one another. In rainfall–intensity–frequency studies it is often the practice to add together the data from several stations, under the assumption that the data are independent—in particular, that the gages are separated far enough that no one intense storm affects

two gages. If there is such dependence between records, the dependence must be taken into account in the use of the combined station-year method.

10-5. Frequency Distributions

After collecting the sample, the next step is to arrange the data systematically for study. An arrangement of data is called a "distribution" or "series." The arrangement may be made in various ways; if arranged

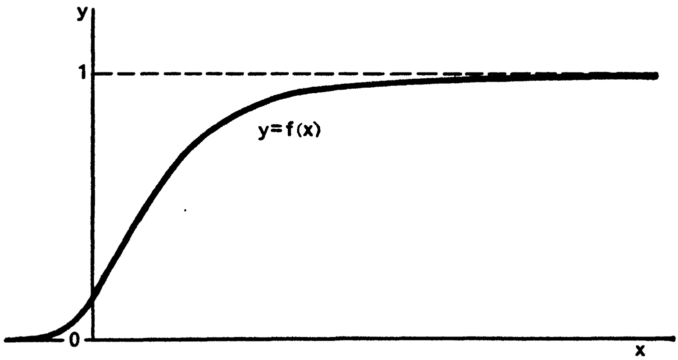

FIG. 10-1A. Distribution function of the continuous type.

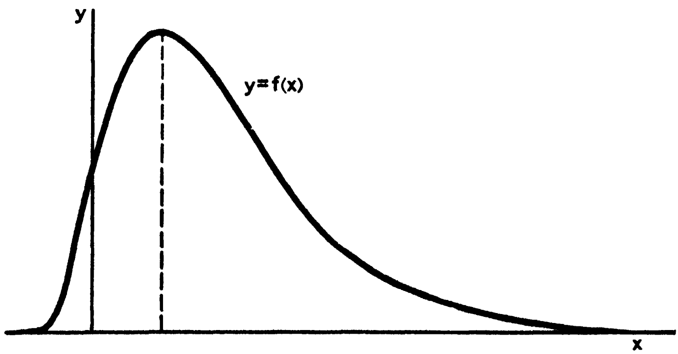

FIG. 10-1B. Frequency function of the distribution in Fig. 10-1A.

according to time or date it is called a "time series," e.g., a hydrograph is the plot of a time series. If arranged according to magnitude or size, the same data would be called a "frequency distribution." Fig. 10-1A is a cumulative frequency distribution, and Fig. 10-1B is the corresponding frequency distribution, the derivative of Fig. 10-1A. The latter is a duration curve (see Fig. 3-19), but with the axes reversed from the engineer's usual method of plotting, that is, the diagram is turned 90 degrees. The derivative of a duration curve, that is, a curve proportional to the slope of a (rotated) duration curve, is a frequency distribution.

Frequency distributions are fundamental to statistics and to its hydrologic applications. The frequency distribution of the sample data is an estimate of the frequency distribution for the population from which the sample is drawn. In other words, the sample is a statistical image of the population—the sample mean is an estimate of the population mean, etc. There are many types of distributions, one of the most important of which is the *normal* or Gaussian distribution. The normal frequency distribution (see Fig. 10-2*A*) is a bell-shaped, symmetrical curve. The mean, median, and modal values* coincide for this type of distribution. The application of the normal curve is broad because so many other distributions tend to the normal form as the size of the sample

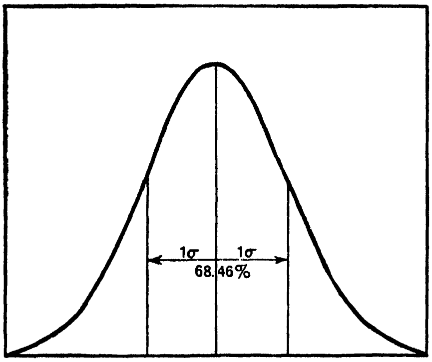

Fig. 10-2*A*. Normal distribution.

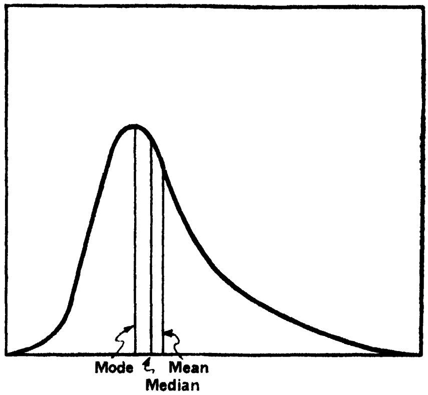

Fig. 10-2*B*. Hypothetical right-skewed frequency distribution, showing position of mode, median, and mean.

is increased. This fact is used in many ways, particularly when it is possible to obtain large samples. The theory of least squares is based on the normal distribution.

Not all symmetrical distributions are normal, and not all distributions are symmetrical. When a curve is asymmetrical, it is said to be "skewed." Fig. 10-2*B* is a right-skewed frequency distribution similar to flood-frequency curves. Note the separation of the mean, mode, and median. This type of curve is typical of many hydrologic data. Statistical tests based on the normal distribution break down if there is much skew to the frequency curve. Other common distributions with which the reader must become familiar if he is to pursue the study of statistics further include the binomial distribution and the Poisson distribution.

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*The reader is referred to any of the elementary textbooks on statistics for definitions of these and other statistical terms.

10–6. The Mean and the Standard Deviation

The normal distribution (as well as others) has a tendency for the values to cluster about the central value, that is, it has the greatest frequency at the center and most of the values close to the center. We therefore need a measure of the central value and also a measure of the dispersion, or scattering, of the values on either side of the center. The arithmetic mean of the observations is a satisfactory measure or estimate of the mean or central value of the population. The arithmetic mean of a sample $x_1, x_2, \ldots, x_n$ is $\bar{x} = [\Sigma(x)]/n$, where $\bar{x}$ is the arithmetic mean of the sample, n the total number of items, and Σ means summation over the whole sample. Then $\bar{x}$ is said to be the best estimate of μ, the population mean.

The standard deviation, s, is used to measure the dispersion. It is calculated by the formula*

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}},$$

or, in words, it equals the square root of the summation of the squares of the deviations from the mean *divided by* by the number of observations less 1. Lower-case s is the *estimate* of σ, the standard deviation of the population. *Sigma* squared (σ^2) is called the "variance," and s^2 is an estimate of the variance.

The standard deviation of a distribution of means, or of any other statistical measure computed from samples, is called the "standard error," s_x, and is equal to the standard deviation of sample divided by the square root of the number of items, thus:

$$s_x = \frac{s}{\sqrt{n}}.$$

The significance of the standard error in normal and near-normal distributions is that it is the error that will be exceeded, on the average, approximately one time out of three. This is indicated on Fig. 10–2*A*.

The relative standard deviation, or coefficient of variation, C, is equal to the standard deviation divided by the mean:

$$C = \frac{s}{\sqrt{n}\,\bar{x}}.$$

*The standard deviation of the *sample*, as contrasted with the estimate of the standard deviation of the *population*, is measured by

$$s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}.$$

FREQUENCY STUDIES

10-7. Introductory

A statistical application that has received much attention from engineers and hydrologists is the estimation of the frequency of rare events, such as excessive rainfalls, floods, and droughts. The problem usually takes some such form as this: Given a record of several years' length, usually short, of the maximum annual floods; to determine the probable return periods for floods of any given size. Solutions to the problem are necessary to the economic design of roads, culverts, bridges, and other engineering works subject to flooding. If a curve of flood magnitude against probable return period (the reciprocal of frequency) can be estimated within reasonable limits, then the cost of flood prevention (by larger bridge, etc.) can be balanced against replacement or repair cost to obtain a design which will be most economical in the long run. As will be shown, extrapolation on a flood frequency curve may give very erroneous estimates, and use of values determined by extrapolation is an exceedingly dangerous practice if loss of life or great property damage is involved, as in the design of large dams.

If floods and other rare events followed the normal law of distribution, then the problem would be a simple one, for the two parameters μ and σ, the mean and the standard deviation, completely define a normal distribution. But flood frequency curves are skew, and therefore another parameter is involved. Much ingenuity has been exerted by many workers in attempts to develop graphical or mathematical means of solving the difficulty of the skewness of flood frequencies. Hazen and Fuller were the pioneers in the field, and they developed the use of "normal probability paper," both arithmetic and logarithmic; Foster, Goodrich, Slade, and Gumbel are others that have made contributions to the mathematical and graphical analysis of skew-frequency data.*

Slade states that "skewness is never a truly significant factor when the sample from which it is computed has less than 140 items." Very few records of annual floods in this country approach this length. The short lengths of records, therefore, reduce the possibilities of obtaining reliable estimates from skew-frequency data. But, aside from this major difficulty, there are others equally unsurmountable.

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*Allen Hazen, *Flood Flows* (New York: John Wiley & Sons, Inc., 1930); H. A. Foster, "Theoretical Frequency Curves," *ASCE Transactions*, Vol. 87 (1924); R. D. Goodrich, "Straight Line Plotting of Skew-Frequency Data," *ASCE Transactions*, Vol. 91 (1927); J. J. Slade, Jr., "An Asymetrical Probability Function," *ASCE Transactions*, Vol. 101 (1936); E. J. Gumbel, "The Return Period of Flood Flows," *Annals Math. Statistics*, Vol. 12 (1941).

10–8. Sampling Errors and Other Complications

Let us assume that we have a long record--10,000 yr. In such a record the magnitude of the 100-yr flood will be well established. The question is: How often will there be no flood as big as the 100-yr flood in a period of 100 consecutive years, how often one, and how often more than one 100-yr flood in 100 yr? We can estimate the sampling errors—that is, we can discover how the 100-yr floods are theoretically distributed throughout the 100 series of 100-yr records—by the binomial distribution.* We are assuming that the 100-yr floods are distributed fortuitously; this may not be exactly correct, but it does offer a reasonable means of estimating sampling errors. The binomial distribution is obtained from the successive terms of the binomial expansion $(p + q)^n$, equal to:

$$p^n + np^{n-1}q + \frac{n(n-1)}{2!}\,p^{n-2}q^2 + \frac{n(n-1)(n-2)}{3!}\,p^{n-3}q^3 + \ldots,$$

where p is the probability of "success," $q = 1 - p$ is the probability of "failure," and n is the number of trials in a series. The numerical value of the first term, p^n, is the probability of n "successes" in a series of n trials; that of the second term is the probability of $n - 1$ "successes" in a series of n trials, and so on.

By taking $n = 100$ (since each series consists of 100 consecutive years), $p = 0.99$ (the probability of occurrence of a flood smaller than a 100-yr flood), and $q = 0.01$ (the probability of occurrence of a flood equal to or greater than a 100-yr flood), we find:

$p^n = (0.99)^{100} = 0.37$ = the probability of occurrence of 100 floods smaller than the 100-yr flood in 100 consecutive years of record. (This is equivalent to the probability of the occurrence of no flood as big as the 100-yr flood.)

$np^{n-1}q = 100 \times (0.99)^{99} \times 0.01 = 0.37$ = the probability of the occurrence of 99 floods smaller than the 100-yr flood in 100 consecutive years of record. (This is equivalent to the probability of the occurrence of just one flood equal to or bigger than the 100-yr flood.)

Similarly, the third term of the binomial expansion evaluates to 0.18, the fourth to 0.06, and the fifth to 0.02, with terms beyond the fifth negligible when the computations are carried only to hundredths.

In terms of the assumed 10,000-yr record, this means that the 100 floods of 100-yr frequency or greater are statistically distributed as shown in Table 10–1.

Another interpretation is this: Suppose that we have a 100-yr record for each of a number of different streams. In 37 per cent of these records,

*R. W. Davenport, discussion of statistical analysis, *ASCE Transactions*, Vol. 108 (1943).

TABLE 10-1

No. of 100-Yr Periods	No. of 100-Yr Floods per Period
37	0
37	1
18	2
6	3
2	4 or more
100	

on the average, there will be no flood as big as the "true" 100-yr flood for the stream represented; and in 26 per cent of these records, on the average, there will be more than one 100-yr flood. (As an exercise, let the student show that 13 per cent of the time there will be not even a 50-yr flood in a 100-yr record.)

The sampling errors of short records can be shown even more dramatically. Fig. 10-3 was prepared in connection with a study of the

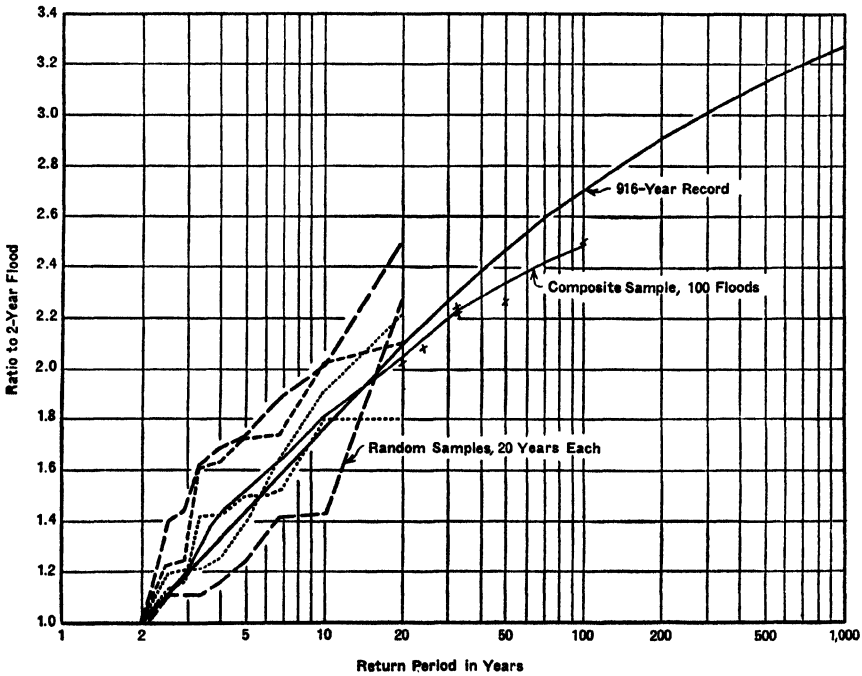

FIG. 10-3. Frequency curves based on random sampling of longer record.

station-year method in which a total of 916 station years were combined. (We are not concerned here with the merits of the station-year method.) The data were in the form of ratios of flood magnitudes to the 2-yr flood magnitude for each stream, so they could be combined into a 916-yr record, as plotted. Five samples of 20 yr each were then selected at random

from the 916-yr record (by a method which effectively simulated the drawing of numbers from a hat), arranged in order of magnitude, and plotted, with ratios less than 1.0 omitted. The five 20-yr samples were also combined into one 100-yr record, labeled "composite" in the figure. The graph clearly indicates that sampling errors may be so large that interpolation on a frequency curve gives only rough estimates of the magnitudes of floods and extrapolation may give extremely erroneous estimates.

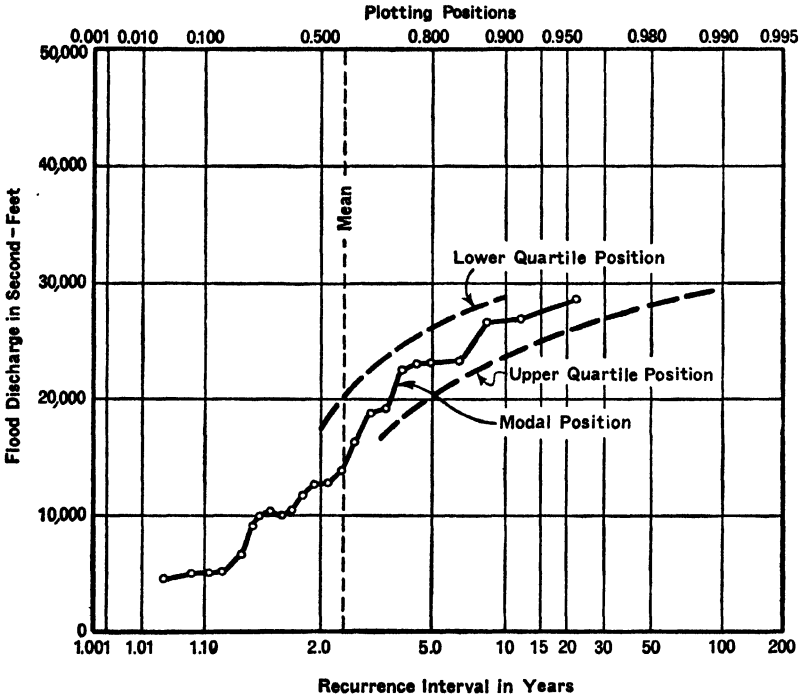

FIG. 10–4. Annual flood frequency data, Licking River, at Toboso, Ohio, showing range of errors. (From William P. Cross, "Floods in Ohio—Magnitude and Frequency," *Bull. 7, Ohio Water Resources Board*, 1946.)

Fig. 10–4 is a diagram of annual floods on the Licking River, at Toboso, Ohio, for 24 yr of record. In addition to the annual floods, shown as dots joined by broken lines, the range of errors has been shown by quartile limits. By an estimate based on the Poisson distribution the probable error has been determined, and the horizontal range (that is, in time) is shown by the quartile limits. There is a 50 per cent chance that the true frequency curve at any given flood discharge is between the limits shown and, of course, a 50 per cent chance that the curve is outside

these limits. For example, the highest flood in 24 yr of record has a 50 per cent probability of being between a 10- and a 90-yr flood.

Another factor that should be considered in interpreting the past performance of a stream as a measure of future probability is the question as to the effect of changes in climate and stream regimen. The changes in rainfall from year to year may or may not be fortuitous; it seems reasonable to assume that chance is the major influence over periods of several years, but there are undoubtedly changes in climatic conditions over several centuries. Because of this, it seems doubtful that a long flood record should be regarded as a sample from one population—the data may not be homogeneous. As for the effect of changes in stream regimen on flood frequency, one need only consider the change that would take place in the frequency of floods of a given size at some point on a stream after the construction of a large storage reservoir upstream.

Despite these difficulties, the application of statistical methods to flood studies is worth while. It is perhaps unfortunate that the pioneer application of statistics to hydrologic problems was in this field, for the complex theoretical discussions have resulted in a belief by some hydrologists that statistical methods generally are of limited value. The use of statistics in other fields of hydrology is usually less difficult and is not only enlightening but also in some cases necessary to a complete understanding of the problems involved.

10-9. The Station-Year Method

Much of the foregoing applies to studies of drought or excessive rainfall rate frequencies as well as to floods. However, rainfall rates are not affected by drainage basin characteristics as floods may be. This suggests the possibility of combining several rainfall records into a longer record—the "station-year method." A longer record obtained in this fashion would make possible more accurate estimates of the frequencies of excessive rainfall rates. This has been done in many studies of excessive rainfalls, often without consideration of the errors which may be involved. To be reliable, (1) the data should be from an area "meteorologically homogeneous," i.e., the expectation of excessive rates should be approximately the same for all stations; and (2) there should be no dependence between records, i.e., no single storm should be counted more than once or, if counted, the combined record should be adjusted for dependence.

It has been suggested that the station-year method of analysis might be applied to flood frequency studies. In order for this to be feasible, the requirements of meteorological homogeneity and independence must be met, plus the requirement that the flood frequency distribution is in no way correlated with any characteristic of the drainage basin. It is possible that these requirements may be substantially met over areas embracing

several drainage basins; the apparent variations in skew between drainage basins may be largely, if not entirely, caused by the sampling errors.*

10–10. Summary

To summarize: (1) Frequency records available now are short in length; (2) longer records, when obtained, may not be homogeneous; (3) sampling errors are large; (4) elaborate theoretical analyses are some-

*The authors have recently had an opportunity to test this on Ohio streams, with encouraging results. The entire state appeared to be reasonably homogeneous meteorologically (insofar as flood-producing storms are concerned). It was assumed that flood frequency distribution might be considered, a priori, to be in no way correlated with any drainage basin characteristic, provided (*a*) no exceptionally permeable or exceptionally impermeable drainage basins were included, and (*b*) only such streams as formed their hydrographs substantially in accordance with unitgraph theory were included.

Suitable for study were some 40 streams with drainage areas ranging from less than 100 to more than 1600 sq mi and having a total of more than 900 station years of record. To put all records on a comparable basis, each flood occurrence was expressed as a ratio to the median flood on the stream on which it occurred. (Median floods were found to be establishable within fairly narrow limits after 16-20 yr of record.) Statistical tests indicated a significant lack of independence "between stations," so that the records could not be added to give the equivalent of a 900-yr record. Actually, the result was only a well-defined 25-yr record. On the other hand, exhaustive tests failed to disclose any evidence for questioning the hypothesis that the deviations of individual stream records from this composite record were attributable to sampling errors alone. In other words, there was no reason for suspecting that the various records were not samples of a single population.

The practical significance of the study is this: As soon as sufficient years of record (16 or so) have accumulated on an Ohio stream to permit a reasonably accurate determination of the median flood, the ratios from the composite frequency graph can be applied to that median; and they will probably give a more accurate expression of the flood potentialities of that stream than any curve that might be developed from the record of the stream itself. To point up the discussion, ratios read from the composite frequency graph are listed below, but with the warning that they are not necessarily, nor even probably, applicable to other areas than the state of Ohio:

Recurrence Interval of Flood (in Years)	Ratio of Instantaneous Peak Discharge to Instantaneous Peak Discharge of Median Flood
2	0.91
5	1.30
10	1.57
20	1.80
50	2.14

An interesting corollary of this study was an attempt to develop a method for estimating the median flood in the absence of a 16-yr record. It was found that there was a reasonably constant relationship between the peak discharge of the 6-hr unitgraph of a drainage basin and the peak discharge of the median flood; specifically, 1.26 times the unitgraph peak gave a fair approximation of the median flood on 19 of the 24 streams on which this relationship was investigated. This suggests that at least a rough approximation of flood frequencies for many Ohio streams can be made as soon as sufficient data have been accumulated for the development or synthesis of a unitgraph.

what futile; and (5) despite the multiplicity of methods of analysis already proposed, additional studies seem desirable and may give fruitful results.

Correlation Analysis

10-11. Definitions and Principles

Every student who has plotted a scatter diagram for two variables in an attempt to determine the relationship between them is familiar with the problem of correlation analysis, although perhaps not by that name. Since many scientific problems consist of the determination of the relation of one variable to one or more other variables, it is not surprising that the methods of correlation analysis are more generally used and understood than any other branch of statistics. Lack of space prevents more than a brief summary of some of the fundamentals, together with a few examples of the pitfalls and limitations of the method.

In statistical parlance, the mean curve or line defined by a scatter diagram is a line or curve of "regression." The regression line or curve is often fitted by the least-squares method.* A measure of the scatter, or variation about the line or curve, is the standard error of estimate, analogous to the standard deviation. The standard deviation measures the scatter about the arithmetic mean and is the quadratic mean of the deviations. Similarly, the standard error of estimate (S_y) is the quadratic mean of the deviations about the line of regression—that is,

$$S_y = \sqrt{\frac{\Sigma(d^2)}{n}},$$

where $\Sigma(d^2)$ is the sum of the squares of the deviations from the line of regression and n is the number of observations.

The size of the standard error of estimate is a measure of the association or correlation between the variables. However, standard errors cannot be directly compared because they are in terms of the units of the independent, or y, variable. To obtain a relative measure of correlation for linear correlations, the standard error is divided by the standard deviation σ_y of the y variable, and the ratio is squared and subtracted from unity. Extracting the square root gives the correlation coefficient (r):

$$r = \sqrt{1 - \frac{S_y^2}{\sigma_y^2}}.$$

It can be shown that

$$r = \sqrt{\frac{\rho}{\sigma_x \sigma_y}},$$

where $\rho = [\Sigma(xy)]/n$ and σ_x and σ_y are the standard deviation of the x and y variables, respectively. The correlation coefficient has a value of

*See Art. 2-11 for an example.

1 for a perfect relationship and a value of 0 for a wholly imperfect correlation. The same general theory may be extended to nonlinear correlation, in which case the measure of correlation is called the "index of correlation." In the case of more than two variables the measure of correlation between one independent variable and the combined effect of all other variables is called the "coefficient of multiple correlation." The "coefficient of partial correlation" is a measure of the association between the dependent variable and a given independent variable, the effects of other variables being eliminated.

10–12. Limitations

The uses of correlation analysis are obvious, the limitations not so easily discerned. Not only is careful thought required to prevent erroneous results, but the elaborate use of correlation coefficients, to the exclusion of all other statistical methods, is subject to criticism on theoretical grounds. Fisher* discusses "the futile elaboration of innumerable measures of correlation" and "the process of abstracting from the field formerly embraced by the correlation coefficient, problems capable of a more direct approach." The hydrologist should guard against the use of the correlation coefficient when another method is more direct, simpler to use, or productive of more definite information. He should also guard against the errors and pitfalls common to the correlation method. Among these, two of the most common are (1) use of a *simple* correlation (i.e., a correlation between *two* variables) when *multiple* correlation is required and (2) the application of correlation methods to variables that do not have a cause-and-effect relationship to one another.

A homely example of the error that may result from omitting to consider an important variable is the following: A farmer, over a period of 3 days, sold 3 dozen eggs to a customer and presented a total bill of $1.02. The purchaser was mathematically inclined and asked how the cost was computed. The farmer said that two dozen eggs were 24¢, one dozen was 36¢, and half a dozen was 42¢. The purchaser took these data home, plotted up number of eggs against cost, and reached the astounding conclusion that the more eggs, the less the total cost! This absurdity results, of course, from neglecting to consider one of the variables involved —namely, the quality of the eggs. In this case, the omission was serious enough to make the remaining variable appear to have an effect opposite to that which it actually exerted. In a less extreme case the relationship might not be reversed but still could be so effectively masked that the results would be of little value. Thus, for example, a correlation between observed wind velocity and observed evaporation would mean little or

*R. A. Fisher, *Statistical Methods for Research Workers* (10th ed; London: Oliver & Boyd, 1946).

nothing, unless the effects of changes in temperature, barometric pressure, etc., were first eliminated.

A correlation between two variables that do not stand in a cause-and-effect relationship to each other is usually open to question, though on occasion two effects of a common cause may be correlated to advantage. Thus the annual runoff of a basin might be correlated, year by year, with the thickness of tree rings growing in the basin, and the indicated regression might be useful in making rough estimates of the runoff from the basin through the lifetime of the oldest trees. This is reasonable, because the amount of runoff and the thickness of the rings are both determined primarily by rainfall and temperature. However, if neither a cause-and-effect relationship nor a common-cause relationship exists, correlation analysis may produce a totally absurd result known as a "non-sense" correlation. An example is this: If *for a single run* with a variable-head permeameter we compute k, the coefficient of permeability of a soil, by Poisseuille's formula, making a series of computations at various heads, we are likely to find that k decreases with h in a very orderly manner, and a correlation analysis would indicate a high degree of correlation between these two variables. Yet there is actually no relationship whatever. The decrease in h results from the way the equipment is constructed and operated, while the decrease in k results from a change in the character of the sample (and possibly from progressive clogging of the filter stone) as the experiment progresses and would still take place even if h were held constant. Thus only a non-sense correlation exists between k and h. To prove this experimentally, it would be necessary only to make several runs in succession on the same sample. However, assuming that only the data for the one run were available, are there not many students who would jump to the conclusion that k is a function of h? It is not the smoothness of the curve but the logic behind it that justifies an attempt to correlate one variable with another.

10–13. An Example of Correlation Analysis

As an example of correlation analysis let us examine the data used in plotting Fig. 4–3 (p. 109), with a view to determining how close a correlation actually exists between mean annual water loss and mean annual temperature. The diagram indicates that there may be a linear relationship between these two variables, for basins with mean annual precipitation in excess of 20 in.

(*a*) Disregarding for the time being the possibility that annual precipitation may affect the correlation, the coefficient of correlation is computed in Table 10–2 and equals 0.929. This would indicate a very close linear correlation. The method shown is that to be used when only two variables enter into a problem.

(*b*) *Assuming that precipitation affects the relation* between water loss and temperature, such effect may be largely eliminated by analyzing only the records

TABLE 10–2*

STUDY OF CORRELATION OF MEAN ANNUAL WATER LOSS (IN.) AND MEAN ANNUAL TEMPERATURE (F)

Gaging Stations	Mean Annual Water Loss (In.) Y	Mean Annual Temperature (F) X	Y Deviations	Y Deviations Squared	X Deviations	X Deviations Squared	Product of Deviations xy
South Branch of Nashua River at Clinton, Mass	22.0	47.8	− 4.1	16.81	− 4.7	22.09	+ 19.27
Sudbury River at Framingham Center, Mass	24.5	47.9	− 1.6	2.56	− 4.6	21.16	+ 7.36
Lake Cochituate Outlet at Cochituate, Mass	23.2	47.9	− 2.9	8.41	− 4.6	21.16	+ 13.34
West River at Newfane, Vt.	21.5	42.3	− 4.6	21.16	−10.2	104.04	+ 46.92
Swift River at West Ware, Mass	23.1	47.9	− 3.0	9.00	− 4.6	21.16	+ 13.80
Middle Branch of Westfield River at Goss Heights, Mass	19.6	46.8	− 6.5	42.25	− 5.7	32.49	+ 37.05
Clearfield Creek at Dimeling, Pa	21.8	50.1	− 4.3	18.49	− 2.4	5.76	+ 10.32
Swatara Creek at Harper Tavern, Pa	21.5	50.7	− 4.6	21.16	− 1.8	3.24	+ 8.28
Upper Little Swatara Creek at Pine Grove, Pa	20.7	50.8	− 5.4	29.16	− 1.7	2.89	+ 9.18
Oconee River near Greensboro, Ga.	30.2	61.1	+ 4.1	16.81	+ 7.6	57.76	+ 31.16
Chattahoochee River near Norcross, Ga	30.0	58.9	+ 3.9	15.21	+ 6.4	40.96	+ 24.96
Conecuh River near Andalusia, Ala	34.1	65.7	+ 8.0	64.00	+13.2	174.24	+105.61
East Fork of Tombigbee River near Fulton, Miss	39.6	63.0	+13.5	182.25	+10.5	110.25	+141.75
Pearl River at Edinburg, Miss	38.8	64.8	+12.7	161.29	+12.3	151.29	+156.21
Red Bank Creek at St. Charles, Pa	19.4	46.4	− 6.7	44.89	− 6.1	37.21	+ 40.87
Miami River at Dayton, Ohio	25.8	51.2	− 0.3	0.09	− 1.3	1.69	+ 0.39
West Fork of White River near Noblesville, Ind	24.2	52.6	− 1.9	3.61	+ 0.1	0.01	− 0.19
Tittabawassee River at Freeland, Mich	20.4	45.0	− 5.7	32.49	− 7.5	56.25	+ 42.75
Red River at Fargo, N.D	20.3	42.4	− 5.8	33.64	−10.1	102.01	+ 58.58
Red River at Grand Forks, N.D	19.7	40.6	− 6.4	40.96	−11.9	141.61	+ 76.16
La Crosse River near West Salem, Wis	20.3	44.8	− 5.8	33.64	− 7.7	59.29	+ 44.66
Kickapoo River at Gays Mills, Wis	21.8	43.8	− 4.3	18.49	− 8.7	75.69	+ 37.41
Blackwater River at Blue Lick, Mo	30.9	55.3	+ 4.8	23.04	+ 2.8	7.84	+ 13.44
South Grand River near Brownington, Mo	30.5	56.1	+ 4.4	19.36	+ 3.6	12.96	+ 15.84
Little Arkansas River at Valley Center, Kans	27.4	56.4	+ 1.3	1.69	+ 3.9	15.21	+ 5.07
Walnut River at Winfield, Kans	27.8	57.2	+ 1.7	2.89	+ 4.7	22.09	+ 7.99
Neches River near Rockland, Tex	35.8	66.2	+ 9.7	94.09	+13.7	187.69	+132.89
Angelina River near Lufkin, Tex	35.0	65.4	+ 8.9	79.21	+12.9	166.41	+114.81
Totals	729.9	1469.1	....	1036.65	....	1654.45	+1215.87
Means	26.1	52.5	....	37.02	....	59.09	+ 43.42

$\sigma_x = \sqrt{59.09} = 7.69$; $\sigma_y = \sqrt{37.02} = 6.08$; $P = 43.42$; $r = 43.42/7.69 \times 6.08 = 0.929$.

* Basic data are from Table 4 of *Water Supply Paper 846.*

TABLE 10-3

STUDY OF CORRELATION OF MEAN ANNUAL WATER LOSS AND MEAN ANNUAL TEMPERATURE*

Gaging Stations	Mean Annual Precip-itation (In.)	Mean Annual Water Loss (In.) Y	Mean Annual Tempera-ture X	Y Deviation	Y Deviation Squared	X Deviation	X Deviation Squared	Product of Deviation xy
South Branch of Nashua River at Clinton, Mass.	43.8	22.0	47.8	− 3.6	12.96	− 5.6	31.36	20.16
Sudbury River at Framingham Center, Mass.	42.8	24.5	47.9	− 1.1	1.21	− 5.5	30.25	6.05
Lake Cochituate Outlet at Cochituate, Mass.	41.9	23.2	47.9	− 2.4	5.76	− 5.5	30.25	13.20
Clearfield Creek at Dimeling, Pa.	42.0	21.8	50.1	− 3.8	14.44	− 3.3	10.89	12.54
Swatara Creek at Harper Tavern, Pa.	42.7	21.5	50.7	− 4.1	16.81	− 2.7	7.29	11.07
Upper Little Swatara Creek at Pine Grove, Pa.	42.0	20.7	50.8	− 4.9	24.01	− 2.6	6.76	12.74
Neches River near Rockland, Tex.	44.9	35.8	66.2	+10.2	104.04	+12.8	163.84	130.56
Angelina River near Lufkin, Tex.	46.0	35.0	65.4	+ 9.4	88.36	+12.0	144.00	112.80
Totals	346.1	204.5	426.8	....	267.59	....	424.64	319.12
Means	43.3	25.6	53.4	....	....	....	....	....

$$r = \frac{\Sigma(xy)}{\sqrt{\Sigma(x^2) \cdot \Sigma(y^2)}} = \frac{319.12}{\sqrt{267.59 \times 424.64}} = 0.947.$$

*Using eight stations with approximately the same average annual precipitation.

with approximately the same annual precipitation. This is done in Table 10–3, where eight stations having mean annual precipitations greater than 41 in. and

TABLE 10–4

ADDITIONAL TABULATION REQUIRED TO COMPUTE PARTIAL CORRELATION COEFFICIENT FOR DATA OF TABLE 10–2

Mean Annual Precipitation Z	Z Deviation	Z Deviation Squared	Product of Deviation xz	Product of Deviation yz
43.8	+ 2.9	8.41	− 13.63	− 11.89
42.8	+ 1.9	3.61	− 8.74	− 3.04
41.9	+ 1.0	1.00	− 4.60	− 2.90
46.5	+ 5.6	31.36	− 57.12	− 25.76
45.4	+ 4.5	20.25	− 20.70	− 13.50
45.6	+ 4.7	22.09	− 26.79	− 30.55
42.0	+ 1.1	1.21	− 2.64	− 4.73
42.7	+ 1.8	3.24	− 3.24	− 8.28
42.0	+ 1.1	1.21	− 1.87	− 5.94
50.7	+ 9.8	96.04	+ 74.48	+ 40.18
58.2	+17.3	299.29	+ 110.72	+ 67.47
53.2	+12.1	146.41	+ 159.72	+ 96.80
58.6	+17.7	313.29	+ 185.85	+238.95
55.5	+14.6	213.16	+ 179.58	+185.42
39.5	− 1.4	1.96	+ 8.54	+ 8.50
37.7	− 3.2	10.24	+ 4.16	+ 0.96
37.5	− 3.4	11.56	− 0.34	+ 6.46
29.7	−11.2	125.44	+ 84.00	+ 63.84
20.8	−20.1	404.01	+ 203.01	+116.58
20.9	−20.0	400.00	+ 238.00	+128.00
30.3	−10.6	112.36	+ 81.62	+ 61.48
31.1	− 9.8	96.04	+ 85.26	+ 42.14
38.6	− 2.3	5.29	− 6.44	− 11.04
38.0	− 2.9	8.41	− 10.44	− 12.76
29.0	−11.9	141.61	− 44.07	− 15.47
32.4	− 8.5	72.25	− 39.95	− 14.45
44.9	+ 4.0	16.00	+ 54.80	+ 38.80
46.0	+ 5.1	26.01	+ 65.79	+ 45.39
Totals 1145.1	....	2591.75	+1294.96	+980.66
Means 40.9	....	....	....	....

$$r_{xy} = 0.929;$$

$$r_{xz} = \frac{1294.96}{\sqrt{1654.45 \times 2591.75}} = 0.633;$$

$$r_{yz} = \frac{980.66}{\sqrt{1036.65 \times 2591.75}} = 0.598;$$

$$r_{xy \cdot z} = \frac{(r_{xy} - r_{xz} \cdot r_{yz})}{\sqrt{(1 - r_{xz}^2)(1 - r_{yz}^2)}} = 0.983.$$

less than 46 in. are studied separately. The correlation coefficient is found to be 0.947, which indicates still closer correlation than (*a*). However, this method is seldom entirely satisfactory, as it reduces the amount of information that can be used in determining the coefficient of correlation. In general, it is better to make use of all the data and apply the formula for the coefficient of partial correlation.

$$r_{xy\cdot z} = \frac{r_{xy} - r_{xz} \cdot r_{yz}}{\sqrt{(1 - r_{xz}^2)^2 (1 - r_{yz}^2)}},$$

where $r_{xy\cdot z}$ is the coefficient of partial correlation between x and y with the effect of z eliminated, and the r's in the right side of the equation are computed as in the previous examples. Table 10–4 shows the additional tabulations and computations for the partial correlation coefficient, which equals 0.983. It is evident that the mean annual precipitation has little effect on the relationship between mean annual temperature and mean annual water loss.

TESTS OF SIGNIFICANCE

10–14. The χ^2 Test

The χ^2 (chi-squared) distribution is a means of testing the agreement between observation and hypothesis in a wide class of problems. It is applied to data consisting of the enumeration of the individuals possessing a certain attribute. Hydrologic data usually are measurements of a continuous variate rather than the enumeration of attributes; nevertheless the χ^2 test is applicable to a variety of hydrologic problems.

The test consists of the comparison of the numbers observed falling into each of a number of classes with the numbers expected in each of those classes, the expected values being based on some hypothesis about the nature of the population. χ^2 is computed as follows:

$$\chi^2 = \sum \frac{(x - m)^2}{m},$$

where x is the observed number in any class and m is the expected number in the same class. That is, χ^2 equals the summation of the ratios of the deviations squared to the expected numbers. The computed χ^2 is then compared with a tabulation of χ^2 (as given, for example, by Snedecor)* to determine whether or not the hypothesis should be rejected. A computed χ^2 so large as to suggest the rejection of the hypothesis is said to be "significant."

Tables of values of χ^2 are set up to be entered with "degrees of freedom" and "probability of a larger value of chi-square." The number of *degrees of freedom* is equal to the number of classes in which frequencies may be filled arbitrarily. In the example below, the expected values in the four classes must add up to the total; if we fill three of the classes arbitrarily, then the fourth is determined by the total. We therefore have three degrees of freedom in this case. (If we had made a hypothesis that the ratio of expected values between two classes was a constant, then there would be only two degrees of freedom, for only two classes could be filled arbitrarily.)

*References are to tables in George W. Snedecor, *Statistical Methods*, (4th ed.; Ames, Iowa: Iowa State College Press, 1946), p. 190. Any other standard statistical textbook would serve as well.

It is usually considered good practice to enter the χ^2 table at 5 per cent probability, and use the 0.05 value for comparison with the computed χ^2. If the computed chi-square is larger than the table value for 0.05, then the hypothesis would be rejected, with one chance in twenty of the decision being incorrect.

As an example, let us examine the relative occurrence of winter and non-winter floods at two stations on the Sandusky River in Ohio. The Sandusky River near Bucyrus, with a drainage area of 89.8 sq mi, had 31 floods in the 7-yr period ending September 30, 1945. The Sandusky River, near Mexico, with a drainage area of 776 sq mi, had 20 floods during the same period. We shall assume that the arbitrary definition of what constitutes a flood at each station will have no effect on the problem, and we shall further arbitrarily assume that a flood occurring between November 15 and March 31 in any year is a winter flood. The hypothesis that we desire to test is that there is no significant difference in the division of floods into winter and non-winter between the two stations. (We might assume that there would be more summer floods on the smaller area, as local intense thunderstorms of the spring and summer would cause high water on small areas but would be too "spotty" and local in character to affect a large area. We have purposely set up the test hypothesis in such a way that we expect to have indications upon which to base a rejection of it.) The observed data are given in Table 10–5.

TABLE 10–5

Station	Winter Floods	Non-winter Floods	Totals
Bucyrus..............	21	10	31
Mexico...............	11	9	20
Totals............	32	19	51

The expected values are computed, in accordance with the hypothesis that we have formulated, on the basis of the ratios of the normal totals (e.g., the expected number of winter floods for Bucyrus is $51 \times 31/51 \times 32/51 = 19.5$). The expected values are given in Table 10–6. The computation of χ^2 can now be made:

TABLE 10–6

Station	Winter	Non-winter	Totals
Bucyrus..............	19.5	11.5	31
Mexico...............	12.5	7.5	20
Totals............	32.0	19.0	51

$$\chi^2 = \frac{(21 - 19.5)^2}{19.5} + \frac{(11 - 12.5)^2}{12.5} + \frac{(10 - 11.5)^2}{11.5} + \frac{(9 - 7.5)^2}{7.5} = 0.79.$$

Looking in the table of χ^2 we find that, for three degrees of freedom, χ^2 would be greater than 2.815 for 5 per cent of the time. We therefore cannot reject the hypothesis. It should be noted that we do not *accept* the hypothesis, at least not on the basis of the χ^2 test. We can merely state that the data which we have examined give no evidence upon which to base a rejection of the hypothesis—that is, they give no evidence that winter and non-winter floods are distributed differently on the two areas.

The above problem is typical of most statistical tests. The steps are: (1) collect the data, (2) make certain assumptions, (3) set up a hypothesis so that the test, if significant, will indicate that the hypothesis should be rejected, (4) make the test, (5) compare the results with the table, and (6) reject or do not reject the hypothesis.

10-15. The *t* Test and the Analysis of Variance

For testing hypotheses involving measurement data rather than the enumeration of attributes discussed in the preceding section, the t test is often applicable. The formula is

$$t = \frac{\bar{x} - m}{\sqrt{\dfrac{s^2}{n}}},$$

where m is the population mean, and the other symbols are as previously defined. A table of t values is given in Snedecor (*op. cit.*, p. 65).

Table 10-7 illustrates the application of the t test to a problem involving stream-flow records. Two stations were operated simultaneously on the same stream for 11 months, a short distance apart. The upper station *apparently* measured more flow than did the downstream station. The question is whether or not there is a significant difference in flow at the two points. We assume that the variable, i.e., the difference between the mean flow for a month at the upstream station and the mean flow for the same month at the downstream station, is distributed normally; and we make the hypothesis that the mean value of this variable is zero—that is, that there is no significant difference in flow at the two stations. Application of the t test indicates no significant difference, and, following the rule that definite statements can be made only when the test indicates significance, we can say that the available evidence fails to show any difference.

If we assume $m = 0$, as in the first example, and solve the t formula for n, we obtain

$$n = \frac{t^2 s^2}{\bar{x}^2}.$$

This expression, modified as follows, is useful in determining the sample size required to estimate the population mean within determined limits:

$$n = \frac{(100)^2 t^2 s^2}{p^2 \bar{x}^2}.$$

In this modified form, n is the sample size required to insure, with a probability approaching as near to certainty as we may please, that the estimate of the population mean will be within p per cent of the true value. In solving the formula, s is computed in the ordinary manner from such

TABLE 10-7

t TEST APPLIED TO DETERMINE SIGNIFICANCE OF MEAN OF A SMALL SAMPLE

Differences in discharge measured at Paint Creek near Bainbridge, Ohio, 779 sq mi, and Paint Creek near Bourneville, Ohio, 808 sq mi.

Month	Discharge in Second-Feet near Bainbridge	Near Bourneville	Difference	$(x - \bar{x})$	$(x - \bar{x})^2$
1939, June......	451	441	− 10	+ 43.8	1918
July......	217	249	+ 32	+ 85.8	7362
Aug.......	190	128	− 62	− 8.2	67
Sept......	14.4	15.3	+ 0.9	+ 54.7	2992
Oct.......	18.9	25.1	+ 6.2	+ 60.0	3600
Nov.......	32.9	28.9	− 4.0	+ 49.8	2480
1939, Dec.......	36.7	30.5	− 6.2	+ 47.6	2266
1940, Jan.......	64.9	90.1	+ 25.2	+ 79.0	6241
Feb.......	1340	1046	−294	−240.2	57696
Mar.......	1591	1397	−194	−140.2	19656
Apr.......	3862	3776	− 86	− 32.2	1037
Totals.....	7818.8	7226.9	−591.9	....	105315

$$\bar{x} = \frac{\Sigma(x)}{n} = -53.8;$$

$$s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} = 10{,}531;$$

$$t = \frac{\bar{x} - 0}{\sqrt{\frac{s^2}{n}}} = \frac{53.8}{\sqrt{957}} = 1.74.$$

From t table (Snedecor) for a probability of 5 per cent and 10 degrees of freedom, $t = 2.23$. Therefore, differences are not significant.

sample data as are already available, and p is set at whatever value may be desired. It is then necessary to define "probability approaching certainty" and choose a value of t accordingly. If we want to be right 19 times out of 20, for example, we would select t from the 5 per cent column of the t table; while if we want to be right 49 times out of 50 we would select it from the 2 per cent column. The value chosen for t is, of course, a preliminary one; for, until the equation is solved for n, we do not know the required sample size and thus can only guess at the number of degrees of freedom to which t should correspond. If, on solving, n turns out to be

such that the number of degrees of freedom is widely different from our first guess, a new value of t can be taken from the table and a corresponding new value of n computed.

As an example, let us examine the existing 23-yr stream-flow record of the Licking River and use it to estimate the sample size (i.e., the length of record) required to insure, with a probability of 19/20, that the estimate of the population mean (i.e., the mean annual discharge) is within 10 per cent of the true value. Here, again, we assume that stream-flow means are normally distributed. All the work is shown in Table 10-8.

TABLE 10-8

DETERMINATION OF LENGTH OF RECORD REQUIRED TO ESTIMATE MEAN ANNUAL DISCHARGE OF LICKING RIVER WITHIN 10 PER CENT

Year	Mean Annual Discharge, Licking River at Toboso, Ohio		
	Discharge in Second-Feet	Deviation from Mean	Deviation Squared
1922	831	+179	32,041
1923	486	−166	27,556
1924	1050	+398	158,404
1925	257	−395	156,025
1926	645	− 7	49
1927	996	+344	118,336
1928	813	+161	25,921
1929	660	+ 8	64
1930	712	+ 60	3,600
1931	188	−464	215,246
1932	669	+ 17	289
1933	737	+ 85	7,225
1934	213	−439	192,721
1935	451	−201	40,401
1936	603	− 49	2,401
1937	1088	+436	190,096
1938	853	+201	40,401
1939	762	+110	12,100
1940	775	+123	15,129
1941	345	−307	94,249
1942	492	−160	25,600
1943	889	+237	56,169
1944	473	−179	32,041
Totals	14,988		1,446,114
Mean	652		

$$\bar{x} = 652; \quad s^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} = 65{,}732; \quad s = 256; \quad P = 10\%; \quad t = 2.0;$$

$$n = \frac{(100)^2 t^2 s^2}{p^2 \bar{x}^2} = \frac{10{,}000 \times 4.00 \times 65{,}732}{100 \times 425{,}104} = 62 \text{ yr.}$$

No second approximation of t was necessary. It is interesting to note the large answer—62 yr. This is another indication that the available hydrologic records are too short to give accurate indications of what may be expected over longer periods.

It can be shown that the t test depends on the fact that the sum of squares of deviations from the mean may be computed from the sum of squares of deviations from zero, less the product of the total and the mean. Expressing this in symbols, and transposing,

$$\Sigma(x^2) = \Sigma(x - \bar{x})^2 + \bar{x}\Sigma(x).$$

Thus the sum of squares of x may be divided into two parts, the first representing variations within the sample, called "random error," and the second representing the deviation of the observed mean from zero. The first subdivision has $n - 1$ degrees of freedom, and the second, only 1. The mean squares may be computed by dividing the sum of squares for each part by the corresponding degrees of freedom, and the ratio of the mean squares then equals t^2. This method of approach, called the "analysis of variance," has wide application to problems more complex than that discussed on page 257. The analysis may be conveniently set forth in the form of Table 10–9.

TABLE 10–9

ANALYSIS OF VARIANCE

	Degrees of Freedom	Sum of Squares	Mean Square
Deviation	1	$\bar{x}\Sigma(x)$	t^2s^2
Random error	$n - 1$	$\Sigma(x - \bar{x})^2$	s^2
Total	n	$\Sigma(x^2)$	....

Introducing the values from the example of Table 10–7, we obtain the values given in Table 10–10. Hence

$$t^2 = \frac{31,844}{10,531} = 3.03, \quad t = 1.74.$$

TABLE 10–10

ANALYSIS OF VARIANCE

	Degrees of Freedom	Sum of Squares	Mean Squares
Random error	10	105,315	10,531
Deviation	1	31,844	31,844
Total	11	137,159	....

This method of subdividing the sum of squares into the component parts may be extended to cases in which there are more than one classification. For normally distributed variables the ratio of the mean squares follows the F distribution (Table 10–7 in Snedecor, *op. cit.*, p. 222). The t

test is, therefore, a special case of the more general F test, in which there is but one classification, the single deviation having one degree of freedom. In this special case $F = t^2$.

The F test may best be shown by examples. For purposes of illustration the data of Table 10-11 will be analyzed.* The basic problem is this:

TABLE 10-11

RAIN GAGE RECORDS, SOUTHWESTERN OHIO

(In. of Rainfall)

Year	Wilmington	Kings Mills	Hillsboro	Washington Court House	Cincinnati
1941	38.04	38.15	38.75	30.91	28.01
1942	39.92	39.80	45.84	32.75	43.80
1943	40.88	38.95	37.99	34.25	32.04
1944	35.30	35.05	33.83	30.32	35.36
1945	58.66	56.34	55.56	47.62	48.96
Mean	42.56	41.66	42.39	35.17	37.63

Given the rainfall records for five rain gages in southwestern Ohio; to determine whether the differences in their means are significant—that is,

TABLE 10-12

t TEST APPLIED TO MEANS OF STATIONS LISTED IN TABLE 10-11

	Mean	$x - \bar{x}$	$(x - \bar{x})^2$
Wilmington	42.56	+2.68	7.18
King's Mills	41.66	+1.78	3.17
Hillsboro	42.39	+2.51	6.30
Washington Court House	35.17	−4.71	22.18
Cincinnati	37.63	−2.25	5.06
Totals	199.41	...	43.89
Means	39.88	...	...

ANALYSIS OF VARIANCE

	Degrees of Freedom	Sum of Squares	Mean Squares
Deviation—stations	1	7952.00	7952
Random error	4	43.89	10.95
Total	5	7996	...

$$F = t^2 = \frac{7952}{10.95} = 726.$$

Therefore, significant differences between the means of the stations exist.

*It should be noted that these records are actually too short for worth-while statistical analysis; however, a typical example of the analysis of variance would be too cumbersome and tedious to follow.

TABLE 10-13

ANALYSIS OF VARIANCE OF RAINFALL RECORDS OF UNEQUAL LENGTH
(One-Way Classification)

Year	Wilmington	King's Mills	Hillsboro	Washington Court House	Cincinnati
1941	38.04	38.15	....	30.91	28.01
1942	....	39.80	45.84	32.75	43.80
1943	40.88	38.95	37.99	34.25	....
1944	35.30	35.05	....	30.32	....
1945	58.66	56.34	55.56	....	48.96
Totals	172.88	208.29	139.39	128.23	120.77

N = Total number of years of record = 19;
x_{ij} = The jth year at the ith station;
x_i = Mean of the ith station ($i = 1 \ldots r$);
r = Number of stations = 5;
n_i = Number of years of record for ith station;
x = Mean of all observations;
X_i = Sum of all observations at ith station;
X = Sum of all observations;
μ = General population mean;
μ_i = Mean of population of ith station minus μ.

Assumption: Expected mean = $\mu_i + \mu$;
Hypothesis: $\mu_1 = \mu_2 = \ldots = \mu_r = 0$;

$$F = \frac{(N - r)}{(r - 1)} \frac{\left(\Sigma \dfrac{X_i^2}{n_i} - \dfrac{X^2}{N}\right)}{\sum\limits_{i=1}^{r} \sum\limits_{j=1}^{k} x_{ij}^2 - \Sigma \dfrac{X_i^2}{n_i}}.$$

	No. of Years	X_i	X_i^2	$\dfrac{X_i^2}{n_i}$	Σx^2_{ij}
Wilmington	4	172.88	29,850	7462	$(38.04^2 + 40.88^2$ etc.) = 7809
Kings Mills	5	208.29	43,400	8680	8958
Hillsboro	3	139.39	19,430	6477	6632
Washington Court House	4	128.23	16,460	4115	4118
Cincinnati	3	120.77	14,600	4867	5103

N	X	X^2/N	$\Sigma X_i^2/n_i$	$\Sigma\, \Sigma x_{ij}^2$
19	769.56	31,170	31,601	32,620

$$F = \frac{14}{4} \times \frac{(31{,}601 - 31{,}170)}{(32{,}620 - 31{,}601)} = \frac{14}{4} \times \frac{431}{1019} = 1.48.$$

ANALYSIS OF VARIANCE TABLE

	Degrees of Freedom	Sum of Squares	Mean Squares
Stations	4	431	108
Errors	14	1019	72.8
Total	18	1450	....

whether they reflect actual climatological differences, or whether they are merely attributable to chance differences between samples drawn from a single population. For derivations of the formulas used in the examples the reader is referred to Snedecor (*op. cit.*).

In Table 10–12 (p. 261) the t test (a special case of the F test) is applied to the station means, and a significant difference is found to exist. Although easy to apply, this test does not make use of all the available data and may give misleading results in other problems. In Table 10–13 some of the years are assumed to be missing, to illustrate a method of analysis useful when records are of unequal lengths. Table 10–14 is a two-way classification, making full use of all the data. It is interesting to compare the result of Table 10–14 ($F = 14.35$, degrees of freedom 16 and 4) with the F of Table 10–12. Although in this case both F's are highly significant, there will be many cases in which the more complete analysis (Table 10–14) will contradict the t test. The one-way classification (Table 10–13) is contradicted.

TABLE 10–14

ANALYSIS OF VARIANCE—5 RAINFALL RECORDS OF EQUAL LENGTH
(Two-Way Classification)

N = Total number of years of record $= rs = 5 \times 5 = 25$;
x_{ij} = The jth year at the ith station;
x_i = Mean of ith station ($i = 1 \ldots r$);
r = Number of stations = 5;
s = Number of years of record = 5;
x_j = Mean of the jth year ($j = 1 \ldots s$);
X_i = The sum of all observations at ith station;
X_j = The sum of all observations for jth year;
X = The sum of all observations;
μ = Population mean;
μ_i = Mean of population for ith station minus general population mean;
s^2 = Estimate of variance based on all observations;
$s^2{}_1$ = Estimate of variance based on stations;
$s^2{}_2$ = Estimate of variance based on years.

Assumption: Expected mean $= \mu_i + \mu_j + \mu$;
Hypothesis: $\mu_1 = \mu_2 = \ldots \mu_r = 0$.

$$s^2 = \frac{1}{(r-1)\,(s-1)}\left[\sum_i \sum_j x^2{}_{ij} - \frac{1}{r}\Sigma X^2{}_j - \frac{1}{s}\Sigma X^2{}_i + \frac{1}{rs}X^2\right];$$

$$s^2{}_1 = \frac{1}{r-1}\left[\frac{1}{s}\Sigma X^2{}_i - \frac{X^2}{rs}\right];$$

$$s^2{}_2 = \frac{1}{s-1}\left[\frac{1}{r}\Sigma X^2{}_j - \frac{X^2}{rs}\right];$$

$$F_1 = \frac{s^2{}_1}{s^2};$$

$$F_2 = \frac{s^2{}_2}{s^2}.$$

(*Table 10–14 continued on next page.*)

TABLE 10-14.—*Continued.*

Station	X_i	X^2_i	Σx^2_{ij}	Year	X_j	X^2
Wilmington	212.80	45,400	9401	1941	173.86	30,300
Kings' Mills	208.29	43,300	8958	1942	202.11	40,800
Hillsboro	211.97	44,900	9275	1943	184.11	33,900
Washington Court House	175.85	31,000	6384	1944	169.86	28,900
Cincinnati	188.17	35,300	7380	1945	267.14	71,300

r	N	X	X^2/N	ΣX^2_i	$\frac{1}{s}\Sigma X^2_i$	ΣX^2_j	$\frac{1}{r}\Sigma X^2_j$	$\Sigma\ \Sigma x^2_{ij}$
5	25	997.08	39,700	199,900	39,980	205,200	41,040	41,398

$s^2 = \frac{1}{16}[41{,}398 - 41{,}040 - 39{,}980 + 39{,}700] = 4.88;$

$s^2_1 = \frac{1}{4}[39{,}980 - 39{,}700] = 70;$

$s^2_2 = \frac{1}{4}[41{,}040 - 39{,}700] = 335;$

$F_1 = 14.35.$

Therefore, the differences between stations are significant.

$F_2 = 68.8$

Therefore, the differences between years are highly significant.

ANALYSIS OF VARIANCE TABLE

	Degrees of Freedom	Sum of Squares	Mean Squares
Deviations—stations	4	280	70
Deviations—years	4	1340	335
Random error	16	78	4.88
Total	24	1698	....

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PROBLEMS, QUESTIONS, AND SPECIAL ASSIGNMENTS

GENERAL ORIENTATION STUDIES

1. Prepare a list of agencies in your state that are concerned in one way or another with applications of hydrology.
2. Prepare a list of engineering projects in your vicinity—complete, under way, approved, or proposed—that involve applications of hydrology. The following check list of project types will help to insure completeness:

Flood control	Pollution control
Navigation	Water supply
Power	Conservation
Irrigation	Recreation
Erosion control	Drainage

3. Prepare a list of the various types of works of man that may contribute to waste of water, stream pollution, increase of flood hazard, reduction in channel depths, surface erosion, or other undesirable effects. From your reading of the newspapers, the technical press, etc., or from personal knowledge, are there any specific projects in your vicinity that have been alleged to cause such effects? Outline the nature of the hydrological studies that would have to be undertaken to determine the actual facts.

SUGGESTED INSPECTION TRIPS

1. A first-order meteorological station.
2. A Soil Conservation Service experiment station or other field laboratory.
3. An irrigation or flood control project or the site of a proposed project.
4. Observation of a U.S. Geological Survey or other experienced stream-gaging party at work.

FIELD PROBLEMS

1. You are instructed to select the most suitable site for a gaging station on a stream in your vicinity. Make a field reconnaissance and prepare your report and recommendations, with photographs and any necessary sketches.
2. Perform the field work in connection with a contracted-opening or slope-area measurement and reduce your data. In preparation for this exercise, the instructor should have visited the selected reach during or immediately after a high-stage period, and permanently fixed a number of high-water marks by means of keel, nails or spikes, or stakes.
3. Determine the infiltration capacity of a soil by means of a concentric-ring infiltrometer. The apparatus can be designed as a class project. The inside ring may conveniently be about 12 in. in diameter, and the outside ring about 24 in. Depth of water on the ground should be maintained at about

¼ in. Design of means for maintaining this level and for measuring the water used in the inner ring should be carefully worked out, and the limits of accuracy of the measurement estimated. Field observations may be extended to cover a wide variety of problems, such as:

(a) Change in infiltration capacity over a period of hours.

(b) Comparison of infiltration capacity of soil with sod cover and soil with cover removed.

(c) Effect on infiltration capacity of soil moisture content at beginning of experiment.

Depending on the problem selected, it may be necessary to take soil samples for soil moisture determination at various depths. It is of interest also to set the equipment up in replicate and study the variance of the results.

MISCELLANEOUS PROBLEMS

1. Construct a set of point rainfall intensity-frequency curves from the records of the nearest first-order meteorological station with at least 30 years of record. (To conserve the students' time, the instructor may wish to abstract the pertinent data from the original record and supply it to the class in mimeograph form.) Answer the following questions:
   (a) From your chart, what rate of runoff would be produced by a once-in-10-year rainfall on a drainage area having a concentration time of 20 min., if the runoff coefficient is 70 per cent?
   (b) Explain fully why this is not necessarily the once-in-10-year runoff for this area.
   (c) Outline the additional studies that would have to be made to arrive at a reasonable estimate of the frequency of a given runoff rate.
2. You are being handed an isohyetal map which contains a number of mistakes and is incomplete in one area. The gages from which the map is developed are spotted on the map, and the recorded rainfall depth is shown for each. Check, indicate revisions, and complete.
3. You are being handed a set of profiles and cross sections adequate for computing the discharge of a stream by the contracted-opening or slope-area measurement. Make the computations.
   (a) Estimate the possible range in results arising from improper selection of n, from inaccuracies in the location of the high-water marks, and from any other sources that you believe may contribute to error.

DRAINAGE BASIN STUDY PROJECT

1. With the assistance of the instructor, select a drainage basin with which you are familiar or can become familiar, for use in the following problems. Ideally, it should be not less than about 100 sq mi nor more than about 1000 sq mi in area; it should not be subject to any high degree of artificial regulation, and it should form its flood hydrographs in reasonable conformity with unitgraph hypothesis; there should be available 10 or more years of stream flow records, including at least a short period of automatic recording gage record; and

there should be an adequate network of precipitation stations within or adjacent to the area.

2. Draw the Thiessen polygons and compute the coefficients applicable to this basin.

   (*a*) Are there any reasons to suspect that the Thiessen method may not be suitable for use with this area?

   (*b*) Are the gages properly distributed to provide a reasonably accurate measure of equivalent uniform depth?

   (*c*) Compute the equivalent uniform depth of annual precipitation over the periods of years for which you have stream-flow records.

3. Compute the annual water loss for the period of record. Then obtain the data from Problem 2(*c*) from a student working on another drainage area in the vicinity of your own, and apply your water-loss data to estimate the runoff of the other area. After making your estimate, compare it with the actual recorded runoff.

   (*a*) What was the percentage of error?

   (*b*) Are there any factors that might have been taken into account that would have contributed to an increase in the accuracy of your estimate?

   (*c*) On the basis of your results, are you justified in either accepting or rejecting this method of estimating stream flow of an ungaged stream?

4. From your data, compute the regression of annual runoff on annual precipitation and the standard error of estimate. What is the significance of the standard error of estimate?

5. By use of mass curves, prepare a graph showing storage required to maintain any given minimum rate of flow. If a reservoir area-capacity curve is available, make use of it and Weather Bureau evaporation records to include allowance for evaporation. If such data are not available, consult your instructor on how to handle evaporation allowance.

6. From the published records of stream flow, prepare a winter depletion curve and a summer depletion curve. Consult the daily precipitation records for the period to make sure you are not systematically including portions of the hydrograph that are being affected by small amounts of rainfall that do not show up as surface runoff; your instructor will advise on the amounts of such rainfall that can be ignored.

   (*a*) Compare the winter and summer curves and explain any differences.

   (*b*) How many days without rain would have to occur to reduce the discharge of the stream to a critically low value, assuming a normal condition to exist at the beginning of the drought? ("Critically low value" and "normal condition" are criteria that should be discussed with your instructor before adoption.)

   (*c*) From an examination of Weather Bureau records, form an opinion as to whether the stream you are studying would be reduced to a critically low discharge often, infrequently, or practically never. Explain what

you understand by "often" and "infrequently." Does your interpretation jibe with that of your classmates?

7. Derive a unitgraph from an isolated storm and from a composite storm, separating the base flow by means of the depletion curves derived in Problem 6. Your procedures must be adapted to the data you have available, and for the purposes of Problem 8 the unit period used must be the same as that of your classmates. If black-line prints of automatic recording gage charts can be obtained and if hourly rainfall records are available, it may be possible for the entire class to develop 6-hr graphs; otherwise it may be necessary to be satisfied with 24-hr graphs.
   (*a*) Compare the unitgraphs derived by the two methods. To which one would you give more weight, taking into account distribution of rainfall, size of storm, accuracy of method, and any other factors that occur to you?
   (*b*) Test your graphs by reconstituting a major flood. Since this is a test of the unitgraph and not of your estimate of runoff coefficients, it is proper to compute the actual volume of surface runoff from the flood hydrograph and reduce the actual rainfall amounts correspondingly, before beginning the reconstitution. Discuss with your instructor whether the net effective rainfall amounts should be computed by applying a fixed percentage, a variable percentage, a uniform deduction from all periods, or a variable deduction.
   (*c*) Using your unitgraphs, predict the hydrograph that would result from a hypothetical major storm assigned by the instructor.

8. Compute Snyder's L_{ca} for your drainage basin by the center-of-gravity method, using cardboard cutouts. Also measure L. Then, using the Snyder equations and the q_p indicated by your unitgraph, compute C_p and C_t. Tabulate also the C_p and C_t values computed by your classmates for other drainage basins. Then:
   (*a*) Compute the mean values of C_p and C_t and note the range in individual values. Is the range small enough that you would feel justified in predicting the unitgraph peak of a neighboring ungaged stream by using the average values? From a study of maps and from round-table discussion, can you set up a rough qualitative classification of drainage basins that would justify your selecting some value of C_p and C_t other than the average, to apply to the ungaged basin?
   (*b*) On the basis of the above-described study, state your preliminary opinion of the applicability of the Snyder method of synthesis to the group of drainage basins studied by the class and outline the additional studies that would be desirable or necessary before a definite conclusion could be reached.

9. Route the hydrograph of Problem 7(*c*) through a detention basin or over a spillway crest. For purposes of comparison, it may be desirable for all members of the class to use the same hydrograph in this problem and the same reservoir capacity curve (to be supplied by the instructor). Individual students may be assigned different sizes of outlet conduit or different lengths

of spillway crest, so that the effect on the outflow hydrograph of varying these dimensions can be observed.

10. For your drainage basin, arrange the maximum annual floods for the period of record in order of magnitude, compute the recurrence interval, and plot, joining the points with a broken line (not a smooth curve). Discharges should be plotted to an arithmetic scale as ordinates, and recurrence intervals to a logarithmic scale, as abscissae. Making use of the binomial theorem, compute the probability (*a*) that the n-year flood did not occur at all in your n years of record and (*b*) that the n-year flood occurred twice during the period. Explain how you would make allowances for this uncertainty in an economic study of a proposed project.

SUBJECT INDEX

INDEX OF NAMES